

Laboratori Nazionali di Frascati

LNF-71/48

M. Greco : MULTIPLE C. M. PROCESSES IN HIGH-ENERGY
 e^-e^+ COLLISIONS

Estratto da : Nuovo Cimento 4A, 689 (1971)

Multiple e.m. Processes in High-Energy e^-e^\pm Collisions.

M. GRECO

Laboratori Nazionali del CNEN - Frascati (Roma)

(ricevuto il 16 Febbraio 1971)

Summary. — The production of electron and muon pairs is studied in high-energy e^-e^\pm collisions. Total and differential cross-sections are discussed with special regard to experimental implications. Hadron production via photon-photon scattering is also studied, and suggestions are made for present and near future experiments.

1. — Introduction.

Very recently some theoretical work has been done ⁽¹⁻⁴⁾ to study the electromagnetic processes produced in both e^+e^- and e^-e^\pm colliding beams, via photon-photon scattering. These processes include reactions in which hadrons are involved, or more simply pure electrodynamic reactions in which one or more pairs of electrons and muons are produced. The total cross-sections relative to both groups of processes increase at least as the third power of the logarithm of the energy. Hadron production via the $\gamma\gamma$ scattering is expected therefore to dominate one-photon processes at sufficiently high energies, in spite of the higher order in α . The experimental study of this kind

⁽¹⁾ F. E. LOW: *Phys. Rev.*, **120**, 582 (1960).

⁽²⁾ S. J. BRODSKY, T. KINOSHITA and H. TERAZAWA: *Phys. Rev. Lett.*, **25**, 972 (1970).

⁽³⁾ V. N. BUDNEV and I. F. GINZBURG: *XV International Conference on High-Energy Physics* (Kiev, 1970); Novosibirsk preprint TP-55 (1970).

⁽⁴⁾ A. JACCARINI, N. ARTEAGA-ROMERO, J. PARISI and P. KESSLER: *Lett. Nuovo Cimento*, **4**, 933 (1970).

of processes should provide information on $\gamma\gamma$ total cross-sections, $C = 1$ meson resonances and, more generally, on the electromagnetic properties of hadrons.

On the other hand, the production of electron or muon pairs according to pure QED, in reactions like $e^+e^- \rightarrow e^+e^- + e^+e^-$ etc., of much less interest from a general point of view, has large total cross-sections, increasing at any considerably high energy as $(\ln s)^{n+2}$, for n pairs produced. For that reason this kind of processes can provide large backgrounds and an appreciable degree of contamination with regard to pure hadronic events, unless special kinematical constraints are imposed on the particles produced. The QED production cross-sections, in fact, attain large values corresponding to very-low-energy pairs emitted in the forward-backward direction. At large angles and for quite large effective masses produced the cross-sections decrease considerably and are reduced by some orders of magnitude.

In the present paper a careful study is made of the main electrodynamic processes in order to make easier the interpretation of the experimental results obtained from colliding electron-positron beams. Total and differential cross-sections are discussed with special regard to the experimental implications, so as to estimate the order of magnitude of the effects in actual experiments. In Sect. 2 we study the energy behaviour of the total cross-sections with production of electron and muon pairs, according to pure QED. The cross-sections are evaluated in different ways, in order to show the degree of confidence of the methods of approximation currently used. In Sect. 3 we discuss the production of any hadronic or pure leptonic state via the photon-photon scattering. We derive the differential cross-sections with respect to the angles and the energies of the final particles. The relevance of the motion of the centre of mass of the two photons on the detection of the particles produced is also discussed. In Sect. 4 we study the production of e.m. pairs of high masses at large angles. Particularly we discuss the production of electrons and muons which might be confused with hadrons, for experiments which do not single out unequivocally the particles detected. Some typical examples are given. In Sect. 5, finally, hadron production via $\gamma\gamma$ scattering is discussed in the assumption of pure QED particles, and of meson resonances which can be produced by the two photons and in turn can decay into a multipion state. Suggestions are also made for present and near future experiments.

2. - QED pair production: total cross-sections.

The total cross-sections for production of electron and muon pairs are derived by using different techniques. The results, which differ in the approximation used, agree within a factor 2 ÷ 3 and give the order of magnitude of the effect. Numerical values for the cross-sections are given in Table I.

TABLE I. - *Total cross-sections for e.m. pair production.* The numerical values are computed from eqs. (7), (9), (17) and (19) respectively.

