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C. Bernardini : GENERALITIES ON THE ELECTROMAGNETIC INTERACTIONS OF HADRONS. -

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GENERALITIES ON THE ELECTROMAGNETIC INTERACTIONS OF HADRONS

(Preliminary version of contribution to TEPP project;
Extended version of 'Lecture Notes' for
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CONTENTS

GENERALITIES ON THE ELECTROMAGNETIC INTERACTIONS OF HADRONS. -

1.1. - The Electrodynamics of Hadrons. -

All that we know of quantum electrodynamics (QED) as a satisfactory theory of electromagnetic interactions of electrons (muons) and photons is based on the possibility of isolating a world consisting of such particles and experimenting on it. Results concerning QED in this world attain quite unusual precision and, in a sense, measure how isolated it is from other fields.

We also known that a much more crowded world of elementary particles, the hadronic, governed by strong and less-understood interactions, possesses charge in common with the charged leptons. The two worlds communicate via the electromagnetic field: so that we expect a natural breakdown of QED on the one hand and the possibility of analysing hadrons by using photons on the other.

Hadrons have a large variety of quantum numbers and a complex structure. Therefore, the adaptation of pure QED to the study of their electromagnetic properties is by no means obvious and easy. The main link with pure QED is provided by the assumption that electric charge behaves similarly in widely different circumstances. When speaking of electric charge, however, it must be made clear to which property we refer in each specific case. Charge has a twofold import(1), since we use it to mean:

i - The strength of an interaction, as in the deflection of a charged particle in an electric field.
ii - A conserved and quantized quantity, as in the comparison of the initial and final state charges in particle collisions.

The two aspects allow a determination of charge and, sometimes, a comparison of the two results is possible. It must be noted that a privilege of charges (as compared to other additive quantum numbers, like the baryonic number) is that strength measurements are possible at the macroscopic level by the use of classical fields. It, however, does not make much sense in many cases, e.g. for the charge of a very unstable particle. At present, not many people feel doubt in attributing to elementary particles charges which are integer multiples of(2)

$$e = (1.6021917 \pm 0.0000070)\, 10^{-19} \text{ coulomb}$$

This statement is mainly based on the assumption that charge is strictly quantized and conserved, but experimental verification of it has not been pursued with great efforts.

Accurate determinations have been done on the charge-equality between protons and electrons and on the charge of the neutron(3). In both cases, deflection type experiments are used giving for the result, in e units, an upper limit of 10^{-19}. It is worth noting that a charge imbalance of this order would make the electric forces between two hydrogen atoms comparable to the gravitational forces; in general, due to the long range of interaction, charge inequality or non-conservation would have deep consequences in cosmology.

Besides charge (which is by no means structureless in the hadronic case) hadrons can have other electromagnetic properties. Best known is the magnetic moment, which is measured with great accuracy in the proton and neutron case by resonance methods on macroscopic samples(4). Higher static moments can play a role with increasing spin (compare 1.4) but their determination is much more difficult.

Questions have been raised as to the occurrence of magnetic monopoles(5) but there is at present no specific connection of them with hadrons.

Some static moments (e.g. the electric dipole) would violate invariance principles which are believed to be valid in hadron physics; the interest in the experimental search for this moments lies in the fact that their presence delineates the limits of the invariance principles. Their use in the physical description of hadron electrodynamics is essentially nil. For example, a dipole electric moment of $e \times 10^{-13}$ e.cm would have an effect on hadron electrodynamics, but the present upper limit for the neutron being many orders of magnitude less than this value indicates a possible relevance only as a test for time reversal invariance(6).

A new problem arises with hadrons in that they can change their nature by electromagnetic transitions (e.g. $\omega^o \rightarrow \pi^o \gamma$, $\Sigma^o \rightarrow \Lambda^o \gamma$ etc.). Transition moments are introduced in this case as a natural extrapolation from atomic physics where they are met in connection with bound systems. The classical picture we usually have in mind helps in fixing the orders of magnitude appropriate for these quantities to be relevant in electromagnetic interactions. Both static and transition multipole moments can be expressed by the product of charge times the l-th power of a length for a 2^l-pole. In the electric case, the Compton wave-length of the pion seems to be most appropriate to measure the size of any hadron; therefore, we think of electric 2^l-poles in units of $e(\hbar/m_\pi c)^l$ in order to appreciate how far from spherical symmetry is the charge distribution. In the magnetic case, however, the Compton wave-length of the hadron itself seems more adequate, on the naive basis of Ampere's theorem and the classical relation between magnetic and mechanical moments. This approach works for nuclei but is somewhat arbitrary in elementary particle physics.

Eventually, we will want to mention the important case in which a strong interacting particle has a peculiar coupling to two photons, like in $\pi^o \to \gamma\gamma$ decay. Here, the common attitude is to exploit to the best invariance principles in order to semplify the way kinematics enter the amplitude; what is left, usually, is an unknown parameter or function of invariant quantities. This parameter acquires a special significance when models are used to calculate it (like in the old $p\bar{p}$ model of π^o decay[7]); or it is expected to have simple properties when comparing different reactions (e.g. $\pi^o \to \gamma\gamma$ versus $\eta^o \to \gamma\gamma$).

1.2. - Quantum Numbers; -

The ignorance on detailed dynamics of strong interactions is alleviated by the wealth of conservation laws (i.e., invariance principles) applicable to the hadrons.

The assignment of good quantum numbers to particles is made as follows: one has to discover, first, which transformation leaves invariant the interaction he deals with. This provides an operator whose eigenstates can be compared with the particle state function. If a particle is an eigenstate, a quantum number can be assigned to it as the proper eigenvalue of the transformation operator.

Often the transformation property is not deeply understood, as in the case of gauge invariance (whence charge conservation and the charge quantum numbers). Nevertheless, the practical use of the charge quantum numbers is quite easy. In general, it looks at present as if the simplest invariance principles (geometric invariance[8]) generate quantum numbers difficult to handle: this is the case of spin and parity because they mix with orbital motion. On the contrary, more sophisticated operations of the gauge type[9] bring in simple additive quantum numbers such as charge, baryon number, hypercharge, etc.

As far as we know, a hadron can be assigned the following quantum numbers:

J, the spin
Q, the charge

B, the baryon number
Y, the hypercharge (Y - B = S, the strangeness)
I, the isospin (I_3 = third component of isospin)
P, the parity

When B = 0, we have a meson. For mesons with Y = 0, G-parity is a useful quantum number[10] combining charge conjugation C and isospin rotation. Mesons with both Y and Q equal zero are also C eigenstates.

The peculiar way hadrons group together into multiplets is now believed to be a manifestation of internal dynamics. This hitherto unknown dynamics is partly guessed on the basis of rules of the following type:

i - dependence of physical parameters on the quantum numbers (e.g. masses)
ii - relation among the quantum numbers.

Basic to the theory of electromagnetic interactions is the famous Gell-Mann, Nakano and Nishijma relation

(1.2.1) $$Q = I_3 + \frac{1}{2} Y$$

actually a definition of the hypercharge. Since half integer charges are not observed, this relation is actually supplemented by the condition

$$2I + Y = 0 \pmod{2}$$

This condition is very similar to the analogous one (half integer spin for baryons)

$$2J + B = 0 \pmod{2}$$

and the analogy is made even more suggestive by remarking that both J and I are related to rotations and B, Y to gauge transformations.

1.3. - Currents. -

We know where the concept of current comes from in classical physics: the charge, an additive quantity, can be transported in space-time. Therefore, a 4-vector current density is introduced in order to describe charge fluxes across any surface. Then, charge conservation leads to a continuity equation saying that the current $j_\mu(x)$ is divergenceless. The procedure is revisited in QED where current operators are generated by means of powerful techniques (e.g. the Noether theorem)[11]. We must be aware of the fact that in simple situations, like QED, there is usually only one conserved 4-vector built up with the ingredients of the theory. As a consequence, the choice leaves no doubt and we just stick the charge to the conserved vector operator.

Going to the hadrons, we see that there are many more conserved additive quantities (Q, B, Y, I_3) which can be associated with currents. The problem to pick out the appropriate current operator is therefore much more intriguing.

The relation (1.2.1) helps in showing that some requirements have to be satisfied by the currents too - a decomposition in two parts, related to I_3 and Y, can a always be made. Such decomposition of the current is commonly postulated to be exactly of the same type as for Q: that is, we require j_μ (x) to behave like $I_3 + (1/2)Y$ on the basis that $\int d^3x\, j_4(x) = Q$ does.

This is the major boldness in attempting a model of electromagnetic interactions of hadrons. It generates a class of interesting models, like the vector meson dominance model[12] or the quark-model approach[13].

