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Comments on Minimum Volume Simple and Compensated Superconducting Coils

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The increasing use of superconducting magnets in many fields of research and technology raises more and more often the question: Which is the cheapest magnet for a given field, subject to constraints such as field homogeneity or radial access? In this paper minimum volume coils of three types are analyzed: simple (Fabry), sixth order compensated (outside notch corrected), split (Helmholtz) of internal radii from 1 to 10 cm, and axial apertures (of the Helmholtz coils) from 0.5 to 10 cm. To obtain a given field with a given effective current density $J\lambda$, about the same amount of wire is necessary for a Fabry coil of internal radius $2a_1$, a sixth order or split coil of internal radii a_1, or a split coil with window aperture a_1.

INTRODUCTION

THE problem of determining the minimum volume of conductor necessary for a prescribed central field H_{0z}, inner radius a_1, and uniform current density $J\lambda$, has been discussed by Fabry[1] for simple cylindrical solenoids, by Girard and Sauzade[2] for sixth order compensated coils with outside notch correction, and by Day[3] for split coils compensated to fourth order by a radial access gap (Helmholtz coils). Fabry's calculations are useful for the design of superconducting generators and motors,[4,5] Girard's and Sauzade's in the design of spectrometers or NMR apparatus, and Day's for high energy physics bubble chamber facilities and superconducting gyroscopes.

Surprisingly enough, to the knowledge of the authors no comparison has been made between the relative wire volumes (and therefore lengths) necessary in each of the three cases. The purpose of this paper is to make such comparison, after first noting the effect of the local field seen by superconducting windings.

THE INFLUENCE OF GEOMETRICAL PARAMETERS ON THE LOCAL FIELD OF SUPER-CONDUCTING WINDINGS

In any magnet, the maximum field seen by the superconductor is higher than the field at the center of the bore, and the maximum superconducting current (critical current, J_c) that can be carried is correspondingly less. Cost optimization (and calculations of magnetic forces acting upon the windings) therefore requires a knowledge of the local magnetic fields.[6]

For a simple cylindrical solenoid the highest field at a distance r from the axis is located on the solenoid midplane. The axial field component $H_z(r)$ at a point r on this plane is related to the central field $H_{0z}=H_{\text{center}}$ by

$$H_z(r)/H_{\text{center}}=r[S_z(b/r,a_2/r)-S_z(b/r,a_1/r)]/a_1[\alpha T(\alpha/\beta)-T(1/\beta)], \quad (1)$$

where a_1 is the magnet inner radius, a_2 the outer radius, b the half-length, α the a_2/a_1, and β the b/a_1. S_z and T, which involve elliptic integrals, can be found in Table I A, B, and C, and in Table II of Hart's book.[7] Figure 1 is a plot of the ratio $H_{\text{deepest layer}}/H_{\text{center}}$, calculated with the help of these tables, as a function of the magnet parameters α and β. This figure, and the critical current vs field characteristic of the superconductor, determine the actual current density which can be carried at a given central field.

FABRY'S MINIMUM VOLUME COILS

It has been well known for nearly a century that for a square ended cylindrical solenoid the axial field at the