	$E = 1 \text{ GeV}$	$E = 3 \text{ GeV}$	$E = 5 \text{ GeV}$	$E = 10 \text{ GeV}$
$\sigma(e^+e^- \rightarrow e^+e^-e^+e^-)$ (7)	$7 \cdot 10^{-27} \text{ cm}^2$	$1 \cdot 10^{-26} \text{ cm}^2$	$1.24 \cdot 10^{-26} \text{ cm}^2$	$1.6 \cdot 10^{-26} \text{ cm}^2$
$\sigma(e^+e^- \rightarrow e^+e^-\mu^+\mu^-)$ (9)	$4.8 \cdot 10^{-32} \text{ cm}^2$	$0.9 \cdot 10^{-31} \text{ cm}^2$	$1.1 \cdot 10^{-31} \text{ cm}^2$	$1.7 \cdot 10^{-31} \text{ cm}^2$
$\sigma(e^+e^- \rightarrow e^+e^-e^+e^+e^-)$ (17)	$1.3 \cdot 10^{-30} \text{ cm}^2$	$1.9 \cdot 10^{-30} \text{ cm}^2$	$2.3 \cdot 10^{-30} \text{ cm}^2$	$3.5 \cdot 10^{-30} \text{ cm}^2$
$\sigma(e^+e^- \rightarrow e^+e^-e^+e^-\mu^+\mu^-)$ (19)	$2.2 \cdot 10^{-34} \text{ cm}^2$	$3.2 \cdot 10^{-34} \text{ cm}^2$	$5.8 \cdot 10^{-34} \text{ cm}^2$	$5.8 \cdot 10^{-34} \text{ cm}^2$

$e^+e^- \rightarrow e^+e^-e^+e^-$. - The first derivation of the total cross-section is due to LANDAU and LIFSHITZ⁽⁵⁾:

$$(1) \quad \sigma = \frac{28}{27} \frac{\alpha^2}{\pi} (z_1 z_2)^2 r_0^2 \log^3 \left(\frac{s}{2m_1 m_2} \right).$$

Equation (1) refers to e^+e^- pair production in a collision of two fast charged particles, of charges $z_1 e$ and $z_2 e$, masses m_1 and m_2 , and total energy s . Equation (1) is calculated by replacing the effect of the colliding particles by an external field. Exchange terms must be taken into account when the colliding particles are electrons or positrons, so that eq. (1) holds only as an order of magnitude.

A perhaps better estimate for σ is obtained by regarding the above processes as pair production in the collision of one electron with the equivalent distribution of photons of the other electron,

$$(2) \quad d\sigma = \int \sigma(k) n(k) dk,$$

where

$$(3) \quad n(k) dk = \frac{2\alpha}{\pi} \log \left[\frac{2E(E-k)}{mk} \right] \frac{E^2 + (E-k)^2}{2E^2} \frac{dk}{k}$$

and

$$(4) \quad \sigma(k) = \frac{28}{9} \alpha r_0^2 \log \frac{2k}{m}.$$

All the energies are expressed in the rest frame of one electron. By approximating $\log(2E(E-k)/mk) \simeq \log(E/m)$ one gets

$$(5a) \quad \sigma = \frac{28}{9} \frac{\alpha^2}{\pi} r_0^2 \log^3 \left(\frac{s}{2m^2} \right),$$

⁽⁵⁾ L. D. LANDAU and E. M. LIFSHITZ: *Phys. Zeits. Sowjet.*, **6**, 244 (1934).

while in the case of a « classical » spectrum of photons

$$n(k) dk \simeq \frac{2\alpha}{\pi} \log\left(\frac{E}{k}\right) \frac{dk}{k}$$

$$(5b) \quad \sigma = \frac{28}{27} \frac{\alpha^2}{\pi} r_0^2 \log^3\left(\frac{s}{2m^2}\right).$$

The method of impact diagrams by CHENG and WU⁽⁶⁾ gives a further and simple way to compute the total cross-section for any production process. The e^+e^- pair production cross-section in particular is related to the forward-scattering amplitude for e^+e^- elastic scattering, via one electron pair, by the optical

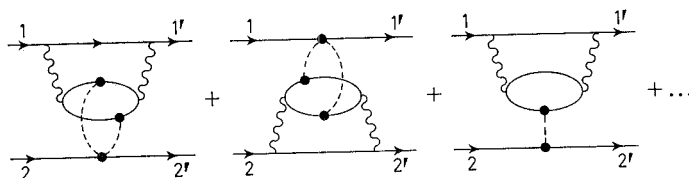


Fig. 1.

theorem. With the notation of ref. (6) the forward-scattering amplitude corresponding to the diagrams of Fig. 1, in the lowest order of e , is given by

$$(6) \quad \mathcal{M}(\mathbf{r}_1 = 0) = 4is \log \gamma \frac{\alpha^4}{\pi^3} \frac{1}{m^2} \delta_{12} \delta_{1'2'} \int \frac{d\mathbf{q}_\perp}{\mathbf{q}_\perp^2} \frac{d\mathbf{q}'_\perp}{\mathbf{q}'_\perp^2} \cdot \left\{ \int_0^1 dx \int_0^1 dy \frac{x(1-x) + y(1-y) - 5x(1-x)y(1-y)}{x(1-x)q_\perp^2 + y(1-y)q'_\perp{}^2 + m^2} \right\} \simeq \frac{28}{9} is \frac{\alpha^4}{\pi} \frac{1}{m^4} \log^3 \gamma \delta_{12} \delta_{1'2'},$$

where r_1 is the total momentum transfer, $\gamma = E/m$, E being the energy of the initial electrons in their centre-of-mass system, and δ_{12} , $\delta_{1'2'}$ are the Kronecker δ 's in spin. By the optical theorem one gets

$$(7) \quad \sigma \simeq \frac{4 \times 28}{9} \frac{\alpha^2}{\pi} r_0^2 \log^3 \gamma.$$