Here, we want to sketch the prominent features of the line of interpretation originating in the postulated behaviour of j_μ (x). Less familiar than I-spin, is the U-spin naturally arising in the SU(3) symmetry scheme.

The point for considering U-spin is that it seems plausible to divide interactions of hadrons in 3 parts:

i - a strong interaction conserving both I-spin and U-spin
ii - a mass-breaking interaction conserving I-spin
iii - an electromagnetic interaction conserving U-spin.

Consider now the well known SU(3) multiplets, usually represented by (Y, I_3) diagrams. As an example, we take the meson octet, shown in Fig. 1.3.1.

The horizontal lines exemplify the isospin multiplets: we expect, for instance, electromagnetic mass differences along these lines. The lines at an angle connecting particles with equal charges show the U-spin multiplets.

Thus K^+, π^+ form a U-spin doublet and have therefore U = = 1/2, with $U_3 = 1/2$, for K^+ and - 1/2, for π^+. In the same way, π^-, $1/2(\sqrt{3}\eta - \pi^0)$ and π^+ form a U-spin triplet, while $-1/2(\sqrt{3}\pi^0 + \eta)$ is the singlet U = 0, and so on.

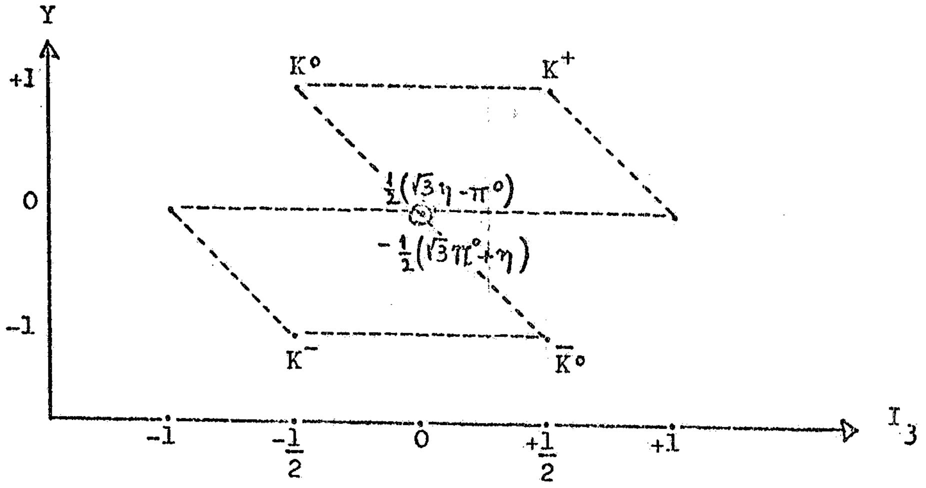

FIG. 1.3.1. - The meson octet.

In other words the decomposition of the hamiltonian for hadrons is such that all particles in an I-spin multiplet behave the same for mass-breaking interactions, all particles in a U-spin multiplet behave the same for electromagnetic interactions.

The last sentence can be put in the following form: any matrix element

$$\langle a \left| j_\mu \; j_\nu \; \cdots \right| b \rangle$$

of products of current operators between hadron states $|a\rangle$, $|b\rangle$ will vanish unless the same U-spin occurs both in $|a\rangle$ and $|b\rangle$. As a simple example, consider(14) the $U_3 = 0$ member of the $U = 1$ triplet in the meson octet Figure 1.3.1. : $1/2(\sqrt{3}\,\eta - \pi^o)$. It will not make transitions to the no-hadron state (2γ decay, $U = 0$) via the electromagnetic interactions. Therefore, the amplitude for $\eta \rightarrow 2\gamma$ will be $1/\sqrt{3}$ that for $\pi^o \rightarrow 2\gamma$. It must be noted that U spin conservation is less restrictive than the assumption $j_\mu \sim I_3 + (1/2)Y$. In fact, U spin conservation prescribes an invariance property in a sub-space of the whole internal space described by SU(3), leaving complete freedom in the choice of the way electromagnetic interactions break the symmetry. Beautiful as it is, the preceeding scheme has not many experimental verifications. There are at present two kinds of departures examined: a first one, questioning the validity of the Gell-Mann, Nakano and Nishijma relation; a second, preserving the above relation for charges, but questioning the postulated structure of the current.

There is no inconsistency at present in the assignment of quantum numbers to hadrons such that $Q - I_3 - (1/2)Y \neq 0$ is required. The doubt originates in a wise attitude against the universality of empirical laws and in the possibility of constructing plausible models in which (1.2.1) does not hold[15].

About the postulated structure of the current operator, however, it must be said that its behaviour in I-space is mainly supported by experimental evidence not very sensitive to departures. Quite recently, the question of how to discover an isotensor component of j_μ has been seriously examined[16] and found to be a difficult problem. The difficulties come from the fact that systems offering the possibility of $\Delta I = 2$ transitions are quite rare in nature, even in nuclear physics. Also, tests are often ambiguous and model dependent; what is more, they have a natural sensitivity limit at the (poorly known) level of 2-photon exchange.

1.4. - Matrix Elements of Current. -

Consideration of specific processes leads to the analysis of matrix elements of the current operator (or products of current operators). The simplest case is

$$\langle a | j_\mu | b \rangle$$

corresponding to the transition $a \rightarrow b + \gamma$. If also a, b represent single particle states, the situation can be analysed in detail to discover what is left unknown.

Particle a has 4-momentum p_a, spin J_a, parity P_a, charge Q_a, isospin I_a, hypercharge Y_a, baryon number B_a. The same notation we use for particle b by replacing a with b. We shall require that $B_a = B_b$ (baryon number conservation), $Q_a = Q_b$ (charge conservation), $Y_a = Y_b$ (hypercharge conservation). By the Gell-Mann, Nakano, Nishijma relation (1.2.1) it can be seen that $I_{3a} = I_{3b}$, the third component of I-spin does not change, But I can change on going from a to b. We omit explicit mention of the conserved numbers in $\langle a | j_\mu | b \rangle$ and rewrite it as

$$\langle J_a, p_a, P_a, I_a | j_\mu | J_b, p_b, P_b, I_b \rangle$$

Let us consider first what can be said on the basis of transformation properties under space-rotations. This analysis is founded on the Wigner-Eckart theorem[17], which for our purposes, amounts to say that given an operator 0_1 behaving like a spherical harmonic of angular moment 1, any matrix element

$$\langle J_a \ldots | 0_1 | J_b \ldots \rangle$$

vanishes unless $|J_a - J_b| \leq l \leq J_a + J_b$. In the case of 0_l associated with electromagnetic interactions, a 2^l multipole is met of the type

electric, when 0_l has the parity $(-1)^l$
magnetic, " 0_l " " " $(-1)^{l+1}$

The conventional names and symbols for the multipoles are: monopoles (E_o, M_o), dipoles (E_1, M_1), quadrupoles (E_2, M_2), octupoles (E_3, M_3), etc.

Consideration of the parities P_a and P_b and explicit requirement of parity conservation leads to the rules:

$(-1)^l = P_a P_b$ electric 2^l-pole allowed
$(-1)^{l+1} = P_a P_b$ magnetic 2^l-pole allowed

and other possibilities forbidden.

The following table helps in a rapid recognition of allowed possibilities (see Table 1.4.1).

Note that, if J is the smaller of J_a, J_b, $2J+1$ multipoles are allowed. M_o is quoted in parenthesis for reasons we discuss below.

How to trace the multipoles in the current matrix element? Consider first the explicit x dependence: since

$$j_\mu(x) = e^{iP\cdot x} j_\mu(0)\, e^{-iP\cdot x}$$

A special reference system can be introduced such that

$$p_a = \left(\frac{\vec{q}}{2}, (m_a^2 + |\vec{q}|^2/4)^{1/2}\right) = \left(\frac{\vec{q}}{2}, E_a\right)$$

$$p_b = \left(-\frac{\vec{q}}{2}, (m_b^2 + |\vec{q}|^2/4)^{1/2}\right) = \left(-\frac{\vec{q}}{2}, E_b\right)$$

This is the so-called Breit-frame, satisfying various simplicity requisites[18]. In the Breit-frame the description of kinematics is reduced to the use of the single vector $\vec{q}$. Current conservation reads

$$\vec{q}\cdot \langle a|\vec{j}(0)|b\rangle = (E_a - E_b)\langle a|j_4(0)|b\rangle$$

TABLE 1.4.1.

