Laboratori Nazionali di Frascati

LNF-71/69

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Estratto da : Lett. Nuovo Cimento 2, 522 (1971)

Photon-Photon Scattering into Hadrons from Finite-Energy Sum Rules.

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(ricevuto il 5 Luglio 1971)

The production of hadrons in high-energy $e^{\pm}e^{-}$ collisions by photon-photon scattering has been recently investigated by several authors ([1-5]). Hadrons are produced in a $C = +1$ state with logarithmically increasing cross-sections which, at high energies, dominate over the more familiar one-photon processes. When the final electrons are detected within small angles $\theta_{1\max}$ and $\theta_{2\max}$ with respect to the initial beam direction ($\theta^2_{1,2\max} \approx m/E$) the cross-section for production of a final state F is

$$d\sigma_{ee\to eeF} = \left(\frac{2\alpha}{\pi}\right)^2 \int \frac{dk_1}{k_1}\frac{dk_2}{k_2}\frac{[E^2+(E-k_1)^2][E^2+(E-k_2)^2]}{4E^2}\cdot$$

$$\cdot\left\{\ln\frac{E(E-k_1)\theta_{1\max}}{mk_1}-\frac{E(E-k_1)}{E^2+(E-k_1)^2}\right\}\left\{\ln\frac{E(E-k_2)\theta_{2\max}}{mk_2}-\frac{E(E-k_2)}{E^2+(E-k_2)^2}\right\}d\sigma_{\gamma\gamma\to F}\,,$$

which is approximately given by

$$\sigma_{ee\to eeF} \simeq \left(\frac{\alpha}{\pi}\right)^2 \ln^2\left(\frac{E}{m}\right)\int_{t_0}^{4E^2}\frac{dt}{t}\,\sigma^{F}_{\gamma\gamma}(t)\,f\left(\frac{t}{4E^2}\right), \tag{1}$$

with $f(y) = -(2+y)^2\ln y - 2(1-y)(3+y)$. Here m, E are the mass and the energy of the colliding electrons and $\sigma^F_{\gamma\gamma}(t)$ is the cross-section for production of the state F by two real photons of total c.m. energy squared t with threshold t_0.

The contribution to eq. (1) of low-energy processes, like π^0 and η production, and creation of pion and kaon pairs according to pure QED have been studied in ref. ([1-5]). Some strong-interaction corrections to $\gamma\gamma\to\pi\pi$ have also been considered. A complete theoretical analysis of $\sigma^F_{\gamma\gamma}(t)$, for any final state F, has not been done. However,

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(1) N. ARTEAGA-ROMERO, A. JACCARINI, J. PARISI and P. KESSLER: *Lett. Nuovo Cimento*, **4**, 933 (1970); **1**, 935 (1971).

(2) S. J. BRODSKY, T. KINOSHITA and H. TERAZAWA: *Phys. Rev. Lett.*, **25**, 972 (1970); and preprint 1971.

(3) V. N. BUDNEV and I. F. GINZBURG: Novosibirsk preprint TP-55 (1970).

(4) M. GRECO: *Nuovo Cimento*, **4** A, 689 (1971).

(5) D. H. LYTH: University of Lancaster preprint (1970).

due to the factor dt/t in the right-hand side of eq. (1), low-mass final states are expected to dominate the production cross-section. Once $\sigma^F_{\gamma\gamma}(t)$ is expressed in terms of the lower-lying resonances one expects, therefore, to get a fairly good description of the production mechanism. Remembering, in addition, that a two-photon system does not couple to a $J^P = 1^-$ state, the final states F therefore have the quantum numbers $J^P = 0^-$, 0^+ and 2^+.

The aim of this paper is to present an evaluation of the different contributions to the total hadronic cross-section from the just-mentioned states. We note that in the case of F being a pseudoscalar meson the situation seems rather firmly established [1,2]. On the contrary, no definite predictions are available for the remaining cases, since the coupling of the two-photon system to a scalar or tensor meson is unknown. In order to estimate these quantities duality and finite-energy sum rules are used in the way described by AVIV and NUSSINOV [6], who first applied these ideas to compute the $\omega \to \pi^0\pi^0\gamma$ decay rate, and, more recently, by GOUNARIS and VERGANELAKIS [7], who successfully studied $\eta \to \pi^0\gamma\gamma$.

