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G. Parisi and F. Zirilli : HARD BREMSSTRAHLUNG IN e^+e^- COLLISIONS

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Hard Bremsstrahlung in e^+e^- Collisions.

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It is well known that there are important radiative corrections to e^+e^- scattering due to bremsstrahlung. It is usually assumed that these corrections are important only for the emission of soft photons; in the case they are hard, the angle between them and the direction of at least one of the incoming or outgoing electrons should be of order m/E_γ.

The aim of this letter is to show that there is an important contribution to the total production cross-section due to the process $e^+e^- \to e^+e^-\gamma$ where the final photon is hard and emitted with a large angle. An approximate evaluation brings to the following typical results. We denote by $\bar{\theta}$ the angle between the incoming and the outgoing e^- in the centre-of-mass frame, by θ_{in} and θ_{out} the angles between the photon and the incoming and outgoing e^-, by η the ratio between the photon energy E_γ and the beam energy E. With this notation the cross-section for the production of $e^+e^-\gamma$ with $135° > \bar{\theta} > 45°$, $\theta_{in}, \theta_{out} > 20°$ is $\sim 0.6\sigma_\mu$, where $\sigma_\mu = \pi\alpha^2/3E^2$ is the total cross-section for the production of a $\mu^+\mu^-$ pair.

This result is brought out in the following way: using an almost real approximation, one gets for the differential cross-section, in the case of zero-mass electrons,

$$\mathrm{d}\sigma = \sigma(\bar{\theta})\,\mathrm{d}\Omega_e \frac{\alpha}{4\pi^2}\left[\frac{1}{\sin^2\theta_{in}} + \frac{1}{\sin^2\theta_{out}}\right]\frac{1+\eta^2}{\eta}\,\mathrm{d}\Omega_\gamma \mathrm{d}\eta\,, \tag{1}$$

where

$$\sigma(\bar{\theta}) = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} - \frac{2\cos^4(\theta/2)}{\sin^2(\theta/2)} + \frac{1+\cos^2\theta}{2}\right] \tag{2}$$

is the differential cross-section for the Bhabha scattering.

This formula is obtained if one treats η, θ_1, θ_2 as small quantities. If we try to use this formula at all angles and energies, we find that the ratio between $\sigma(\bar{\theta})$ and the cross-section for the production of $e^+e^-\gamma$ with the same $\bar{\theta}$ and $\eta > 0.1$, $\theta_1, \theta_2 > 20°$ is ~ 40.

By a simple integration over $\bar{\theta}$ we obtain the required cross-section. In order to test the reliability of this result, we have to check the validity of formula (1) in the large-angle region.

To do this we have performed the exact calculation of the differential cross-section, not using the trace theorems, but calculating the helicity amplitudes and summing their squared absolute value ([1]). The results are in perfect agreement with the trace calculations of ref. ([2]).

The ratio between the exact value and formula (2) is plotted in Figs. 1 and 2.

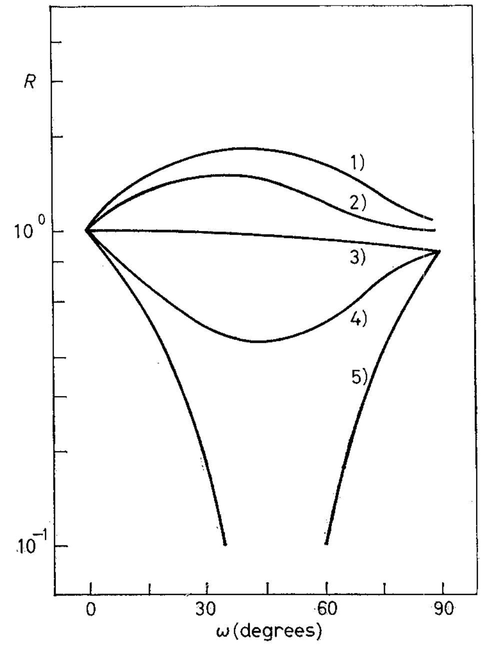

Fig. 1.

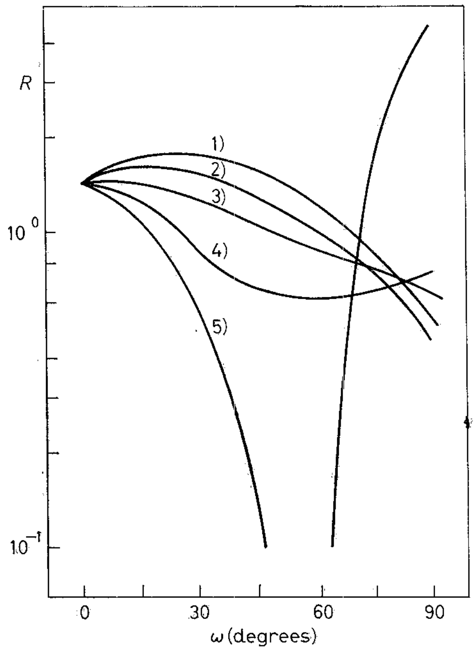

Fig. 2.

Fig. 1. – R is the ratio between formula (1) and the exact cross-sections. The angle φ has the following values: curve 1) $\varphi = 0$, curve 2) $\varphi = 45°$, curve 3) $\varphi = 90°$, curve 4) $\varphi = 135°$, curve 5) $\varphi = 180°$. $\eta = 0.1$, $\bar{\theta} = 90°$.

Fig. 2. – R is the ratio between formula (1) and the exact cross-sections. The angle φ has the following values: curve 1) $\varphi = 0$, curve 2) $\varphi = 45°$, curve 3) $\varphi = 90°$, curve 4) $\varphi = 135°$, curve 5) $\varphi = 180°$. $\eta = 0.4$, $\bar{\theta} = 90°$.

The cross-section is calculated at $\bar{\theta} = 90°$, ω and φ are the latitude and longitude of a polar co-ordinate system where the $\omega = 0$, $\varphi = 0$ axis is in the beam direction and the $\omega = 90°$, $\varphi = 0$ axis is in the direction of the outgoing electron.

One can easily see that our approximation is not too bad: eq. (2) gives, in the average, the correct result. The most prominent departure is given by a destructive interference which causes a zero at $\varphi \sim 180°$, $\omega \sim 45°$.

We can conclude that our result is essentially correct and, therefore, the process $e^+e^- \to e^+e^-\gamma$ gives a relevant contribution to the multiple production of noncollinear particles at large angle in e^+e^- colliding beams.

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([1]) The details of this method can be found in G. PARISI and F. ZIRILLI: to be published.

([2]) S. M. SWANSON: *Phys. Rev.*, **154**, 1601 (1967).