J_b		0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
0	$P_a = P_b$	E_o	-	M_1	-	E_2	-
	$P_a = -P_b$	$[M_o]$	-	E_1	-	M_2	-
$\frac{1}{2}$	$P_a = P_b$		E_o, M_{-1}	-	M_1, E_2	-	E_2, M_3
	$P_a = -P_b$		$[M_o], E_1$	-	E_1, M_2	-	M_2, E_3
1	$P_a = P_b$			E_o, M_1, E_2	-	M_1, E_2, M_3	-
	$P_a = -P_b$			$[M_o], E_1, M_2$	-	E_1, M_2, E_2	-
$\frac{3}{2}$	$P_a = P_b$				E_o, M_1 E_2, M_3	-	$M_1, E_2,$ M_3, E_4
	$P_a = -P_b$				$[M_o], E_1,$ M_2, E_3	-	$E_1, M_2,$ E_3, M_4
2	$P_a = P_b$					$E_o, M_1, E_2,$ M_3, E_4	-
	$P_a = -P_b$					$[M_o], E_1, M_2,$ E_3, M_4	-
$\frac{5}{2}$	$P_a = P_b$						$E_o, M_1, E_2,$ M_3, E_4, M_5
	$P_a = -P_b$						$[M_o], E_1, M_2,$ E_3, M_4, E_5

and in the elastic case when $m_a = m_b$ the orthogonality of $\vec{q}$ and $\langle a | \vec{j}(0) | b \rangle$ follows.

In general we can now write[19]

$$\langle a | j_\mu(0) | b \rangle = J_\mu(\vec{q})$$

and decompose $\vec{J}$ into a transverse and longitudinal part with respect to $\vec{q}$:

$$\vec{J}(\vec{q}) = \vec{J}_t(\vec{q}) + \vec{J}_l(\vec{q})$$

where

$$\vec{J}_t(\vec{q}) \cdot \vec{q} = 0; \qquad |\vec{q}| J_l(\vec{q}) = (E_a - E_b) J_4(\vec{q})$$

Therefore, $\vec{J}_t$ and J_4 are not related by current conservation which only helps in eliminating J_l. In particular, a vector $\vec{M}(\vec{q})$ can be introduced such that

$$\vec{J}_t(\vec{q}) = i\vec{q} \times \vec{M}(\vec{q})$$

which explicitly exhibits the transversality of $\vec{J}_t$. This definition amounts to say that $\vec{J}_t$ is the curl of a vector $\vec{M}$, just like the case of a classical current generated by a magnetization density.

We can now proceed to identify the multipole moments in full observance of the classical non relativistic limits whence the names have been borrowed.

Let us indicate by $\hat{q}$ a unit vector in the direction of $\vec{q}$. We introduce also two transverse axes in the plane orthogonal to $\vec{q}$ and use the notation $A_\pm = 1/\sqrt{2}(A_1 \pm iA_2)$ where A_1 and A_2 are components of a vector $\vec{A}$ along these transverse axes. The following expansion in spherical harmonics is possible:

$$J_4(\vec{q}) = 4\pi \quad \Sigma_l (-i)^l Y_l^0(\hat{q}) C_l'(|\vec{q}|^2)$$

$$M_\pm(\vec{q}) = 4\pi \quad \Sigma_l (-i)^l Y_l^{\pm 1}(\hat{q}) C_l''(|\vec{q}|^2)$$

The presence of $Y_l^0(\vec{q})$ in the development of J_4 results from invariance under rotations around the $\vec{q}$ direction. In the same way $Y_l^{\pm 1}(\hat{q})$ appears in the development of $M_\pm(\vec{q})$ because rotations by an angle φ around $\hat{q}$ will change $M_\pm$ into $e^{\pm i\varphi} M_\pm$.

Note that the sum for M_+ will not contain the $l = 0$ term. The coefficients $C_l(|\vec{q}|^2)$ are just matrix elements of the type $\langle a|0_l|b\rangle$, where 0_l behaves like a spherical harmonics with angular momentum l. Their parity, however, will not be simply $(-1)^l$, because of the intrinsic parities of a and b. Assuming now that under space inversion

$$j_4 \rightarrow j_4, \quad \vec{j} \rightarrow -\vec{j}$$

we easily see that the definition of electric multipoles corresponds to the C_l' terms; that for magnetic multipoles to the C_l''.

With the assumed space-inversion properties of j_μ, no magnetic monopole term will appear: in fact it cannot originate from j_4 (which allows $l = 0$) unless it has anomalous inversion properties.

1.5. - Static Moments. -

On the basis of the discussion in section 1.4 and of the analogy with classical systems, we can now proceed to recognise the static moments. When $m_a = m_b$ we will have the moments of a given particle; when $m_a = m_b$ transition moments will met. The static limit corresponds to $\vec{q} = 0$ for $a = b$; to $q^2 = 0$ for the $a \neq b$ case.

For $a = b$ we can imagine that we study the interaction of a particle with a classical external field; for $a \neq b$ we are considering transitions $a \rightarrow b + \gamma$ in which a real γ is emitted.

For example, take the case of a $a = b =$ proton, which is known to be a $J^P = (1/2)^+$ particle. By the above analysis we expect it can only have a charge and a magnetic dipole moment. Also, consider a ϱ^+ meson, $J^P = 1^-$: it will have charge, magnetic dipole and electric quadrupole. The decay $\omega^0 \rightarrow \pi^0 + \gamma$, in which the ω^0 meson has $J^P = 1^-$ and the π^0 meson has $J^P = 0^-$, will be accounted for by a transition magnetic dipole (compare Table 1.4.1).

The way moments are introduced in classical physics is well known[20]: starting from an interaction energy density

$$\mathcal{H}_{int}(x) = j^\mu(x) A_\mu^{ext}(x)$$

for a current in an external field $A_\mu^{ext}(x)$ one calculates the interaction energy

$$W_{int} = \int d^3\vec{x}\, j^4(x) A_4(x) - \int d^3\vec{x}\, \vec{j}(x) \cdot \vec{A}(x)$$

When $\vec{j}(x) = \text{curl}\,\vec{M}(x)$, the second term in the r.h.s. can be transformed so that

$$W_{int} = \int d^3\vec{x}\, j^4(x) A_4(x) - \int d^3x\, \vec{M}(x)\cdot\vec{H}(x)$$

where $\vec{H}(x)$ is the magnetic field. Then by expanding in powers of the space coordinates

$$A_4(x) = V_o + \vec{x}\cdot\vec{E}_o + \ldots\ldots; \qquad \vec{H}(x) = \vec{H}_o + (\vec{x}\cdot\nabla)\vec{H}_o + \ldots.$$

where the subscript o indicates that the field derivatives are computed at $\vec{x} = 0$, one gets

$$W_{int} = Q V_o + \vec{D}\cdot\vec{E}_o + \ldots. - \vec{\mu}\cdot\vec{H}_o - M_{lk}\left(\frac{\partial H_l}{\partial x_k}\right)_o - \ldots\ldots$$

It is well known that Q is the charge, $\vec{D}$ the electric dipole, $\vec{\mu}$ the magnetic dipole, M_{lk} the magnetic quadrupole tensor. Clearly:

$$Q = \int d^3\vec{x}\, j_4(x); \quad \vec{D} = \int d^3\vec{x}\,\vec{x}\, j_4(x); \quad \vec{\mu} = \int d^3\vec{x}\,\vec{M}(x)$$

$$M_{lk} = \int d^3\vec{x}\, M_l(x)\, x_k \qquad (l, k = 1, 2, 3)$$

These multipoles can be generated as follows: consider for instance

$$j_4(\vec{q}) = \int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}} j_4(x)$$

Then $Q = \lim_{\vec{q}=0} j_4(\vec{q})$, $\vec{D} = \lim_{\vec{q}=0}\left[-i\,\nabla q\, j_4(\vec{q})\right]$ and so on.

In the same way we generate the magnetic multipoles from

$$\vec{M}(\vec{q}) = \int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}}\vec{M}(x)$$

The procedure can be extended to the case that j_4 and $\vec{M}$ are operators in hadron space(21): we only need take matrix elements of the operators involved, just as we did in the preceding section. Here it can be seen that, since

$$e^{i\vec{q}\cdot\vec{x}} = 4\pi \sum_{l,m} (-i)^l f_l(|\vec{q}||\vec{x}|)\, Y_l^m(q)\, Y_l^{m*}(\hat{x})$$

the well known expansion for a plane wave, we find

$$C_l'(|\vec{q}|^2) = \int d^3\vec{x}\, f_l(|\vec{q}||\vec{x}|)\, Y_l^o(\hat{x})\, j_4(x)$$

and

$$C_l''(|\vec{q}|^2) = \int d^3\vec{x}\, f_l(|\vec{q}||\vec{x}|)\, Y_l^{\pm 1}(\hat{x})\, M_{\pm}(x)$$

Remember that $f_l(z) \xrightarrow[z=0]{} \frac{z^l}{(2l+1)!!}$ showing that C_l behaves like $|\vec{q}|^l$ as $|\vec{q}| \to 0$ and expressing again the fact that static 2^l moments are obtained by derivating l-times over $\vec{q}$.