The eqs. (1), (5a), (5b) and (7) are in a quite good agreement within a factor $2 \div 3$. As one can see from Table I, the cross-sections are of order $(10^{-26} \div 10^{-27}) \text{ cm}^2$ for c.m. energies ranging within $(1 \div 10) \text{ GeV}$. Such large cross-sections reduce, however, by many orders of magnitude in the region of large angles and high masses of final electrons, as will be shown in the Sect. 4.

(6) H. CHENG and T. T. WU: *Phys. Rev. D*, **1**, 1069, 2775 (1970), and previous articles quoted there.

$e^+e^- \rightarrow e^+e^-\mu^+\mu^-$. - The total cross-section for this process can be computed by the method of impact diagrams together with the optical theorem, as above, by calculating the forward-scattering amplitude for e^+e^- elastic scattering, via one muon pair.

One gets

$$(8) \quad \mathcal{M}(\mathbf{r}_1 = 0) \simeq \frac{28}{9} i s \frac{\alpha^4}{\pi} \frac{1}{m^2 \mu^2} \log \frac{E}{\mu} \log^2 \frac{E}{m} \delta_{12} \delta_{1'2'}$$

and

$$(9) \quad \sigma \simeq 4 \times \frac{28}{9} \frac{\alpha^4}{\pi} \frac{1}{\mu^2} \log \frac{E}{\mu} \log^2 \frac{E}{m},$$

where m and μ are the electron and muon masses respectively.

A different approach can also be used, which is related to the high-energy photon-photon scattering. This approach will be discussed in the next Section, where we derive the following formula for the total cross-section:

$$(10) \quad \sigma \simeq \left(\frac{\alpha}{\pi}\right)^2 \log^2 \frac{E}{m} \int_{4\mu^2}^{4E^2} \frac{ds}{s} \sigma_{\gamma\gamma \rightarrow \mu^+\mu^-}(s) \cdot \left\{ \left(2 + \frac{s}{4E^2}\right)^2 \log \frac{4E^2}{s} - 2 \left(1 - \frac{s}{4E^2}\right) \left(3 + \frac{s}{4E^2}\right) \right\},$$

where

$$(11) \quad \sigma_{\gamma\gamma \rightarrow \mu^+\mu^-}(s) = \frac{4\pi\alpha^2}{s} \left[\left(1 + \frac{4\mu^2}{s} - \frac{8\mu^4}{s^2}\right) \log \frac{1 + \sqrt{1 - 4\mu^2/s}}{1 - \sqrt{1 - 4\mu^2/s}} - \sqrt{1 - \frac{4\mu^2}{s}} \left(1 + \frac{4\mu^2}{s}\right) \right] \simeq \frac{4\pi\alpha^2}{s} \sqrt{1 - 4\mu^2/s}.$$

The last approximate equality holds because the integral in the r.h.s. of (10) is dominated by the region $s \sim 4\mu^2$. One finds

$$(12) \quad \sigma \simeq \frac{2}{3} \frac{\alpha^4}{\pi\mu^2} \log^2 \left(\frac{E}{m}\right) \left(8 \log \frac{E}{\mu} - 6\right).$$

Equations (9) and (12) at high energies differ of about a factor of 2. Numerical values for σ are given in Table I.

$e^+e^- \rightarrow e^+e^- + 2(e^+e^-)$. - The total cross-section for this process has been computed by SERBO^(7,8), using the method of equivalent photons,

$$(13) \quad \sigma(e^\pm e^- \rightarrow e^\pm e^- + 2(e^+e^-)) = \frac{1}{6} \left(\frac{\alpha}{\pi}\right)^2 \sigma(\gamma\gamma \rightarrow 2e^+e^-) \log^4 \left(\frac{s}{m_e^2}\right),$$

(7) V. G. SERBO: *JETP Lett.*, **12**, 39 (1970).

(8) J. J. SAKURAI: invited paper presented at *Balaton Symposium on Hadron Spectroscopy* (1970).

where

$$(14) \quad \sigma(\gamma\gamma \rightarrow 2e^+e^-) = \frac{\alpha^4}{9\pi m_e^2} \left(\frac{175}{4} \zeta(3) - \frac{19}{2} \right) \simeq 6 \cdot 10^{-30} \text{ cm}^2.$$

Equation (13), however, should be taken only as an order of magnitude, because of the identity of the target with the particles produced. We give therefore a second derivation for the total cross-section, using again the method of impact

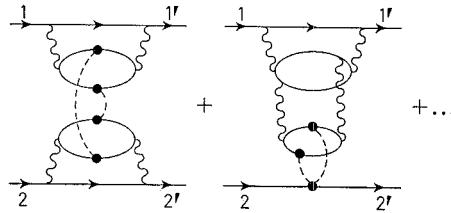


Fig. 2.