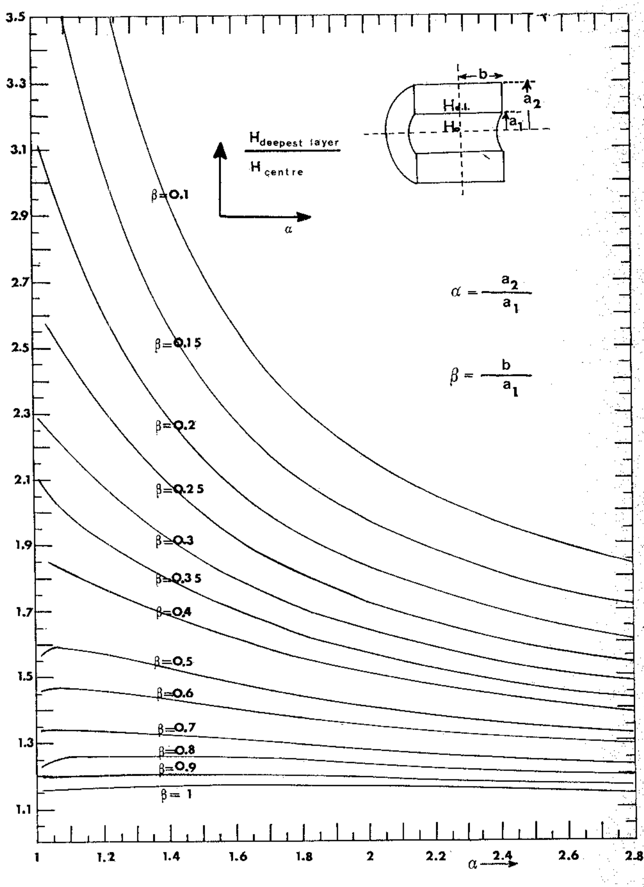

FIG. 1. Influence of the coil parameters on the ratio $H_{\text{deepest layer}}/H_{\text{center}}$.

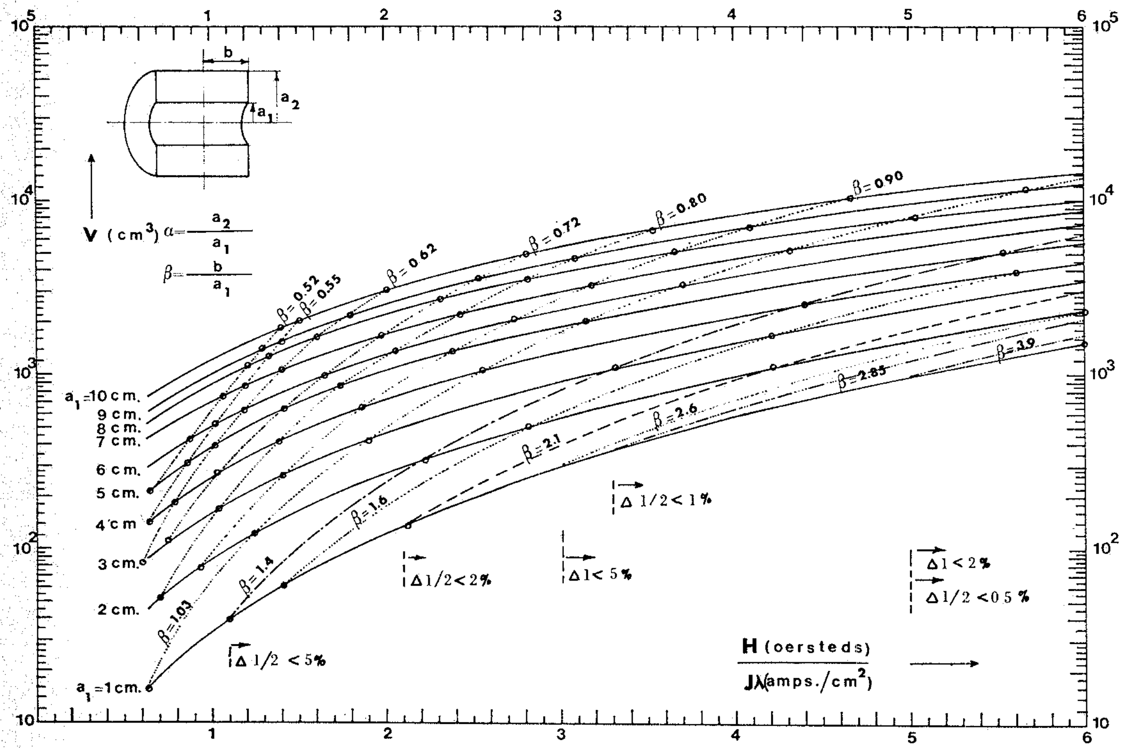

FIG. 2. Minimum volume (cm^3) necessary to obtain a magnetic field H with a current density $J\lambda$ and inner radius a_1, for a simple cylindrical solenoid.

center H_{0z} is given by

$$H_{0z}=(4\pi/10)bJ\lambda \log_e\{[\alpha+(\alpha^2+\beta^2)^{\frac{1}{2}}]/[1+(1+\beta^2)^{\frac{1}{2}}]\}, \quad (2)$$

where λ = filling factor = conductor cross section/total cross section, and $J\lambda$ = effective current density. The magnetic field strength is in oersteds when dimensions are in centimeters and the current density is in amperes per square centimeter.