All we have said above applies in the case of $m_a = m_b$ thanks to the Breit frame which allows a straightforward extension of non relativistic results(19). The case $m_a \neq m_b$, corresponding to $a \to$ $\to b + \gamma$ is not so simple because of $q^4 = E_a - E_b \neq 0$.

To get manifest relativistic invariance it is convenient to start from

$$j_4(\vec{q}) = \int d^3\vec{x}\, e^{-iq\cdot x}\, j_4(x); \qquad \vec{M}(\vec{q}) = \int d^3\vec{x}\, e^{-iq\cdot x}\, \vec{M}(x).$$

where now $-q\cdot x = \vec{q}\cdot\vec{x} - (E_a - E_b)t$. We still write $\vec{q}$ for the argument of the quantities on the l.h.s. since, given m_a, m_b, it is $\vec{q}$ that completely determines the kinematics.

The emission of a real γ-ray corresponds to $q^2 = (E_a - E_b)^2 - q^2 = 0$, which determines the value $|\vec{q}| \neq 0$ of the photon momentum in the Breit frame

$$|\vec{q}|^2 = \frac{(m_a^2 - m_b^2)^2}{2(m_a^2 + m_b^2)}$$

A special consideration must be given to the charge operator Q and to electric monopole transitions when $m_a \neq m_b$. Actually, Q being conserved, it's commutator with the total hamiltonian H vanishes

$$[H, Q] = 0$$

Therefore

$$\langle a | [H, Q] | b \rangle = (E_a - E_b) \langle a | Q | b \rangle = 0$$

and $\langle a | Q | b \rangle = 0$ follows when $E_a \neq E_b$.

For higher moments, one has just to follow thel ine indica ted for the $m_a = m_b$ case, replacing $|\vec{q}|^2$ by the equivalent variable $q^2 = (E_a - E_b)^2 - |\vec{q}|^2$ in order to make more transparent the limit $q^2 \to 0$ corresponding to the emission of real photons.

1.6. - Experiments on Static Moments. -

Only a few of the moments have been measured in the hadron case, the main difficulty being that most particles are short lived. Apart from the spin 0 mesons which can only have a charge, the first interesting case is that of $j = 1/2$ baryons, exhibiting a magnetic moment.

The proton and neutron magnetic moments (μ_p and μ_n) are best known because of their abundance in nature as stable particles. Actually, measurements of μ_p and μ_n are among the most accurate in physics and belong to the realm of nuclear magnetic resonance techniques[4].

The unit generally adopted for these moments is the proton magneton, 1/2 e (proton Compton wavelenght) = $e\hbar/m_p c$. It is very useful to remember that a magnetic moment of one proton magneton corresponds to a precession frequency

$$\omega = (4.789484 \pm 0.000027) \times 10^3 \text{ rad/sec}$$

in a field of 1 Gs.

The values as determined by the most accurate experiments are

$$\mu_p = (2.792782 \pm 0.000017) \text{ proton magnetons}$$
$$\mu_n = (-1.913148 \pm 0.000066) \text{ proton magnetons}$$

One sees that these moments have a large anomalous part since, in these units, one would expect $\mu_p = 1$ and $\mu_n = 0$ for a point Dirac particle (apart from higher order electromagnetic corrections).

Note that μ_p-1 is not very different from $-\mu_n$, that is the anomalous moments of p and n are comparable. Recently[22], the magnetic moments of Λ^o and Σ^+ have been determined and found to be

$$\mu_\Lambda = -0.73 \pm 0.16 \text{ proton magnetons}$$
$$\mu_{\Sigma^+} = 2.57 \pm 0.52 \text{ proton magnetons}$$

Since Σ^+ belongs to the same U spin doublet containing the proton, we expect $\mu_{\Sigma^+} = \mu_p$ and this is roughly true. For the Λ^o particle the problem is a little bit more difficult because it is $1/2(\sqrt{3}\,\Sigma^o + \Lambda^o)$ that belongs to a U-spin singlet whereas $1/2(\sqrt{3}\,\Lambda^o - \Sigma^o)$ is in a U-spin triplet with n and Ξ^o. Therefore we can only say that, on the basis of U-spin conservation, μ_{Σ^o}, μ_Λ are related to the transition moment $\mu_{\Sigma^o\Lambda}$ of the decay $\Sigma^o \rightarrow \Lambda^o + \gamma$. If however the explicit form suggested by the Gell-Mann, Nakano and Nishijima relation for j_μ is assumed, we find $\mu_\Lambda = 1/2\ \mu_n$ in slight disagreement with the experimental value.

The techniques used to measure μ_Λ and μ_{Σ^+} are quite different and less accurate than in the case of p an n. One has to produce particles whose spins point in a known direction and obtain the precession frequency in a magnetic field by observing asymmetries in the decay products at various distances from the production point.

No other case is known, besides Λ^o and Σ^+, neither among baryons nor among mesons (an attempt to measure μ_{Ξ^-} gave -0.1 ± 2.1 proton magnetons[22]).

As for the transition moments, the following radiative decays have been observed:

a) Mesons: $\omega^o \longrightarrow \pi^o\gamma$ $(1^- \rightarrow 0^- + \gamma)$ M_1 transition

$X^o \longrightarrow \varrho^o\gamma$ $(0^- \rightarrow 1^- + \gamma)$ M_1 transition

Also, upper limits are known for $\varrho \rightarrow \pi\gamma$, $\phi \rightarrow \pi\gamma$, $\eta\gamma$, $\omega\gamma$, $\varrho\gamma$; $V(\varrho^o, \omega, \phi) \rightarrow \pi^+\pi^-\gamma$.

b) Baryons: $\Delta(1236) \rightarrow N + \gamma$ $(\frac{3}{2}^+ \rightarrow \frac{1}{2}^+ + \gamma)$ $M_1 + E_2$ transition

$\Sigma^o \rightarrow \Lambda^o + \gamma$ $(\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + \gamma)$ M_1 transition

$\Lambda^o(1520) \rightarrow \Lambda^o + \gamma$ $(\frac{3}{2}^- \rightarrow \frac{1}{2}^+ + \gamma)$ $E_1 + M_2$ transition

All together the information on static electromagnetic properties of hadrons is very poor and some transparent difficulties (competition of strong effects, short mean lifetimes, small production cross sections and polarizations, small decay parameters, neutral decays) discourage the experimentalist.

1.7. - Role of the Isospin. -

The decomposition of j_μ according to the Gell-Mann, Nakano an Nishijma rule

$$j_\mu (x) = j_\mu^S (x) + j_\mu^V (x)$$

corresponds to the introduction of a scalar $\left(j_\mu^S (x) \right)$ and a vector $\left(j_\mu^V (x) \right)$ part in isospin space. Both are separately conserved like Y and I_3 are. We can enlighten the meaning of this decomposition by considering the behaviour of the current under G-parity transformations[10].

The operation

$$G = C \exp(i \pi I_2)$$

generates both a rotation by an angle π around the I_2 axis and conjugation of the charge.

The charge conjugation operator C changes a particle with quantum numbers Q, B, Y, I_3 into the antiparticle having $-Q, -B, -Y, -I_3$. Therefore, unless $B = Y = 0$, a particle cannot be an eigenstate of G. Usefulness of G is restricted to the case of non-strange mesons. The reason for considering it is that it has a larger class of eigenstates than C, because of the rotation in I-space which allows the inclusion of charged mesons.

Take now a state $|I, I_3, Q\rangle$ with $Y = B = 0$. The operation $\exp(i \pi I_2)$ changes this state into $(-1)^{I+I_3} |I, -I_3, -Q\rangle$ (because of $Q = I_3$, (1.2.1.), employing the Condon and Shortley convention for phases[23]. This gives $\pi^\pm \rightarrow \pi^\mp$, $\pi^0 \rightarrow -\pi^0$ in the case of π mesons. Also

$$C|I, I_3, Q\rangle = \eta |I, -I_3, -Q\rangle$$

where $\eta = \pm 1$. We see that when $Q = I_3 \neq 0$, the state cannot be an eigenstate of C or $e^{i \pi I_2}$; but it is an eigenstate of G. Since C transforms $\pi^\pm \rightarrow -\pi^+$, $\pi^0 \rightarrow \pi^0$, it follows that the pions are eigenstates of G with G-parity -1. Also, from the definition, it follows that a collection of n-pions has G-parity $(-1)^n$. Because of invariance of the strong interactions under C and I-spin rotations, G-parity is conserved in strong processes. Therefore, the decays $\rho \rightarrow \pi\pi$, $\omega^o \rightarrow \pi^+ \pi^- \pi^o$, $\phi \rightarrow \pi^+\pi^- \pi^o$ display the G-partities of the 3 vector mesons, +1 for the ρ, -1 for ω and ϕ. Now, j^s (x), being an I-spin scalar, changes sign under G

$$G \, j^s_\mu (x) G^{-1} = -j^s_\mu (x)$$

because of $C \, j_\mu (x) C^{-1} = -j_\mu (x)$. On the other hand,

$$G \, j^v_\mu (x) G^{-1} = j^v_\mu (x)$$

Therefore, transitions in which the electromagnetic interactions change the G-parity will be induced by the isoscalar part of the current, whereas the isovector current will leave the G-parity unchanged.