Briefly the procedure is as follows: consider Compton scattering from pseudoscalar mesons

$$\mathrm{P}(q_1) + \gamma(-k_1) \to \mathrm{P}(q_2) + \gamma(k_2) \tag{2}$$

and with the kinematical invariants $s = (q_1 - k_1)^2$, $t = (k_1 + k_2)^2$, $u = (q_1 - k_2)^2$, $\nu = \frac{1}{4}(s - u)$ and $Q = \frac{1}{2}(q_1 + q_2)$, write the Feynman-invariant amplitude as

$$F = \varepsilon_1^\mu \varepsilon_2^\nu T_{\mu\nu} \tag{3}$$

with the gauge-invariant tensor given by

$$\begin{aligned} T_{\mu\nu} = {} & A(\nu, t)\{g_{\mu\nu}(k_1k_2) - k_{1\nu}k_{2\mu}\} + \\ & + B(\nu, t)\{(Qk_1)Q_\nu k_{2\mu} + (Qk_2)Q_\mu k_{1\nu} - (Qk_1)(Qk_2)g_{\mu\nu} - (k_1k_2)Q_\mu Q_\nu\}\,. \end{aligned} \tag{4}$$

For low values of ν the amplitudes $A(\nu, t)$ and $B(\nu, t)$ are described in terms of contributions from nearby singularities in the s and u channels, namely the ρ, ω and B mesons. The high-energy behaviour, on the other hand, is parametrized by the Regge trajectories exchanged in the t-channel, which, according to the two isospin possibilities, are P′ and ε trajectories or A_2 and δ. The usual lowest-momentum FESR is then applied to calculate the Regge residues. The details of this calculation are accurately discussed in ref. [6,7]. Once the on-shell Regge residues are known it is easy to obtain the representation of the A and B amplitudes in the t-channel, where the $\gamma\gamma$ system is coupled to the scalar and tensor mesons.

For the processes $\gamma\gamma \to \delta$, $A_2 \to \eta\pi$, the result is

$$A^\delta\,(\nu, t) = \frac{2\beta_A^\delta}{t - m_\delta^2 + i m_\delta \Gamma_\delta}\,, \tag{5a}$$

$$A^{A_2}(\nu, t) = \frac{2\beta_A^{A_2}}{t - m_{A_2}^2 + i m_{A_2}\Gamma_{A_2}}\,\frac{1}{24}\,\lambda(m_{A_2}^2, m_\eta^2, m_\pi^2)\,P_2(\cos\theta_t)\,, \tag{5b}$$

$$B^{A_2}(\nu, t) = \frac{2\beta_B^{A_2}}{t - m_{A_2}^2 + i m_{A_2}\Gamma_{A_2}}\,, \tag{5c}$$

(7) R. AVIV and S. NUSSINOV: *Phys. Rev. D*, **2**, 209 (1970).
(6) G. J. GOUNARIS and A. VERGANELAKIS: CERN preprint TH-1310, March 1971.

where

$$\lambda(m_1^2, m_2^2, m_3^2) = [m_1^2 - (m_2 + m_3)^2][m_1^2 - (m_2 - m_3)^2] ,$$

$$\beta_A^\delta = 0.118 , \quad \beta_A^{A_2} = -0.128 , \quad \beta_B^{A_2} = 0.0154 , \quad \nu = \tfrac{1}{4}\{(t - m_\eta^2 - m_\pi^2)^2 - 4m_\eta^2 m_\pi^2\}^{\frac{1}{2}} \cos\theta_t$$

and θ_t is the production angle of the $\eta\pi$ system in the c.m. frame of the two photons. The masses appearing into $\lambda(m_1^2, m_2^2, m_3^2)$ are expressed in GeV. In order to take account of the finite width of the δ and A_2 resonances which occur at the physical region in the t-channel, we have phenomenologically added an imaginary part to $\alpha_\delta(t)$ and $\alpha_{A_2}(t)$. Notice also that the δ pole contributes only to $A(\nu, t)$.

Consider now the reaction $\gamma\gamma \to \pi^0\pi^0$ with the ε-meson as intermediate state. From the smallness of the $\varphi \to \pi^+\pi^-\gamma$ decay rate ($\Gamma_{\varphi\to\pi^+\pi^-\gamma} < 0.16$ MeV) (8) and considering the ε-meson as a member of an octet, one gets from VMD $g_{\varepsilon\gamma\gamma} = (-\frac{3}{5})(e/f_\rho)g_{\varepsilon\omega\gamma}$. This relation together with the results from ref. (6) yields

$$A^\varepsilon(\nu, t) = \frac{2\beta_A^\varepsilon}{t - m_\varepsilon^2 + im_\varepsilon\Gamma_\varepsilon} , \tag{6}$$

where $\beta_A^\varepsilon = 1.12(e/f_\rho)$. It is worth noting that SU_3 symmetry gives for the ratio $R = g_{\varepsilon\omega\gamma}^2 g_{\varepsilon\pi^0\pi^0}^2 / g_{\delta\gamma\gamma}^2 g_{\delta\eta\pi}^2$ the value $3f_\rho^2/4e^2$ which agrees well with $R = 210$ as can be easily deduced from (5a) and (6). The fact provides a good check for the above calculations.