diagrams. From ref. (6), the amplitude relative to the diagrams of Fig. 2 is given by

$$(15) \quad \mathcal{M}(\mathbf{r}_1 = 0) = 2 is \log^2 \gamma (2\pi)^{-6} \int \frac{dq_{1\perp}}{q_{1\perp}^4} \int \frac{dq_{2\perp}}{q_{2\perp}^4} \int \frac{dq_{3\perp}}{q_{3\perp}^4} \cdot K(0, q_{1\perp}, q_{2\perp}) K(0, q_{2\perp}, q_{3\perp}) \frac{1}{4} e^4 m^{-2} \delta_{11'} \delta_{22'},$$

where

$$(16) \quad K(0, q_{1\perp}, q_{2\perp}) = \frac{4e^4}{(2\pi)^3} q_{1\perp}^2 q_{2\perp}^2 \cdot \int_0^1 dx \int_0^1 dy \frac{x(1-x) + y(1-y) - 5xy(1-x)(1-y)}{x(1-x)q_{1\perp}^2 + y(1-y)q_{2\perp}^2 + m^2}.$$

We have approximately

$$(17) \quad \sigma \simeq \frac{6\alpha^4 r_0^2}{\pi^3} \log^4 \left(\frac{s}{m^2} \right),$$

which is a factor 2.8 higher than (13). In the Sect. 4 we shall discuss the factor $1/m_e^2$, which fixes the scale of the cross-section for this process.

$e^+e^- \rightarrow e^+e^- + e^+e^-\mu^+\mu^-$. — An estimate of the cross-section for this process can be obtained using the method of equivalent photons, from the knowledge of $\sigma(\gamma\gamma \rightarrow e^+e^-\mu^+\mu^-)$. This cross-section has been obtained by MASUJIMA (9),

(9) M. MASUJIMA: *Nucl. Phys.*, **24** B, 182 (1970).

using the Cheng and Wu techniques:

$$(18) \quad \sigma(\gamma\gamma \rightarrow e^+e^-\mu^+\mu^-) = \frac{2\alpha^4}{27\pi} \frac{1}{m_\mu^2} \left\{ 7 \log^2 \left(\frac{m_\mu^2}{m_e^2} \right) + \frac{103}{3} \log \frac{m_\mu^2}{m_e^2} + \frac{985}{9} \right\} \simeq 2.82 \cdot 10^{-33} \text{ cm}^2.$$

In analogy with (13) one gets

$$(19) \quad \sigma(e^\pm e^- \rightarrow e^\pm e^- + e^+e^-\mu^+\mu^-) \simeq \frac{2.82}{6} \frac{\alpha^2}{\pi^2} \log^4 \left(\frac{s}{m_e^2} \right) \cdot 10^{-33} \text{ cm}^2,$$

which is more than a thousand times smaller than $\sigma(e^\pm e^- \rightarrow e^\pm e^- + 2(e^+e^-))$.

All the cross-sections for the processes studied increase with the energy with some power of $\ln s$. More generally, the cross-section for n pair production in $e^{(\mp)}e^-$ collisions goes as $(\ln s)^{n+2}$ in the lowest order of perturbation theory. If further diagrams with fermion loops are included, the Froissart bound has been shown⁽¹⁰⁾ to be saturated at infinite energy. More explicitly, the perturbation series loses its meaning when $\alpha^2 \ln(s/m_e^2) \sim 1$, which is however far enough from any conceivable high-energy future experiment.

3. - Production of any state f via $\gamma\gamma$ scattering.

Let's discuss the production of any state f via the photon-photon scattering, according to the diagram of Fig. 3. For the validity of the formulae below, if the state f contains one or more pairs of electrons, they should be kinema-

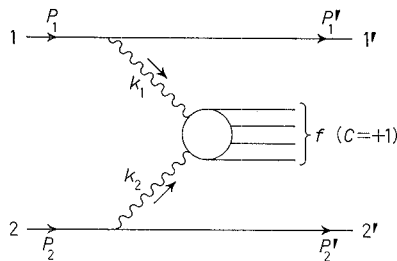


Fig. 3.

tically separable from the particles $1'$ and $2'$, which are strongly peaked in the forward-backward direction. The cross-sections of processes of the type shown in Fig. 3 in fact, are large only where the virtual photons k_1 and k_2 have small masses; this means that the initial electrons tend to proceed in the beam direction.

⁽¹⁰⁾ H. CHENG and T. T. WU: *Phys. Rev. Lett.*, **24**, 1456 (1970).