Examples are given by $\omega^o \rightarrow \pi^o \gamma$ in which only j^v_μ (x) has a role (note that $\omega^o \rightarrow \pi^+ \pi^- \pi^o$); in $\rho^o \rightarrow \pi^o \gamma$, however, the isoscalar part will be present. Also, the vacuum will be connected to 2, 4, 6, ... pions by j^v_μ (x); to 3, 5, 7, ... by j^s_μ (x) (1 pion states are not allowed by angular momentum conservation).

The similarity in the behaviour of j^s_μ (x) and the ω^o, ϕ^o fields is the basis of the theories originating the electromagnetic interactions of hadrons in the transitions $\gamma \rightarrow V$, where V is a $J^{PC} = 1^{--}$ vector meson.

1.8. - Classification of the Cross Sections. -

When working in the lowest electromagnetic order (one-photon approximation) the main ingredient of cross sections or transition rates of processes in which a photon (real or virtual) causes a hadron state $|a\rangle$ to go into another, $|b\rangle$, is the tensor

$$\langle a | j_\mu (0) | b \rangle \langle b | j_\nu (0) | a \rangle = R_{\mu\nu}$$

The detailed structure of this second rank tensor is generally exceedingly complicated, as can be deduced from the fact that it must be built up from combinations of all the momenta and spins of the particles participating in the reaction.

The situation however greatly simplifies when:

i - a is a single (real) particle with momentum p_a and mass m_a; $p_a^2 = m_a^2$.

ii - We sum over all the momenta in b allowed by 4 momentum conservation

iii - We sum over the spins of initial and final particles.

Let us call q the 4-momentum of the photon. Then $P_b = p_a + q$ is the total momentum of the hadrons in b. The prescriptions (i, ii, iii) correspond to the introduction of

$$\tilde{R}_{\mu\nu} = \sum_{\text{spins}} \int \frac{d^3 P_{bl}}{E_{bl}} \cdots \delta^{(4)}(P_b - p_a - q) R_{\mu\nu}$$

where P_{bi} is the 4-momentum of the b_i particle in b. Now, by Lorentz invariance, $\tilde{R}_{\mu\nu}$ can only be constructed from q_μ, $p_{a\mu}$, and the metric tensor $g_{\mu\nu}$. However, the only invariant quantities are clearly q^2 and $q \cdot p_a$. We write $q \cdot p_a = m_a \nu$. Therefore

$$\tilde{R}_{\mu\nu} = A_1 q_\mu q_\nu + A_2 g_{\mu\nu} + A_3(q_\mu p_{a\nu} + q_\nu p_{a\mu}) +$$
$$+ A_4(q_\mu p_{a\nu} - q_\nu p_{a\mu}) + A_5 p_{a\mu} p_{a\nu} \qquad \text{where } A_i = A_i(q^2, \nu)$$

Current conservation requires

$$q^\mu \tilde{R}_{\mu\nu} = q^\nu \tilde{R}_{\mu\nu} = 0$$

which reduces the five A_i functions to two (note that current conservation is a condition on 4-vectors; A_4 is readily seen to be zero, the remaining vector equation can be saturated multiplying by q and p_a). The two unknown functions can be chosen so that

$$\tilde{R}_{\mu\nu} = W_1(q^2, \nu)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2(q^2, \nu) \frac{1}{m_a^2} \times$$
$$\times \left(p_{a\mu} - \frac{m_a \nu}{q^2} q_\mu\right)\left(p_{a\nu} - \frac{m_a \nu}{q^2} q_\nu\right)$$

Further details on the structure of the two functions W_1 and W_2 will be given in various chapters in the book (photoproduction, photoabsorption, electro-production, e-p scattering, etc). We want now to use the above decomposition of the tensor $R_{\mu\nu}$ to classify experiments on electromagnetic processes in a simple pictorial way.

Consider a bi-dimensional plot (Fig. 1.8.1) in which $-q^2$ is the ordinate and ν is the abscissa. Reactions involving real γ rays (like radiative decays, photoproduction and photoabsorption) will correspond to some intervals of the ν axis, because of $q^2=0$. Reactions involving virtual γ rays with $q^2 \leq 0$ (e.g., electron scattering) will cover an area in the upper half-plane. Reactions which have an electron or muon pair (corresponding to $q^2 \geq 0$) in the final state (like $\pi^- p \to n + e^+e^-$, $\Sigma^0 \to \Lambda^0 + e^+e^-$) will be represented by regions or lines in the lower half plane.

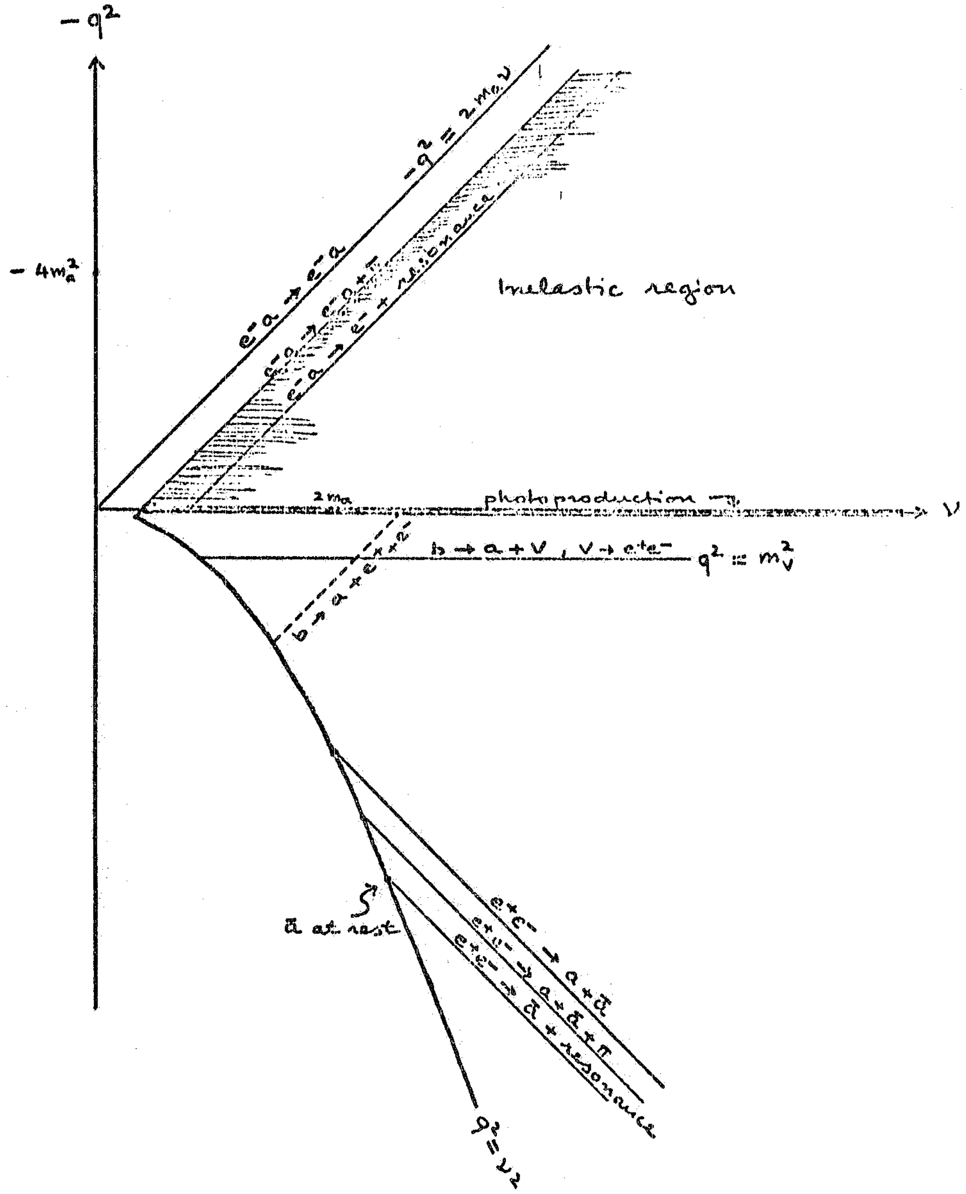

FIG. 1.8.1

To make a plot containing most of the representative situations, consider the case of a "target" particle which enters into radiative decays and allows scattering experiments to be performed on it.