For the reaction $\gamma\gamma \to f \to \pi^0\pi^0$ we obtain from ref. (6) and SU_3 symmetry the following results for the invariant amplitudes $A(\nu, t)$ and $B(\nu, t)$:

$$A^f(\nu, t) = \frac{2\beta_A^f}{t - m_f^2 + im_f\Gamma_f}\,\frac{1}{24}\,\lambda(m_f^2, m_\pi^2, m_\pi^2) P_2(\cos\theta_t) , \tag{7a}$$

$$B^f(\nu, t) = \frac{2\beta_B^f}{t - m_f^2 + im_f\Gamma_f} , \tag{7b}$$

where $\beta_A^f = \sqrt{3}\beta_A^{A_2} - \frac{4}{3}(e/f_\rho)\,4.7$, $\beta_B^f \simeq \sqrt{3}\beta_B^{A_2}$ and $\nu = \frac{1}{4}t(1 - 4m_\pi^2/t)^{\frac{1}{2}} \cos\theta_t$.

The evaluation of the cross-sections for two-body processes is straightforward. From eq. (3) we have

$$\sigma_{\gamma\gamma\to PP'}(t) = \frac{1}{32\pi^2}\,\frac{1}{t}\int \frac{d^3q_1}{2q_{10}}\,\frac{d^3q_2}{2q_{20}}\,\delta(k_1 + k_2 - q_1 - q_2)\,T_{\mu\nu}T^{\mu\nu} , \tag{8}$$

where the photon polarizations have been explicitly averaged and $T_{\mu\nu}$ is determined through eqs. (5), (6) and (7). An equivalent expression for $\sigma_{\gamma\gamma}^F(t)$, valid near a resonance R of angular momentum J, is

$$\sigma_{\gamma\gamma\to R\to PP'}(t) = 8\pi(2J + 1)\,\frac{\Gamma(R \to \gamma\gamma)\Gamma(R \to PP')}{(t - m_R^2)^2 + m_R^2\Gamma_R^2} . \tag{9}$$

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(8) PARTICLE DATA GROUP: *Rev. Mod. Phys.*, **43**, No. 2, Part 2 (1971).

Comparing eqs. (8) and (9), we find

$$\Gamma(\delta \to \gamma\gamma)\Gamma(\delta \to \eta\pi) = 3.7\ (\text{MeV})^2 , \tag{10a}$$

$$\Gamma(\varepsilon \to \gamma\gamma)\Gamma(\varepsilon \to \pi\pi) = 1.5\ (\text{MeV})^2 , \tag{10b}$$

$$\Gamma(A_2 \to \gamma\gamma)\Gamma(A_2 \to \eta\pi) = 5\cdot 10^{-3}\ (\text{MeV})^2 , \tag{10c}$$

$$\Gamma(f \to \gamma\gamma)\Gamma(f \to \pi\pi) \simeq 0.1\ (\text{MeV})^2 , \tag{10d}$$

where in the $\varepsilon, f \to \pi\pi$ decays all the charged-pion states are included. Taking $\Gamma(\delta \to \eta\pi) \simeq 70$ MeV, $\Gamma(\varepsilon \to \pi\pi) \simeq 250$ MeV, $\Gamma(A_2 \to \eta\pi) \simeq 15$ MeV and $\Gamma(f \to \pi\pi) \simeq = 120$ MeV (8), we also obtain

$$\begin{cases} \Gamma(\delta \to \gamma\gamma) = 50\ \text{keV} , & \Gamma(\varepsilon \to \gamma\gamma) = 6\ \text{keV} , \\ \Gamma(A_2 \to \gamma\gamma) = 0.3\ \text{keV} , & \Gamma(f \to \gamma\gamma) \simeq 0.8\ \text{keV} . \end{cases} \tag{11}$$

The production cross-sections are easily evaluated by substituting eq. (8) into (1). They are shown in Fig. 1 in the case of δ and ε mesons. The contribution coming from A_2 and f turns out to be negligible. For $\pi^+\pi^-$ production we have shown the pure QED predictions and the complete cross-section which is deduced from

$$\sigma_{\gamma\gamma}^{\pi^+\pi^-}(t) = \frac{1}{32\pi}\frac{\beta}{t}\left\{4e^4\left[1+\frac{4m_\pi^2}{t}\left(1-\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right)+\frac{8}{\beta}\frac{m_\pi^4}{t^2}\ln\frac{1+\beta}{1-\beta}\right]+ + \frac{t^2}{(t-m_\varepsilon^2)^2+m_\varepsilon^2\Gamma_\varepsilon^2}\beta_A^{\varepsilon^2}+\frac{8m_\pi^2}{\beta}e^2\beta_A^\varepsilon\frac{m_\varepsilon^2-t}{(t-m_\varepsilon^2)^2+m_\varepsilon^2\Gamma_\varepsilon^2}\ln\frac{1+\beta}{1-\beta}\right\}, \tag{12}$$

where $\beta = (1-4m_\pi^2/t)^{\frac{1}{2}}$.