We have checked out the validity of the double Weizsäcker-Williams approximation by studying the photon-photon scattering in the representation of the helicity amplitudes. The main contribution to the cross-sections come from transverse photons, and in the case in which the final electrons are detected within small angles $\theta_{1\max}$ and $\theta_{2\max}$ ($\theta_{1,2\max}^2 \approx m/E$) with respect to the initial beam direction the result is

$$(20) \quad d\sigma(e^{(\pm)}e^- \rightarrow e^{(\pm)}e^-f) = \left(\frac{2\alpha}{\pi}\right)^2 \frac{dk_1}{k_1} \frac{dk_2}{k_2} \cdot \\ \cdot \left\{ \log \frac{2EE'}{mk_1} + \frac{1}{2} \log \left(\frac{1 - \cos \theta_{1\max}}{2} \right) - \frac{EE'}{E^2 + E'^2} \right\} \cdot \\ \cdot \left\{ \log \frac{2EE''}{mk_2} + \frac{1}{2} \log \left(\frac{1 - \cos \theta_{2\max}}{2} \right) - \frac{EE''}{E^2 + E''^2} \right\} \cdot \\ \cdot \frac{E^2 + E'^2}{2E^2} \frac{E^2 + E''^2}{2E^2} d\sigma(\gamma\gamma \rightarrow f),$$

where E , $E' = (E - k_1)$ and $E'' = (E - k_2)$ are the energies of the initial and final electrons respectively in the c.m. system.

An approximate formula which gives the order of magnitude of the cross-sections is obtained by taking only the leading term $\log(E/m)$ from the full logarithmic dependence. In these approximations we get for the total cross-section

$$(21) \quad \sigma(e^{(\mp)}e^- \rightarrow e^{(\mp)}e^-f) \simeq \left(\frac{\alpha}{\pi}\right)^2 \log^2 \left(\frac{E}{m}\right) \int_{s_{\text{th}}}^{4E^2} \frac{ds}{s} \sigma_{\gamma\gamma}^f(s) \cdot \\ \cdot \left\{ \left(2 + \frac{s}{4E^2}\right)^2 \log \frac{4E^2}{s} - 2 \left(1 - \frac{s}{4E^2}\right) \left(3 + \frac{s}{4E^2}\right) \right\},$$

where $\sigma_{\gamma\gamma}^f(s)$ is the cross-section for production of f by the two photons of c.m. energy squared s . Equation (21) is valid if the experimental apparatus does not impose cuts on the velocity of the centre of mass of the two photons. If this is the case eq. (21) becomes

$$(22) \quad \sigma(e^{(\mp)}e^- \rightarrow e^{(\mp)}e^-f) \simeq \left(\frac{\alpha}{\pi}\right)^2 \log^2 \left(\frac{E}{m}\right) \int_{s_{\text{th}}}^{4E^2} \frac{ds}{s} \sigma_{\gamma\gamma}^f(s) \cdot \\ \cdot \left\{ (2 + x_0)^2 \log \sqrt{x_0} \sqrt{\gamma} - 2 \sqrt{x_0} \sqrt{\gamma} (2 + x_0) + \right. \\ \left. + x_0 \gamma + 2x_0(2 + x_0) \frac{1}{\sqrt{x_0} \sqrt{\gamma}} - \frac{x_0}{\gamma} \right\}_{\gamma_{\min}}^{\gamma_{\max}},$$

where $\{F(\gamma)\}_{\gamma_{\min}}^{\gamma_{\max}} \equiv \{F(\gamma_{\max}) - F(\gamma_{\min})\}$, $x_0 = s/4E^2$ and $\gamma \equiv (1 + \beta)/(1 - \beta) = 4k_1^2/s$, with $\beta = (k_1 - k_2)/(k_1 + k_2)$. In the absence of cuts, the range of variability of γ is given by $\gamma_{\max} = 1/\gamma_{\min} = 4E^2/s$.

4. - Large-angle and high-mass pair production.

In Sect. 2 we have studied the total cross-sections for the main processes of electron and muon pair production and we have shown that they attain values comparable to and in some cases much larger than the production of single pairs via the one-photon channel. These large cross-sections, however, correspond to very-low-energy pairs emitted mainly in the forward-backward direction. In this Section we estimate the order of magnitude of the effect of these processes on experiments which observe hadron production at large angles as those performed by the Frascati groups ^(11,12). Let us consider first the production at large angles of two particles coplanar with the beam direction. We shall use the results of the preceding Section.

For the reaction $e^{(\mp)}e^- \rightarrow e^{(\mp)}e^- + e^+e^-$, which has the largest total cross-section, the angular distribution of the pair produced in the c.m. by the two virtual photons is given by

$$(23) \quad d\sigma = \frac{2\pi\alpha^2}{s} \beta \frac{1 - \beta^4 \cos^4 \theta + 8(m^2/s)\beta^2 \sin^2 \theta}{(1 - \beta^2 \cos^2 \theta)^2} \sin \theta d\theta,$$

where $\beta^2 = 1 - 4m^2/s$. By integrating (23) over the range of interest, for example $60^\circ \leq \theta \leq 120^\circ$, and combining with (21), the result is

$$(24) \quad \sigma_{\text{exp}}^{e^+e^-} \simeq \frac{\alpha^4}{\pi} \log^2 \left(\frac{E}{m} \right) \frac{1}{s_{\text{th}}} \left(4 \log \frac{4E^2}{s_{\text{th}}} - 6 \right).$$