We will utilize the two invariants q^2, the 4-momentum of the photon involved, and $\nu = p_a \cdot q/m_a$, p_a, m_a being 4-momentum and mass of the target. For an example, take the case that the target is a neutron.

The reaction $e^- + a \rightarrow e^- + a$, elastic electron scattering, corresponds to the line $-q^2 = 2m_a\nu$ extending from the origin to infinity. When, in electron scattering, a new particle b of definite (or nearly definite) mass m_b is produced (resonance production), a similar line is obtained:

$$e^- + a \rightarrow e^- + b; \qquad -q^2 = 2m_a\nu - (m_b^2 - m_a^2)$$

Usually $m_b^2 > m_a^2$. The line starts at $q^2 = 0$ but now it crosses the ν axis at

$$\nu_{min} = \frac{m_b^2 - m_a^2}{2\,m_a}$$

corresponding to the reaction $\gamma + a \rightarrow b$ (photoproduction of a resonance).

Photoproduction lies on the ν axis, starting from the first threshold which corresponds to $\gamma + a \rightarrow a + \pi$ and

$$\nu_t = m_\pi \left(1 + \frac{m_\pi}{2m_a}\right)$$

Also, the boundary of electroproduction of pions, $e^- + a \rightarrow e^- + a + \pi$ is a line parallel to the elastic scattering line. The triangular region limited by the photoproduction and electroproduction lines corresponds to the area that can be reached by inelastic $e^- + a$ scattering. It includes resonances as well.

Consider now reactions in which a hadronic system b can go into $a + \gamma$. Here b can be a single particle (mass m_b) or a group of particles; also, γ can be a real γ ray or the virtual γ linking the hadrons to a e^+e^- pair (or $\mu^+\mu^-$). In general we use the notation $s = P_b^2$. Now

$$s = m_a^2 + 2m_a\nu + q^2$$

Where $q^2 \geq 0$ (by neglecting the electron mass).

When $q^2 = 0$, real γ rays, we come back tothe photoproduction line (we actually use here the inverse reaction). At a given s, we find the continuation of the lines in the inelastic region of the upper half plane (with the same slope). This continuation cannot pass, however, the parabola $q^2 = \nu^2$ corresponding to the extreme q^2, ν values.

Clearly the maximum q^2 occurs when the a particle is at rest in the total center of mass system and the electron and the positron fly 180^o apart. It is easily shown that in this case $q^2 = (\sqrt{s} - m_a)^2$, $\nu = \sqrt{s} - m_a$.

There could be a line intermediate between the elastic and the electroproduction boundary lines when a particle b exists such that $b \rightarrow a + \gamma$ (e^+e^-), $m_b - m_a < m_\pi$ (as in the case of $\Sigma^o \rightarrow \Lambda^o + \gamma$).

It is customary to utilize the same plot for reactions in which a is replaced by $\bar{a}$, the antiparticle of a. These are colliding-beam reactions

$$e^+ + e^- \rightarrow \bar{a} + b$$

Again, the two invariants q^2 and $q \cdot p_{\bar{a}} = m_{\bar{a}} \nu = m_a \nu$ can be used; it is convenient to refer to the center of mass system of the e^+e^- to readily compute the relevant kinematical quantities. Since now

$$q^2 + m_a^2 - 2m_a \nu = P_b^2 ,$$

a straight line is obtained with a slope reversed with respect to that of the e - a scattering case. Given q^2, the minimum ν corresponds to particle $\bar{a}$ at rest: in fact, in the c.m.s.

$$p_a \cdot q = \sqrt{q^2} \cdot E_a = m_a \nu \geq m_a \sqrt{q^2} ,$$

that is, $\nu^2 \geq q^2$. The production of the a $\bar{a}$ pair corresponds to the line $q^2 = 2m_a \nu$ starting from $\nu = 2m_a$, $q^2 = 4m_a^2$; this line is the equivalent of the line for $e^- + a \rightarrow e^- + a$ in the upper half plane. If a particle b (lighter than a) can be produced in

$$e^+ e^- \rightarrow \bar{a} + b ,$$

we obtain a line corresponding to

$$q^2 = 2\, m_a \nu \; - (m_a^2 - m_b^2) \; ,$$

above the line for $\bar{a}$ a production. Lines for the production of $\bar{a}$ b with b heavier than a (a case similar to resonance production) lie below the a $\bar{a}$ line.

In any case the $q^2 = \nu^2$ parabola is an universal boundary (independent of target) in the lower half plane.

An important feature is exhibited by the lines $q^2 = m_V^2 > 0$ corresponding to the masses of vector mesons with $J^{PC} = 1^{--}$. These mesons could account for most of the electromagnetic interactions of hadrons because of $\gamma \rightarrow V$ transition with high strength(12), as shown by measurements on $e^+e^- \rightarrow V(\rho, \omega, \phi)$ with colliding beams(24). The location of the lines $q^2 = m_V^2$ indicates, by comparison of the distance of a point with the widths of the vector mesons, the regions where large electromagnetic cross sections are found.

1.9. - Photon Polarization Problems. -

Electromagnetic processes in which polarized photons occur are often encountered today, although much of the physical information in the field still has to be provided.

1.9.1. - Polarized Photons (description). -

The polarization 4-vector of a real photon, whose 4-momentum is $k = (k, \vec{k})$ can be represented in terms of a suitable basis:

$$e(i) = (0,\ \vec{e}\,(i)), \qquad \vec{e}\,(i) \cdot \vec{k} = 0, \qquad i = 1, 2, \qquad \vec{e}\,(1) \cdot \vec{e}\,(2) = 0$$

$$e(3) = (0,\ k\ ), \qquad \vec{k} = k\,\hat{k}, \qquad e(4) = (1,\ \vec{0}\ )$$

1, 2 are usually called transverse vectors, 3 longitudinal. Thanks to gauge invariance and the Lorentz condition the polarization, e_μ can be written in general as

$$e_\mu = \varepsilon_1\, e_\mu\,(1) + \varepsilon_2\, e_\mu\,(2)$$

with

$$\left|\varepsilon_1\right|^2 + \left|\varepsilon_2\right|^2 = 1$$

Linear polarization corresponds to the case in which $\varepsilon_1 = \cos\lambda$,

$\varepsilon_2 = \sin\lambda$, λ being the angle of the electric field with the $\vec{e}(1)$ axis in the plane orthogonal to $\vec{k}$.

Circular polarization corresponds to

$$\varepsilon_1 = \frac{1}{\sqrt{2}}, \qquad \varepsilon_2 = \pm \frac{i}{\sqrt{2}}$$

where $\pm$ refer to right and left circularly polarized photons respectively.

In the calculation of cross sections or decay rates e_μ appears in the tensor combination $e_\mu \, e^x_\nu$:

$$e_\mu e^x_\nu = |\varepsilon_1|^2 e_\mu(1) e^x_\nu(1) + \varepsilon_1 \varepsilon^x_2 e_\mu(1) e^x_\nu(2) + $$
$$+ \varepsilon_2 \varepsilon^x_1 e_\mu(2) e^x_\nu(1) + |\varepsilon_2|^2 e_\mu(2) e^x_\nu(2)$$

The case of unpolarized photons corresponds to averaging over the value of λ :

$$\langle \varepsilon_i \varepsilon^x_j \rangle = \frac{1}{2} \delta_{ij}$$

Partial linear polarization P is defined by

$$|\varepsilon_1|^2 = \frac{1}{2}(1-P) + P\cos^2\lambda \, , \qquad |\varepsilon_2|^2 = \frac{1}{2}(1-P) + P\sin^2\lambda$$
$$\varepsilon_1 \varepsilon^x_2 = \varepsilon^x_1 \varepsilon_2 = P \sin\lambda \cos\lambda$$

(corresponding to $\varepsilon_1 = \sqrt{1-P}\cos\lambda' + \sqrt{P}\cos\lambda$ and $\varepsilon_2 = \sqrt{1-P}\sin\lambda' + \sqrt{P}\sin\lambda$ and average over λ').