For the sake of completeness we have also plotted in Fig. 2 the cross-section for production of π^0, η and η'. They have been obtained using $\Gamma(\pi^0 \to \gamma\gamma) = 7.8$ eV, $\Gamma(\eta \to \gamma\gamma) = 1$ keV (8) and $\Gamma(\eta' \to \gamma\gamma) = 55$ keV. The latter partial width has been deduced from SU_3 symmetry assuming a quadratic mass formula for the η-η' mixing (9,10).

Let us briefly discuss our results. In the reaction $ee \to ee\pi^+\pi^-$ the Born terms practically dominate over all other exchanges. Similar conclusions have been drawn by LYTH (5) using dispersion relations and unitarity. On the contrary, our result is smaller by a factor of about 8 than that obtained by BRODSKY, KINOSHITA and TERAZAWA (2). A reason for such a discrepancy can be traced to the fact that in the superconvergent sum rule used by these authors only the ε-resonance was taken into account. If we had used only the ε and δ poles to explain the $\omega \to \pi^0\pi^0\gamma$ and $\eta \to \pi^0\gamma\gamma$ decays, our results ((10a), (10b)) would have increased by more than a factor 10.

For high-multiplicity production we find that the δ and η' intermediate states dominate over all others. Adding together the different $\sigma_{ee\to eeF}$ for any state F we show in Fig. 2 the total cross-section for hadron production by $\gamma\gamma$ scattering. It can be seen that the predicted rate for hadron production by two-photon mechanism would exceed 10^{-32} cm^2 at $E = 1.5$ GeV and is, therefore, comparable with the e^+e^- annihilation cross-

(9) H. HARARI: *Proceedings of the Vienna Conference* (Vienna, 1968).
(10) A. BARACCA and A. BRAMÓN: *Nuovo Cimento*, **69** A, 613 (1970).

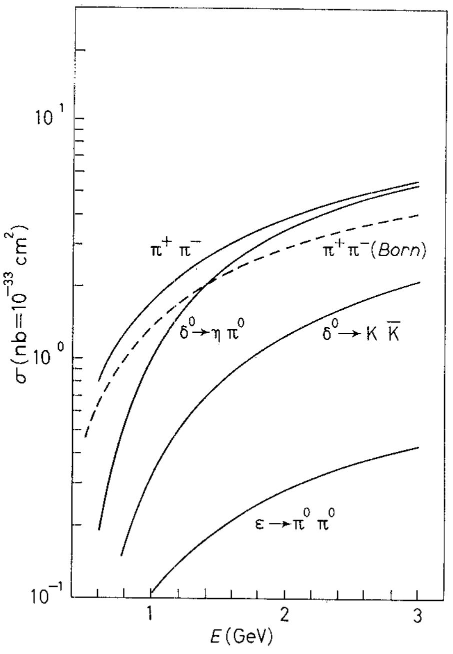

Fig. 1.

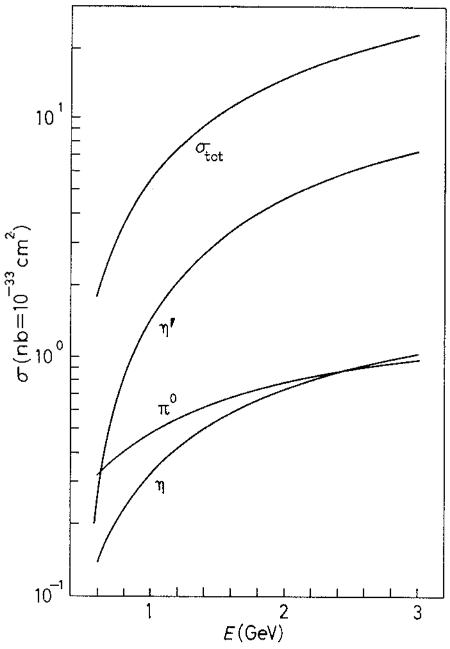

Fig. 2.

Fig. 1. – Total cross-sections for δ and ε production. The values $m_\varepsilon = 0.8$ GeV and $m_\delta = 1$ GeV have been used.

Fig. 2. – Total cross-section for production of π^0, η and η'. σ_{tot} refers to the total hadronic two-photon cross-section.

sections. It follows that the use of tagging systems is necessary to separate the one-photon from the two-photon processes.

After this work was completed we have received the paper « Finite-Energy Sum Rules and the Reactions ee → eeε(750) and ee → eef(1260) » by B. SCHREMPP-OTTO, F. SCHREMPP and T. F. WALSH, DESY 71/20, where some of the arguments discussed above are studied in a similar way.

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One of us (A.B) acknowledges G.I.F.T. for financial support.