An additional factor $\frac{1}{4}$, due to the identity of the final particles, has been included in (21). At $E = 1 \text{ GeV}$, and $s_{\text{th}} = 4 \cdot 10^{-2} (\text{GeV})^2$ one has $\sigma_{\text{exp}}^{e^+e^-} \simeq$

⁽¹¹⁾ G. BARBIELLINI, B. BORGIA, M. CONVERSI, M. GRILLI, A. MULACHIÈ, G. NICOLETTI, L. PAOLUZI, W. SCANDALE, P. SPILLANTINI and R. VISENTIN: paper submitted to the *XV International Conference on High-Energy Physics* (Kiev, 1970), and Frascati report LNF-70/38 (1970), p. 3; G. BARBIELLINI, B. BORGIA, F. CERADINI, F. GRIANTI, M. GRILLI, L. PAOLUZI, R. SANTONICO and R. VISENTIN: paper submitted to the *XV International Conference on High-Energy Physics* (Kiev, 1970) and Frascati report LNF-70/38 (1970), p. 21; G. BARBIELLINI, M. CONVERSI, M. GRILLI, A. MULACHIÈ, M. NIGRO, L. PAOLUZI, S. SPILLANTINI, R. VISENTIN and G. T. ZORN: paper submitted to the *XV International Conference on High-Energy Physics* (Kiev, 1970) and Frascati report LNF-70/38 (1970), p. 31.

⁽¹²⁾ B. BARTOLI, B. COLUZZI, F. FELICETTI, G. GOGGI, G. MARINI, F. MASSA, D. SCANNICCHIO, V. SILVESTRINI and F. VANOLI: paper submitted to the *XV International Conference on High-Energy Physics* (Kiev, 1970); *Nuovo Cimento*, **70** A, 615 (1970).

$\simeq 0.6 \cdot 10^{-32} \text{ cm}^2$. In the case of an experimental cut on the velocity of the c.m. of the two photons, eq. (24) has to be replaced by

$$(25) \quad \sigma_{\text{exp}}^{e^+e^-} \simeq \frac{\alpha^4}{\pi} \log^2 \left(\frac{E}{m} \right) \frac{1}{s_{\text{th}}} 4 \log \gamma_{\text{max}}.$$

For example, for $|\beta| \leq 0.5$, at $E = 1 \text{ GeV}$ and $s_{\text{th}} = 4 \cdot 10^{-2} (\text{GeV})^2$ one gets $\sigma_{\text{exp}}^{e^+e^-} \simeq 0.2 \cdot 10^{-32} \text{ cm}^2$.

Quite similar arguments hold for the reaction $e^{(\mp)}e^- \rightarrow e^{(\mp)}e^-\mu^+\mu^-$. For example, at $E = 1 \text{ GeV}$ for $60^\circ < \theta < 120^\circ$, and $200 \text{ MeV} \leq E_\mu \leq 700 \text{ MeV}$, one has $\sigma_{\text{exp}}^{\mu^+\mu^-} \simeq 0.3 \cdot 10^{-32} \text{ cm}^2$. Furthermore, for $|\beta| \leq 0.5$ and $\beta < 0.05$ one gets $\sigma_{\text{exp}}^{\mu^+\mu^-} = 0.2 \cdot 10^{-32} \text{ cm}^2$ and $0.2 \cdot 10^{-33} \text{ cm}^2$ respectively. The latter situation corresponds to almost collinear muons in the colliding-beam frame.

We have only studied up to now the production of e.m. pairs coplanar with the beam direction. In order to estimate the effect of noncoplanar pairs we consider the process shown in Fig. 4, where the incoming photon belongs

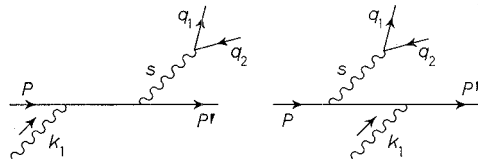


Fig. 4.

to the bremsstrahlung spectrum of the other electron. By integrating over the whole of the solid angle of the pair and the field electron, one has

$$(26) \quad d\sigma = \frac{\alpha^3}{12} \frac{ds}{s^2} \frac{1}{\omega^2} \frac{4\omega^2 - s}{4\omega^2} \log \frac{4E'^2}{m^2} \left\{ \omega^2 (4\beta_q + 3 - \beta_q^2) + s \left(\frac{\beta_q}{2} - 2 - \frac{\beta_q^2}{2} \right) - \frac{s^2}{8\omega^2} \left(\frac{7}{2} + \beta_q + \frac{\beta_q^2}{2} \right) + \frac{4\omega^2 s}{4\omega^2 - s} \right\},$$