Often, the e (1) direction is chosen such that $\lambda = 0$ for linear polarization. Also, a photon-polarization density matrix is defined by

$$e_i e^x_j = \varrho_{ij} \qquad (i, j = 1, 2, 3),$$

taking advantage of the fact that both in the real and virtual photon case, the conditions of Lorentz invariance and current conservation allow the elimination of time-components.

Therefore, for partial linear polarization P in the $\vec{e}(1)$ direction

$$\varrho = \begin{vmatrix} \frac{1}{2}(1+P) & 0 & 0 \\ 0 & \frac{1}{2}(1-P) & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$P = 1$ for complete polarization, $P = 0$ for the unpolarized case. For circular (right) polarization

$$\varrho = \begin{vmatrix} \frac{1}{2} & -\frac{i}{2} & 0 \\ \frac{i}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

and so on.

The case of virtual photons is somewhat more complicated[25]. Here e_μ is replaced by

$$\frac{1}{q^2} \langle 1' | j_\mu | 1 \rangle$$

where 1, 1' are electrons (muons) as in

$$1 + a \rightarrow 1' + b$$

We indicate by 1_μ , $1'_\mu$ the electron 4-momenta, and neglect the electron mass in the following. Thus $1 = 1' + q$. We assume 1, 1' are unpolarized and take advantage again of the fact that j_μ is a conserved current to eliminate time components. Then take the 3 axis along $\vec{q}$; the 1, 3 axis in the plane containing $\vec{1}$, $\vec{1}'$; and call θ the scattering angle of the lepton. Define also

$$P = \left(1 + 2 \frac{|\vec{q}|^2}{q^2} \, tg^2 \frac{\theta}{2}\right)^{-1}$$

Since the two leptons are free point Dirac particles, it can be shown by standard techniques that in this case

$$(q^2)^2 e_\mu e_\nu^{*} = -(1_\mu 1_\nu^{*} + 1'_\mu 1_\nu + \frac{1}{2} q^2 g_{\mu\nu})$$

or

$$q^2 \varrho_{ij} = \frac{l_i l'_j + l'_i l_j}{q^2} + \frac{1}{2} \delta_{ij}$$

With a little manipulation we then obtain

$$q^2 \varrho_{11} = \frac{1}{2} \frac{1+P}{1-P}, \qquad q^2 \varrho_{22} = \frac{1}{2},$$

$$q^2 \varrho_{33} = \frac{1}{2} \frac{P}{1-P} \left(1 + \frac{|\vec{q}|^2}{q^2}\right),$$

$$q^2 \varrho_{13} = q^2 \varrho_{31} = \frac{1}{\sqrt{2}} \frac{[P(1+p)]^{1/2}}{1-P} \left(1 + \frac{|\vec{q}|^2}{q^2}\right)^{1/2}$$

$$q^2 \varrho_{12} = q^2 \varrho_{21} = q^2 \varrho_{23} = q^2 \varrho_{32} = 0.$$

This matrix, however, does not reduce to the one we used in the real photon case when $q^2 \to 0$; it is, moreover, singular in this limit. Since, for the product of two conserved current matrix elements,

$$j_\mu J^\mu = j_4 \cdot J^4 - \vec{j} \cdot \vec{J} = -j_1 J^1 - j_2 J^2 - \frac{q^2}{q^2 + |\vec{q}|^2} j_3 J^3$$

because of $q^4 j_4 = |\vec{q}| j_3$, $q^4 J_4 = |\vec{q}| J_3$, we see that it is possible to include a factor $q^2/q^2 + |\vec{q}|^2$ in the lepton current component j_3. That is, by defining

$$j'_{1,2} = j_{1,2}, \qquad j'_3 = \frac{q^2}{q^2 + |\vec{q}|^2} j_3, \qquad j'_4 = 0$$

we can construct a density matrix ϱ'_{ij} with the desired property of having the proper limit when $q^2 \to 0$:

$$\varrho'_{11} = q^2 \varrho_{11}(1-P) = \frac{1}{2}(1+P), \qquad \varrho'_{22} = q^2 \varrho_{22}(1-P) = \frac{1}{2}(1-P),$$

$$\varrho'_{33} = q^2 \left(\frac{q^2}{q^2+|\vec{q}|^2}\right)^2 (1-P) \quad \varrho_{33} = \frac{q^2}{q^2+|\vec{q}|^2} \, P \; ,$$

$$\varrho'_{12} = \varrho'_{31} = \frac{q^2}{q^2+|\vec{q}|^2} \, q^2 \quad \varrho_{13}(1-P) = \frac{1}{\sqrt{2}} \left(\frac{q^2}{q^2+|\vec{q}|^2}\right)^{1/2} \left[P(1+P)\right]^{1/2}$$

$$\varrho'_{12} = \varrho'_{21} = \varrho'_{23} = \varrho'_{32} = 0$$

Finally, the longitudinal polarization is defined by

$$P_L = \frac{q^2}{q^2+|\vec{q}|^2} \quad P$$

Note that in the case of elastic scattering, the use of the Breit-frame for the scattered hadron gives the particularly simple result

$$P_L = \frac{1}{2} \, P = \frac{1}{1+tg^2 \frac{\theta_B}{2}}$$

where θ_B is the electron scattering angle.

This result is obviously not Lorentz invariant.

The introduction of the matrix ϱ'_{ij} allows the calculation of cross sections for $1+a \rightarrow 1'+b$. Let us call J the matrix element of the hadronic current in the transition $a \rightarrow b$. Then the cross section is proportional to

$$|M|^2 = \frac{e^4}{1-P} \; \frac{1}{q^4} \; \varrho'_{ij} J_i J_j^{*}$$

(only phase-space is left out).

Finally, the case of the crossed reaction

$$1+\bar{1} \rightarrow \bar{a}+b$$

can be considered[26]. Here, by the same procedure, it can be seen that by taking the electron momentum in the center of mass system along the 1 axis, we get the simple result

$$\varrho_{11} = 0, \qquad \varrho_{22} = \varrho_{33} = \frac{1}{2}, \qquad \varrho_{ij} = 0 \qquad (i \neq j)$$

The 2 and 3 axes cannot be distinguished in this case (unpolarized leptons) because of $\vec{q} = 0$. The result expresses the fact that the current of the leptons is orthogonal to their line of flight in the limit of zero mass. Therefore, by letting $\vec{J}_{\perp}$ indicate the component of the hadron current orthogonal to the electron space-momentum we get in this case

$$|M|^2 = \frac{1}{2} \frac{e^4}{q^4} \vec{J}_{\perp} \cdot \vec{J}_{\perp}^{*}$$

exhibiting the well known $\sin^2\theta$ dependence upon the angle between $\vec{J}$ and $\vec{l}$ in the c.m.s.

1.9.2. - Polarized photons (experimental techniques). -

We saw in the preceding section that in the case of electron scattering (virtual photons) the polarization is completely determined by kinematics of the leptons. This is, therefore, a case where polarization is easily produced and precisely known. The use of polarized real photons is somewhat more difficult. A powerful technique exploited in the last years[27] consists in obtaining the polarization by the mono-crystal method. This method is based on the fact that coherent electron bremsstrahlung in crystals shows both intensity enhancement and partial linear polarization maxima (an example is given in Fig. 1.9.1). Also, circular polarized bremsstrahlung has been used[28] by sampling suitable angular regions in ordinary bremsstrahlung beams.

With a partial linear polarization P, the quantity directly measured by experiments is the ratio of the number of events whose production plane is orthogonal to the polarization axis, $n_{\perp}$, to the number of events whose production plane contains the polarization axis, $n_{\parallel}$. The two numbers refer to the same number of photons initiating the reaction. If $\sigma_{\perp}$ and $\sigma_{\parallel}$ are the cross sections for the two cases, we have

$$n_{\perp} \sim (1+P)\sigma_{\perp} + (1-P)\sigma_{\parallel}, \qquad n_{\parallel} \sim (1-P)\sigma_{\perp} + (1+P)\sigma_{\parallel}$$

whence

$$\Sigma = \frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}} = \frac{1}{P} \frac{n_{\perp} - n_{\parallel}}{n_{\perp} + n_{\parallel}}$$

Σ is the so called "asymmetry" giving information complementary to the unpolarized cross section $\sigma_{unpol} = 1/2(\sigma_{\perp} + \sigma_{\parallel})$. The statistical error on Σ will be large unless both P and Σ are large (a general feature of these up-down types of analysis): typically in photomeson production at low energies both P and Σ are $\sim .3$ so that 10% differences have to be measured.