If S is the cross section of the wire, the volume V of

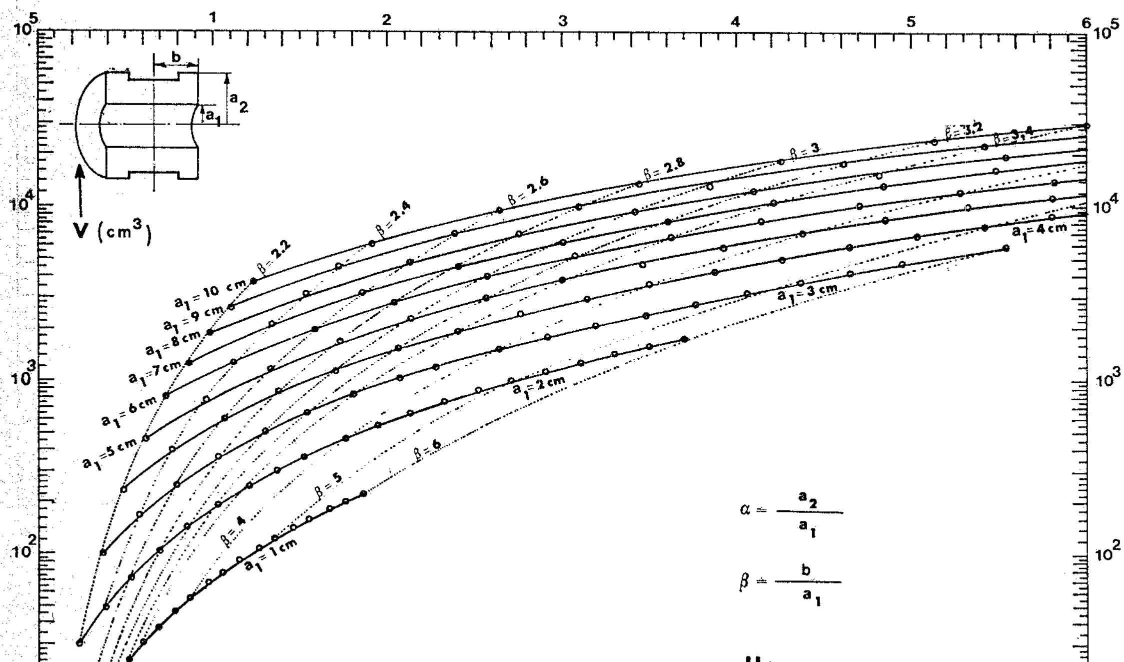

FIG. 3. Minimum volume (cm^3) necessary to obtain a central field H with a current density $J\lambda$ and inner radius a_1, for a sixth order compensated solenoid (outside notch correction).

