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On the Possibility of Testing C Conservation in the Positronium Three-Photon Decay Mode.

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It is well known that a bound state of an electron and a positron can exist, in the ground state and in the absence of any perturbation, in two spin states: $S = 0$ ($^1S_0$-parapositronium) or $S = 1$ ($^3S_1$-orthopositronium).

If we let form the bound state in the presence of a microwave magnetic field, we can obtain a quantum mixture of para- and orthopositronium (this latter with $S_z = 0$). The perturbing term due to the microwave magnetic field can be written as

$$V = \mu_0(\sigma_{ez} - \sigma_{pz}) \frac{H}{2} \left[\exp[-i(\omega t + \varphi)] + \exp[i(\omega t + \varphi)]\right], \tag{1}$$

where

$\mu_0 = \dfrac{e\hbar}{2m_e c} = 0.5788 \cdot 10^{-14}\ \text{MeV G}^{-1}$,

σ_{ez}, σ_{pz} = Pauli spin operators,

H = magnetic-field amplitude,

ω, φ = pulsation and phase of H.

General dispersion theory leads to the experimentally confirmed view that the influence of the alternating field will be very small off resonance. We see that in (1) only the term $\exp[-i(\omega t + \varphi)]$ can give rise to a resonance phenomenon. The second term produces only dispersion effects which average out during the time of observation (1) and, therefore, can be dropped.

The only nonvanishing matrix element of V is

$$\langle 10|V|00\rangle = D \exp[-i\omega t], \qquad D = 2\mu_0 H \exp[-i\varphi],$$

(1) O. HALPERN: *Phys. Rev.*, **94**, 904 (1954).

and causes a transition from a singlet state to the triplet state with $S_z = 0$. So the superposition principle allows to write the general state of the system in the form

$$|\mu\rangle = b_0(t) \exp[-g_1 t/2]|10\rangle + b(t) \exp[-gt/2]|00\rangle ,$$

where $g = 8 \cdot 10^9\,\text{s}^{-1}$ and $g_1 = 7.2 \cdot 10^6\,\text{s}^{-1}$ are the total transition amplitudes for the $|10\rangle$ and $|00\rangle$ state, respectively.

By solving the Schrödinger equation we find that the vector $|\mu\rangle$ is the sum of two parts $|\mu_1\rangle$ and $|\mu_2\rangle$ with definite lifetimes. Indeed these can be written, apart from unessential phase factors, like

$$\text{(2)} \qquad \begin{cases} |\mu_1\rangle \propto \big[ik \exp[i(vt + \psi_1)]|10\rangle + |00\rangle\big] \exp[-G_1/2t] , \\ |\mu_2\rangle \propto \big[i \exp[i(vt + \psi_2)]|10\rangle + k|00\rangle\big] \exp[-G_2/2t] , \end{cases}$$

where v is the difference between the proper pulsation of the system and that of the microwave field,

$$G_1 = \frac{g_1 k^2 + g}{1 + k^2} , \qquad G_2 = \frac{g_1 + k^2 g}{1 + k^2} ,$$

ψ_1 and ψ_2 are angles directly related to the phase of the perturbing term (1) and k is a factor depending on the magnetic-field strength H.

If H is of the order of 10 G as can be done with the present techniques, $k^2 \ll 1$ and we have

$$G_1 \simeq g , \qquad G_2 \simeq g_1 + k^2 g .$$

Let us now suppose that there exist a C-violating term in the Hamiltonian responsible of the three-photon decay, with coupling constant h. Consequently, the three-photon decay amplitude of $|\mu_1\rangle$ and $|\mu_2\rangle$ consists of two coherent terms, one C-forbidden from the singlet and one C-allowed from the triplet.

These terms can interfere, and the interference term is linear in the coupling constant h. The fact of dealing with terms in h instead of h^2 greatly enhances the sensitivity of this method with respect to the usual one, consisting in the search of the decay ${}^1S_0 \to 3\gamma$.

It is straightforward to notice that $|\mu_1\rangle$ and $|\mu_2\rangle$ become the imperturbed states $|00\rangle$ and $|10\rangle$ when the magnetic-field amplitude H is vanishing and that, if a C-violating term exists, an interference term can be observed starting from either positronium states.

The squared matrix element for the decay can be written as (H is the total Hamiltonian)

$$\text{(3)} \qquad \begin{aligned} |\langle 3\gamma|H|\mu_1\rangle|^2 \propto \{&k^2|\langle 3\gamma|H|10\rangle|^2 + |\langle 3\gamma|H|00\rangle|^2 - \\ &- 2k \,\text{Re}\,[\langle 3\gamma|H|10\rangle^* \langle 3\gamma|H|00\rangle] \sin(vt + \varphi_1)\} \exp[-G_1 t] |\langle 3\gamma|H|\mu_2\rangle|^2 \propto \\ &\propto \{|\langle 3\gamma|H|10\rangle|^2 + k^2|\langle 2\gamma|H|00\rangle|^2 - \\ &- 2k \,\text{Re}\,[\langle 3\gamma|H|10\rangle^* \langle 3\gamma|H|00\rangle] \sin(vt + \varphi_2)\} \exp[-G_2 t] \end{aligned}$$

We see that the amplitude of the interference term and the amplitude of the allowed term are in the ratio k^{-1} and k, respectively, for the two states $|\mu_1\rangle$ and $|\mu_2\rangle$. As typically $k \ll 1$, the first possibility gives a better sensitivity than the second, but this advantage must be paid by dealing with a lifetime G_1^{-1} of $\simeq 150$ ps. We also notice that the pulsation of a possible interference term can be changed varying the ω parameter of the microwave magnetic field and that in the static case ($\omega = 0$) it becomes the proper pulsation of the system.

Moreover, care must be taken with the phase φ_1 because it is a random number, which may average out the interference term. Presumably this method may lower the present experimental limit (2) on C violation in $^1S_0 \to 3\gamma$ decay of several orders of magnitude.

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(2) A. MILLS and S. BERKO: *Phys. Rev. Lett.*, **18**, 420 (1967).