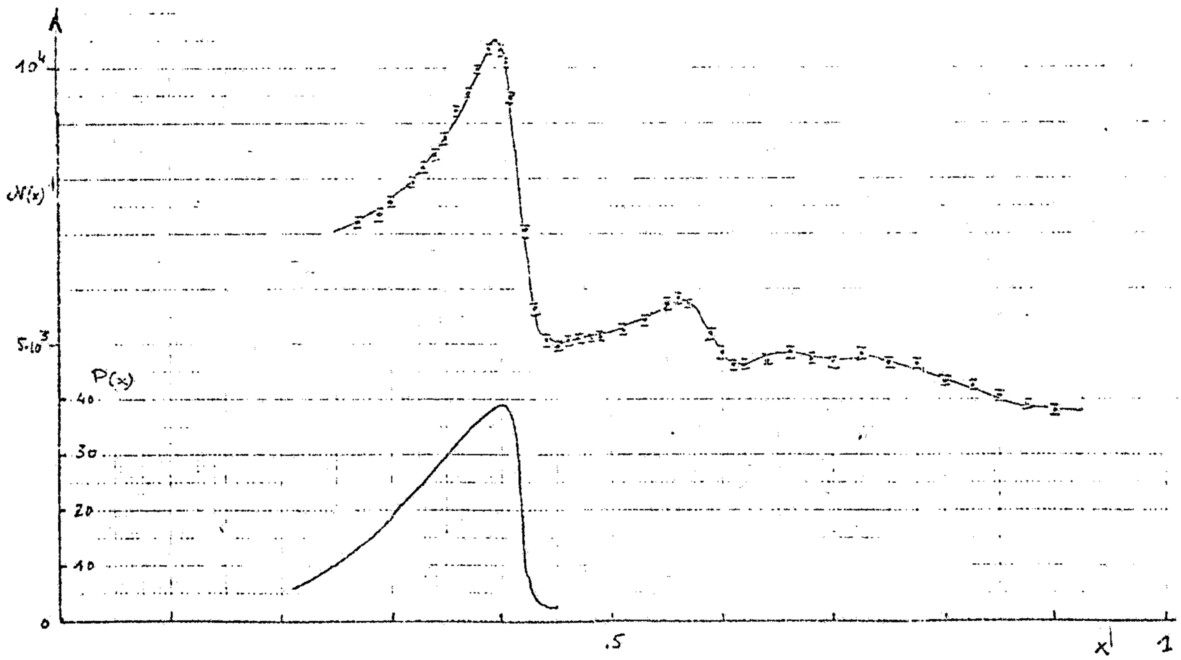

FIG. 1.9.1

1.10. - Two-photon Reactions. -

We shall distinguish the case in which a two-photon process occurs, the one-photon process being forbidden (genuine two-photon) from the case in which the two-photon process occurs as a correction to the first order, one-photon contribution. The reason we tell the second case a correction is that we assume perturbation theory to work in electromagnetic interactions; therefore we also usually expect the two-photon amplitude to be a factor $\alpha = 1/137$ less than the one-photon amplitude. This last statement is by no means obvious because of the possible role of kinematics factors: there are at present examples of the cases in which one must take care to get rid of second order terms(29). In the genuine two-photon processes the matrix elements of $j_{\mu}(x)\, j_{\nu}(y)$, a product of current operators, have the relevant role. For instance, in $\pi^0 \to \gamma\gamma$ decay one must take the matrix element of $j_{\mu}\, j_{\nu}$ between the π^0 and the vacuum states.

Lorentz-covariance and space-time properties allow one to write the appropriate general form of the matrix elements for genuine two-photon processes, leaving to specific models the task of computing unknown scalar parameters. For instance, in the $\pi^0 \rightarrow \gamma\gamma$ case we dispose of the two polarization vectors ε_1, ε_2 of the photons and of the three particle moments. We also know that by momentum conservation only two momenta (the photon momenta k_1 and k_2, say) are independent. Moreover, the π^0 has odd intrinsic parity and the amplitude must be odd under space-inversion. Finally, gauge invariance tells that the replacement $k_i \rightarrow \varepsilon_i$ must set the amplitude to zero. These conditions imply that the amplitude can be written

$$T = g\ \varepsilon_{\alpha\beta\gamma\delta}\ k_1^{\alpha}\ \varepsilon_1^{\beta}\ k_2^{\gamma}\ \varepsilon_2^{\delta}$$

where g is the unknown constant and $\varepsilon_{\alpha\beta\gamma\delta}$ is the completely antisymmetric 4-tensor. g is related to the width of the decay, Γ, by

$$\Gamma = \frac{m_{\pi}^3}{32\pi}\ g^2$$

It is a simple and instructive exercise to show that the photons have opposite circular polarizations, thus checking the intuitive feeling that this must happen in the decay of a spin-zero particle.

The same form is expected to hold in the case of $\eta \rightarrow \gamma\gamma$. Here we see that

$$g_{\eta}^2 \sim \frac{\Gamma_{\eta \rightarrow \gamma\gamma}}{m_{\eta}^3} \qquad\qquad g_{\pi}^2 \sim \frac{\Gamma_{\pi^0 \rightarrow \gamma\gamma}}{m_{\pi}^3}$$

so that, on the basis of U-spin conservation (compare section 1.3) giving $g_{\eta}^2 = (1/3)\, g_{\pi}^2$, we expect

$$\Gamma_{\eta \rightarrow \gamma\gamma} = \frac{1}{3}\ \frac{m_{\eta}^3}{m_{\pi}^3}\ \Gamma_{\pi^0 \rightarrow \gamma\gamma}$$

Actually(2), $\Gamma_{\pi \rightarrow \gamma\gamma} = (1.0 \pm 0.26)$ keV, $\Gamma_{\pi^0 \rightarrow \gamma\gamma} = (8.6 \pm 1.9) \cdot 10^{-3}$ keV, showing a discrepancy by a very large factor (≈ 5.5).

Doubts are cast on the U-spin result by the large π - η mass difference: it is by no means obvious that the quantities to compare are g_{η} and g_{π}. Also, the role of the $\eta'(X^0)$ in the symmetry scheme is not clear at present.

The case of two-photon contributions to processes dominated by one-photon exchange is quite intriguing also. Separation of the two-photon amplitude can be made on the basis of the different behaviour under charge conjugation of the one - and two - photon terms. In general, C is even or odd according to $(-1)^n$ where n is the number of photons involved. Therefore, in the case of $e^{\pm}$-hadron scattering (which is the best known example) the amplitude up to second order will be

$$-eT_1 + e^2T_2 \text{ for } e^-\text{-hadron}, \qquad +eT_1 + e^2T_2 \text{ for } e^+\text{-hadron}$$

where e^nT_n is the n-photon contribution.

Assuming $|eT_2| \ll |T_1|$ and calling $\sigma_{\pm}$ the cross section for $e^{\pm}$ scattering we have

$$\frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = 2\, e\, \frac{\mathrm{Re}\, T_1 T_2^x}{|T_1|^2}$$

On the same basis we can show that in $e^+e^- \rightarrow$ hadrons it is possible to distinguish the one-and two-photon contributions by recognition of the final charges.

Also, other specific tests concerning the two photon amplitudes can be devised by exploiting the peculiar behaviour of the polarization of the particles or the angular dependence at fixed momentum transfers as in the Rosenbluth plot (compare chapter).

Not much is known in general on the second order electromagnetic contributions; people usually hope they are small since the analysis of the data is much easier when the first order dominates. This is, however, not generally true and a word of warning must be said, as the following example better suggests. Consider an electron positron collision and a process which, in graphical form is shown in Fig. 1.10.1

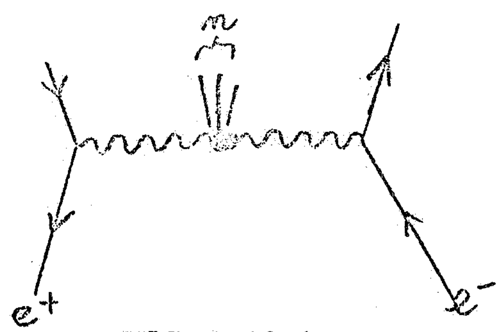

FIG. 1.10.1

Here n is a hadron state having C-conjugation +.

These "Low-graph" can originate from the emission of two quasi-real γ's according to the semi-classical Weizsächer-Williams procedure. Therefore, the process is very similar to $\gamma\gamma \rightarrow n$ with real γ's and in fact it can be shown that the cross section can be factorized into the probability of γ emission by each electron times the cross section for $\gamma\gamma \rightarrow n$, $\sigma_{\gamma\gamma}$ (30). Now, the probability for γ emission contains, besides the charge, a factor $\ln(E/m)$ where E and m are the energy and mass of the $e^{\pm}$. This log factor can be quite large at high energy making the process dominate over other lowest order processes.

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