COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-71/67
4 Ottobre 1971

M. D'Eramo, G. Parisi and L. Peliti : THEORETICAL PREDICTIONS FOR CRITICAL EXPONENTS AT THE λ-POINT OF BOSE LIQUIDS.

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-71/67
4 Ottobre 1971

M. D'Eramo[x], G. Parisi and L. Peliti[x]: THEORETICAL PREDICTIONS FOR CRITICAL EXPONENTS AT THE λ-POINT OF BOSE LIQUIDS. -

In a previous work[1] we pointed out an opportunity for theoretical predictions of critical exponents at the λ-point of Bose liquids. Some non-linear integral equations which determine the critical behaviour can be given a solution covariant under conformal group transformations. This fact allows to transform the integral equations into equations in which the only unknown are the indices, requiring only the evaluation of some integrals.

We have performed the calculation and we have found the following values for the exponents:

$$\nu = .6666; \qquad \eta = .2 \times 10^{-6} \tag{1}$$

Both these indices are not directly measurable at the time being. Scaling predictions[2] for ν, based upon the logarithmic singularity of the specific heat at the λ-point, yield $\nu = 2/3$. This value is in very good agreement with our result. No measurement related to η is so far available in superfluid Helium. We remark however that our enormously small value of η is related to the very small value of the difference $\nu - 2/3$ through a relationship of the kind

(x) - Istituto di Fisica dell'Università di Roma.

(2) $$\eta \cong \text{const} \left(\nu - \frac{2}{3}\right)^2$$

where the constant is of order $.5 \times 10^2$. Taking the example of three--dimensional Ising model, where ν is known from computer calculation to be of order .62-.63, this relationship gives for η an estimate of order 5×10^{-2}, i.e. noticeably different from 0, in agreement with computer estimations which yield $\eta = .041$[3].

In our calculation we neglected some more complicated diagrams. An evaluation of the error which may have been introduced in this way, as well as an application of the same theory to problems in high-energy physics, is in progress.

The integral equation involved is the following:

(3) $$V(X,Y,Z) = \int d^3x_1\, d^3x_2\, d^3y_1\, d^3y_2\, d^3z_1\, d^3z_2 \; V(X,y_1,z_1)\, V(x_1,Y,z_2)\, V(x_2,y_2,Z) \; D(x_1-x_2)\, G(y_1-y_2)\, G(z_1-z_2)$$

where V is the vertex part and G and D the particle and "phonon" propagator respectively[1]. Denoting by $\bar{V}$, $\bar{G}$, $\bar{D}$ the analytic continuations the Fourier transforms of V, G, D respectively to negative "momentum" squares, we add to (3) the following unitarity equations[4]:

(4) $$\operatorname{Im} \bar{G}^{-1}(p^2) = (2\pi)^{9/2} \int d^3K\, d^3q \; \delta^3(p-K-q) \left|\bar{V}(K^2,p^2,q^2)\right| \times \theta(q_o) \operatorname{Im}\bar{G}(q^2)\, \theta(K_o) \operatorname{Im}\bar{D}(K_o)$$

$$\operatorname{Im} \bar{D}^{-1}(p^2) = (2\pi)^{9/2} \int d^3K\, d^3q \; \delta^3(p-K-q) \left|\bar{V}(p^2,K^2,q^2)\right|^2 \theta(q_o) \operatorname{Im}\bar{G}(q^2)\, \theta(K_o) \operatorname{Im}\bar{G}(K^2)$$

In (1) we emphasized that these equations allow for solutions of the form:

(6) $$G(\underline{x}) = g\left|\underline{x}\right|^{-1-\eta}; \qquad D(\underline{x}) = d\left|\underline{x}\right|^{-6+2/\nu};$$

$$V(\underline{x};\underline{y},\underline{z}) = c\left|\underline{x}-\underline{y}\right|^{-1/\nu} \left|\underline{x}-\underline{z}\right|^{-1/\nu} \left|\underline{y}-\underline{z}\right|^{-5+\eta+1/\nu}$$

Letting eq. (6) into (3, 4, 5) respectively we obtain the following equation:

(7) $$1 = f(\eta,\nu)(c^2 g^2 d); \qquad 1 = g(\eta,\nu)(c^2 g^2 d); \qquad 1 = h(\eta,\nu)(c^2 g^2 d)$$

where f, g, h are obtained by direct evaluation of the integrals. We note that the functions (6) may be not directly Fourier-transformable, in which case their Fourier transforms are to be understood 85 distributions.

The function f has been evaluated in exact way. It turned out to be

$$f(\eta,\nu) = \pi^9 \left[C(-\tfrac{1}{\nu})\right]^3 \left[C(-1-\eta)\right]^4 \left[C(-5+\eta+\tfrac{1}{\nu})\right]^4 \left[C(-2+\eta-\tfrac{1}{\nu})\right]^3 \times$$

$$\times C(-3+\tfrac{1}{\nu})\, C(-6+\tfrac{2}{\nu})\, C(2-\eta-\tfrac{1}{\nu})\, C(1-2\eta)$$

where

$$C(x) \equiv \Gamma\left(\frac{x+3}{2}\right)\left[\Gamma(-x/2)\right]^{-1}$$

The main tool involved was the identity, if $a+b+c=-6$

(9) $$\int d^3 t\, |t-x|^a |t-y|^b |t-z|^c = \pi^{3/2}\, C(a)\, C(b)\, C(c) \times$$
$$\times |x-y|^{-3-c} |y-2|^{-3-a} |z-x|^{-3-b}$$

The functions g and h were obtained by computer calculation with sufficient precision in reasonable time. We note that the constant takes the value $\lambda = 7 \times 10^{-12}$ on the solution.

The authors are grateful to Prof. G. Preparata and Dr. F. Zirilli for useful suggestions. They also thank Profs. G. Altarelli, M. Cassandro, F. De Pasquale, C. Di Castro and F.P. Ricci for interesting discussions.

REFERENCES. -

(1) - G. Parisi, L. Peliti, Lettere al Nuovo Cimento 2, 630 (1971).
(2) - L. P. Kadanoff et al., Rev. Mod. Phys. 39, 395 (1967).
(3) - M. A. Moore, D. Jasnow, and M. Wortis, Phys. Rev. Letters 22, 940 (1969).
(4) - A. A. Migdal, Sov. Phys.-JETP 28, 1036 (1969); AM. Polyakov, Sov. Phys.-JETP 28, 533 (1969).