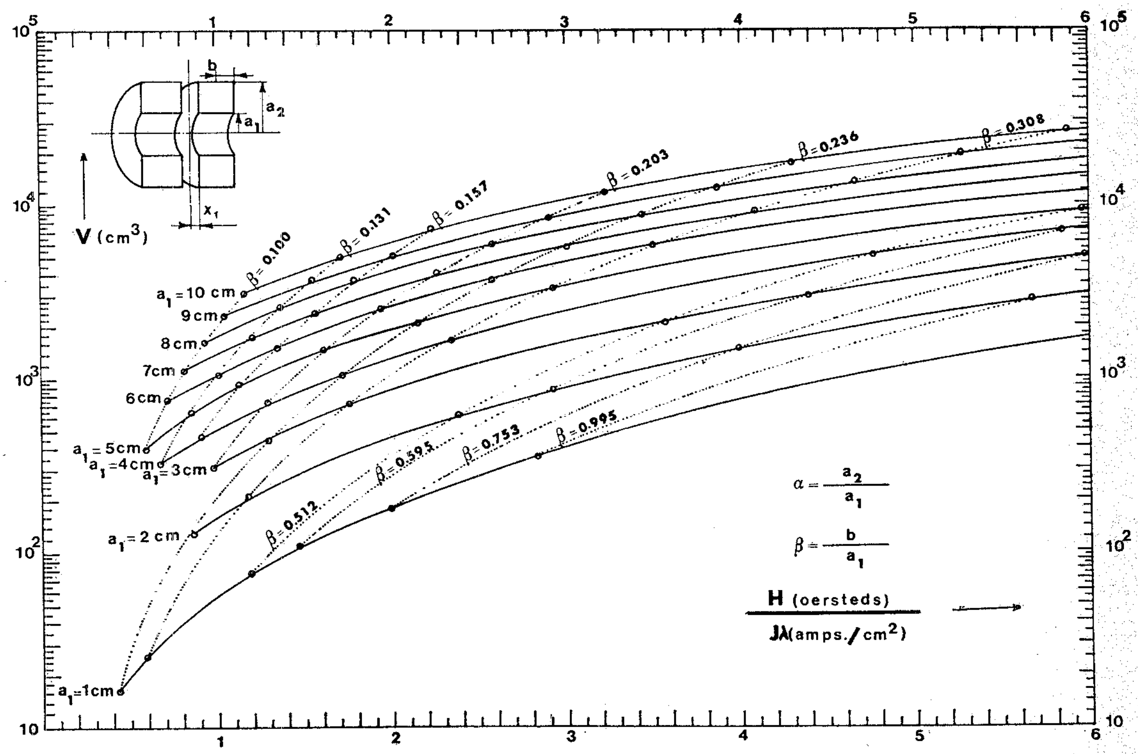

FIG. 4. Minimum volume (cm^3) necessary to obtain a magnetic field H at the center of the gap, with a current density $J\lambda$ and an inner radius a_1, for a fourth order compensated (Helmholtz type) split coil.

material is

$$V = 2\pi a_1^3(\alpha^2-1)\beta = a_1^3\nu, \tag{3}$$

and V/S is the required length of wire. Fabry[1] gave a table showing, for a range of values of ν between 1 and 10^5, values of α and β for which a maximum magnetic field is obtained for a given amount of material and fixed inner radius a_1; i.e., he maximized

$$H_{0z}/a_1J\lambda = (2\pi/5)\beta \log_e\{[\alpha+(\alpha^2+\beta^2)^{\frac{1}{2}}]/[1+(1+\beta^2)^{\frac{1}{2}}]\} \tag{4}$$

subject to constant ν.[8] We have repeated his calculations (see Fig. 2) and added 40 new optimal values extending the range of ν down to 0.4, for the design of large bore magnets of small size.

Figure 2 is used as follows: Assume a magnet is wanted having an inner radius $a_1 = 5$ cm and a central field $H_{0z} = 6\times10^4$ Oe. Then, with superconducting wire carrying 3×10^4 A/cm^2 at a field of 6×10^4 Oe and a filling factor $\lambda = \frac{2}{3}$, $H/J\lambda = 3$, and 1900 cm^3 is necessary; with 0.25 mm diam wire this means a length of about 39 km. Dotted lines in Fig. 2 are β-constant lines. $\Delta 1$ and $\Delta\frac{1}{2}$ are the axial homogeneities over distances a_1 and $a_1/2$, respectively; for all magnets of $\beta > 2.1$, $\Delta\frac{1}{2} < 2\%$; and for all with $\beta > 3.9$, $\Delta 1 < 2\%$ and $\Delta\frac{1}{2} < 0.5\%$.

If the required field is very high, the coil should be constructed in two concentric parts; for a coil with $H_{0z} = 1.2\times10^5$ Oe, $2a_1 = 12$ cm, and $J\lambda$ at 1.2×10^5 = only 1×10^4 A/cm^2, $H_{0z}/J\lambda = 12$, and the minimum volume Fabry's coil is $\alpha = 3$ and $\beta = 1.8$. For a single coil the total volume V of wire required would be $V \approx 19\,500$ cm^3. However, the field seen by the windings at 9 cm from the center is only 88 kOe. Between 9 and 18 cm from the center, therefore, cheaper (or higher current density) materials can be used. The volume of expensive, very high field superconducting material is thereby reduced to 15% of the volume otherwise required. One promising new moderate field material is glass fibers impregnated with Pb–Bi, which can carry critical current densities of 3×10^3 A/cm^2 at a field of 60 kOe.[9]