where $E' = (4\omega^2 - s)/4\omega$, $\beta_q = (4\omega^2 - s)/(4\omega^2 + s)$ and ω is the energy of the photon in the electron-photon c.m. system. For fixed ω , the cross-section will be maximized at small s , and therefore, by neglecting s with respect to ω^2 , we have

$$(27) \quad d\sigma \simeq \frac{\alpha^3}{2} \frac{ds}{s^2} \log \frac{4\omega^2}{m^2} = \frac{\alpha^3}{2} \frac{ds}{s^2} \log \frac{4Ek_1}{m^2},$$

where now E and k_1 are the energies of the electron and the photon in the electron-positron c.m. system. By integrating over the photon spectrum one

gets

$$(28) \quad \sigma(e^\pm e^- \rightarrow e^\pm e^- (e^+ e^-)_{n. cop.}) \simeq \frac{2\alpha}{\pi} \log\left(\frac{E}{m}\right) \frac{\alpha^3}{2} \cdot$$

$$\int_{s_{\min}}^{4E^2} \frac{ds}{s^2} \int_{s/4E}^E \frac{E^2 + (E-k)^2}{2E^2} \frac{dk}{k} \log \frac{4Ek}{m^2} = \frac{2\alpha^4}{\pi} \log^2\left(\frac{E}{m}\right) \frac{1}{s_{\min}} \log\left(\frac{4E^2}{s_{\min}}\right) \simeq 3 \cdot 10^{-33},$$

where the last equality is obtained at $E = 1$ GeV and $s_{\min} = 4 \cdot 10^{-2} (\text{GeV})^2$. Equation (28) gives an upper limit to the large-angle noncoplanar pair production, because of the integration over all the solid angle of the pair.

As far as the double electron pair production is concerned (this process is represented by Fig. 5) one can note the following. From eq. (17) it follows

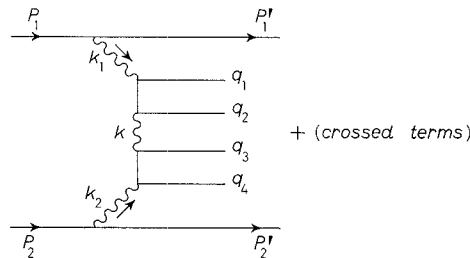


Fig. 5.

that large values of the cross-section, in spite of sixth powers of α , come from a factor m_e^{-2} together with a fourth power of $\log(E/m)$. The factor m_e^{-2} comes from the internal photon propagator. Now it is easily seen that k^2 can attain values comparable with m_e^2 only if the pairs are produced with small mass squared and along the beam direction. Both these conditions cannot be satisfied if one is interested at quite large masses produced at large angles. For the same reason it is very reasonable to lose a factor $\log^2(s/m^2)$, in favour of $\log(s/s_1) \cdot \log(s/s_2)$ in the best case, where s_1 and s_2 are the quadratic masses of the two pairs. We expect, therefore, in the kinematical conditions of actual experiments, a contribution of the type

$$(29) \quad \sigma \sim \frac{\alpha^6}{\pi^3} \frac{1}{s_{\min}} \log^2\left(\frac{E}{m}\right) \log\left(\frac{s}{s_1}\right) \log\left(\frac{s}{s_2}\right),$$

which gives values $10^{-5} \div 10^{-6}$ smaller than (17).

From all the above considerations it follows that the main contribution to large-angle events comes from an electron or muon pair with a rate

$$(30) \quad \sigma \sim \frac{\alpha^4}{\pi} \frac{1}{s_{\min}} \log^2\left(\frac{E}{m}\right),$$

which is quite negligible for the Frascati experiments. The threshold dependence of (30), together with the analysis of radiative corrections to Bhabha scattering can be useful to recognize and reduce this kind of effects.

5. - Hadron production in $\gamma\gamma$ collisions.

Hadron production in e^+e^- collision does occur via the $\gamma\gamma$ interaction as discussed previously by several authors⁽¹⁻⁴⁾, with a rate given by (21) or (22), once $\sigma_{\gamma\gamma}^{\text{hadrons}}(s)$ is given. In the simplest case of $\pi^+\pi^-$ production, assuming pointlike pions one gets

$$(31) \quad \sigma_{\text{tot}}(e^+e^- \rightarrow e^+e^-\pi^+\pi^-) \simeq \frac{1}{3} \frac{\alpha^4}{\pi m_\pi^2} \log^2\left(\frac{E}{m}\right) \left(8 \log\left(\frac{E}{m_\pi}\right) - 6\right),$$

which is $3 \cdot 10^{-33} \text{ cm}^2$ at $E = 1 \text{ GeV}$. This value becomes $\sim 2 \cdot 10^{-34} \text{ cm}^2$ when $E_\pi \geq 250 \text{ MeV}$, $60 \leq \theta_{\text{c.m.}} \leq 120$ and $|\beta| \lesssim 0.5$.