SIXTH ORDER COMPENSATED MINIMUM VOLUME COILS

From Girard's and Sauzade's calculations[2]

$$H_{0z}/J\lambda a_1 = (2\pi/5)\alpha(M_0 - M_0'), \tag{5}$$

where M_0 is given by

$$M_0 = (\beta/\alpha) \log_e\{[\alpha+(\alpha^2+\beta^2)^{\frac{1}{2}}]/[1+(1+\beta^2)^{\frac{1}{2}}]\}, \tag{6}$$

and M_0' is Eq. (6) with α and β replaced by α' and β', the parameters of the outside notch correction. $M_0 - M_0'$ is the quantity N_0 given in the table at the end of Ref. 2. The total volume of such a coil is obtained by multiplying $(H_{0z}/J\lambda)^3$ by the coefficient Z of the same table. It is in this way that Fig. 3 was obtained.

MINIMUM VOLUME HELMHOLTZ COILS

For split coils compensated to fourth order, Day[3] calculated the parameters minimizing the volume as a

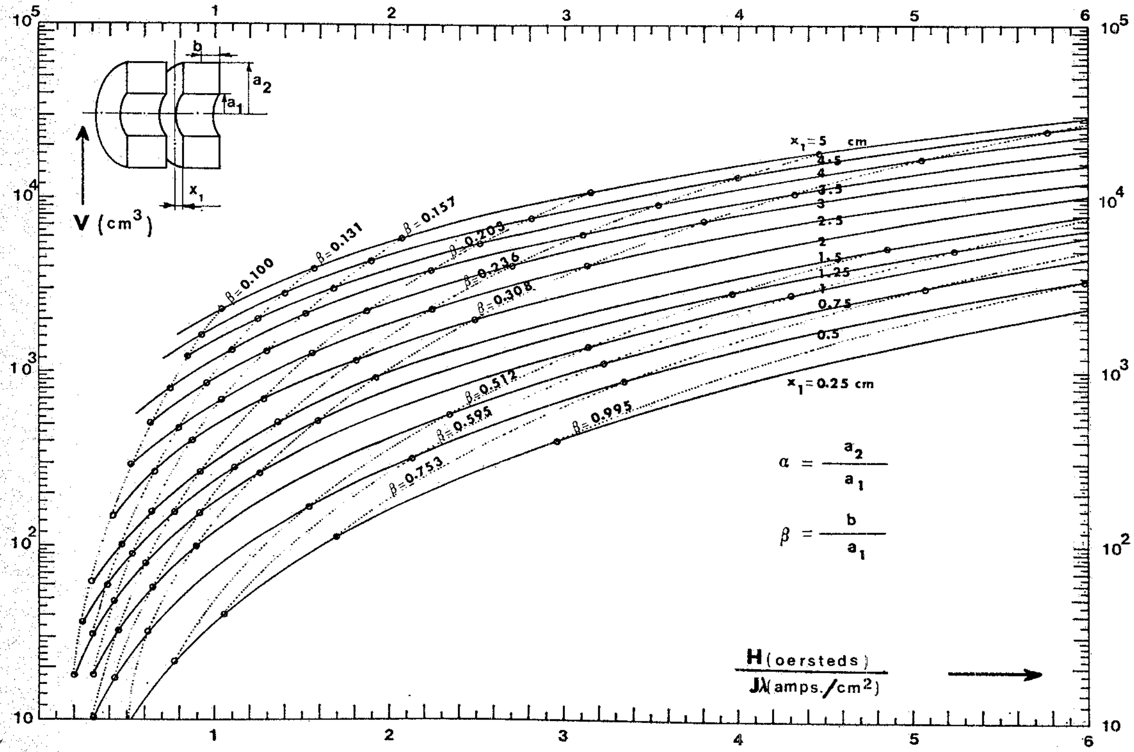

FIG. 5. Minimum volume (cm³) necessary to obtain a magnetic field H with a current density $J\lambda$ and half-width of the window, for a fourth order compensated split coil.

function of a reduced length L defined as

$$L=H_{0z}/4\pi J\lambda, \quad (7)$$

where H_{0z} is the field at the center of the split coil.[10] He defines the additional parameters: $\xi_1=x_1/L$, $\xi_2=x_2/L$, $\alpha_1=a_1/L$, and $\alpha_2=a_2/L$, where x_1, a_1, and a_2 are defined in Figs. 4 and 5, and $x_2-x_1=2b$.