The possibility that a strong resonance with $J = 0$ or 2 and $C = +1$ can be produced by the two almost-real photons and which could subsequently decay, gives a source of multipion final states and offers a simple way to measure partial and total decay rates⁽¹⁾. This can be seen from simple considerations based on a Breit-Wigner description in the vicinity of the resonance:

$$(32) \quad \sigma_{\gamma\gamma}^{\text{R}}(s) = 8\pi(2J+1) \frac{\Gamma_{2\gamma} \Gamma}{(s-M^2)^2 + \Gamma^2 M^2}$$

where $\Gamma_{2\gamma}$ is the decay rate of the resonance of mass M and width Γ into 2γ . Combining (21) and (32), one sees that at least in principle the width can be measured by detecting in coincidence the outgoing electrons scattered at very small angles, by measuring the energy loss and plotting the events against s . In any actual experiment, however, the measured quantity will be the integral of the cross-section in a region determined by the experimental resolution and if the resonance is very narrow, with a width much smaller than the experimental energy resolution one gets from (21) and (32)

$$(33) \quad \sigma(e^{(\mp)}e^- \rightarrow e^\pm e^- \text{R}) \simeq \frac{8\alpha^2}{M^3} (2J+1) \Gamma_{2\gamma} \log^2\left(\frac{E}{m}\right) \cdot \left\{ \left(2 + \frac{M^2}{4E^2}\right)^2 \log \frac{4E^2}{M^2} - 2 \left(1 - \frac{M^2}{4E^2}\right) \left(3 + \frac{M^2}{4E^2}\right) \right\}.$$

At $E = 1 \text{ GeV}$, for example, for the η -meson, one has $\sigma \simeq 3.2 \cdot 10^{-34} \text{ cm}^2$, and, for the $X^0(958)$ or η' , $\sigma \simeq 2.70 \cdot 10^{-35} \text{ cm}^2 (\text{keV})^{-1} \Gamma_{2\gamma}$. For the η' a direct measurement of the $\eta' \rightarrow \gamma\gamma$ branching ratio has been made and turns out to be

~ 0.1 ⁽¹³⁾. It follows, therefore, that an observation of the above effect will lead to a direct measure of the total width, or at least to reduce the present upper limit of 4 MeV. Similar arguments hold for all mesonic resonances with the 2γ quantum numbers.

The $\gamma\gamma$ total cross-section into hadrons can be obtained from (21) and (22) by detecting in coincidence the final electrons. Due to the form of the mass spectrum dM^2/M^2 , the direction of the photon momenta, and the small transverse-momentum distribution, which is a characteristic of all high-energy hadronic reactions, the hadrons produced will more usually be along the beam direction. A theoretical estimate of $\sigma_{\gamma\gamma}^{\text{total}}$ is highly model dependent. A pure Q.E. analogy will give asymptotically constant cross-sections, in agreement with a crude factorization hypothesis of strong cross-sections at high energies. A logarithmic dependence of the total cross-sections $\sigma(e^+e^- \rightarrow e^+e^- \text{ hadrons})$ from the threshold energy s_{th} is the experimental observable consequence of the constancy of $\sigma(\gamma\gamma \rightarrow \text{hadrons})$. For production of high hadron masses at large angles however, it is quite conceivable to expect a rate of the order of (30), as in pure e.m. pair production. It follows that at sufficiently high energies one expects in electron-positron collisions a large contamination between hadrons produced via the one-photon channel and the $\gamma\gamma$ mechanism. It would therefore be highly desirable to have both electron-electron and electron-positron experiments running in the next future rings in order to separate the contribution of the $\gamma\gamma$ interaction and subtract it out in studying the deep e^+e^- annihilation into hadrons.

* * *

We wish to thank Prof. C. BERNARDINI for comments and for a critical reading of the manuscript. We are grateful also to the experimental groups of the Frascati Storage Ring ADONE for discussions on the status of the experimental situation.

⁽¹³⁾ M. ROOS, C. BRICMAN, A. BARBARO-GALTIERI, L. R. PRICE, A. RITTENBERG, A. H. ROSENFELD, N. BARASH-SCHMIDT, P. SÖDING, CHIH YUNG CHIEN, C. G. WOHL and T. LASINSKI: *Phys. Lett.*, **33** B, 1 (1970).

● RIASSUNTO

Si studia la produzione di coppie di elettroni e muoni nelle collisioni di e^-e^\pm ad alta energia. Si discutono le sezioni d'urto totali e differenziali con particolare riguardo alle implicazioni sperimentali. Si studia anche la produzione di adroni tramite lo scattering fotone-fotone, e si danno alcuni suggerimenti riguardanti esperimenti presenti e del prossimo futuro.

Множественные электромагнитные процессы в e^-e^\pm соударениях при высоких энергиях.

Резюме (*). — Исследуется рождение электронных и мюонных пар в e^-e^\pm соударениях при высоких энергиях. Обсуждаются полное и дифференциальное поперечные сечения с точки зрения экспериментальных приложений. Также исследуется рождение адронов через фотон-фотонное рассеяние. Высказываются предположения относительно экспериментов в настоящем и ближайшем будущем.

(*) *Переведено редакцией.*