The winding volume for a split coil is given by

$$V=2\pi(x_2-x_1)(a_2^2-a_1^2)=4\pi a_1^3(\alpha^2-1). \quad (8)$$

With Day's symbols

$$\alpha=\alpha_2/\alpha_1, \quad \beta=(\xi_2-\xi_1)/2\alpha_1. \quad (9)$$

With the inner radius of a magnet fixed, α_1 of Day's table gives $H_{0z}/J\lambda$ and a volume

$$V=2\pi a_1^3((\alpha_2/\alpha_1)^2-1)(\xi_2-\xi_1)/\alpha_1$$
$$=10^3(H_{0z}/J\lambda)^3((\alpha_2/\alpha_1)^2-1)(\xi_2-\xi_1)/32\pi^2. \quad (10)$$

Figure 4 is analogous to Figs. 2 and 3.

Alternatively, once x_1, the half-aperture of the radial access window of the magnet, has been fixed, $H_{0z}/J\lambda$ can be obtained from the α_1 of Day's table and the corresponding volume calculated, as in Fig. 5. The inner radius a_1 may then be calculated from $\xi_1=(10Hx_1/4\pi J\lambda)$ and $\alpha_1=(10Ha_1/4\pi J\lambda)$ in Fig. 2 of Ref. 3. Obviously, $\alpha_1/\xi_1=a_1/x_1$.

ANALYSIS OF RESULTS

Inspection of Figs. 2–5 leads to several conclusions. For identical values of $H/J\lambda$, the volumes of the three types of minimum volume coils are approximately the same if the inner radii for the Fabry coil, sixth order compensated coil, and Helmholtz coil are, respectively, $2a_1$, a_1, and a_1, or if the coil separation is $2x_1=a_1$.

The sixth order coils, however, generally are much longer than the other types. Because of the effect shown in Fig. 1, these sixth order coils will carry currents more nearly approaching the critical value $J_c(H_{0z})$. Thus the wire in a high homogeneity sixth order coil may cost no more than twice that in a comparable Fabry's coil of much poorer field homogeneity. The present large price difference between the uncompensated coils with field homogeneities $\Delta 1$ of perhaps a few percent, and sixth order outside notch compensated coils must be due to difficulties in winding, and not primarily to differences in wire cost.

Furthermore, the price differential for sixth order solenoids compensated by an inside notch should be even less.

While slight misalignment of the turns in such a coil is more detrimental to field homogeneity, our preliminary calculations indicate that the amount of wire might be the same as for a Fabry coil of radius only 50–60% larger.

ACKNOWLEDGMENTS

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[2] B. Girard and M. Sauzade, Nucl. Instrum. Methods **25**, 269 (1964).

[3] J. D. A. Day, J. Sci. Instrum. **40**, 583 (1963).

[4] A. D. Appleton, Cryogenics **9**, 147 (1969).

[5] Speed control of motors having stators with continuous fields can be accomplished by varying armature voltage via a reversing thyristror converter. If the rotor of such a motor is fitted directly to the periphery of a mill drum, then currently used mechanical transmissions, such as gears and couplings, are completely dispensable. A recent review of dc motor driven grinding mills in the cement and mineral industries has been published by A. C. Spurgeon in Elec. Eng. (Australia) **47**, 34 (1970).

[6] We limit our discussion to coils wound with wire or not too thin tape; e.g., single or multiple wires of Nb–Ti or Nb–Ti rich alloys imbedded in a matrix of copper for purposes of stabilization. In the case of very thin tape, such as Nb_3Sn strip, diamagnetic supercurrents result, according to Graham and Hart, in distortion of the local magnetic field seen by the windings, and the uniform current density assumption is no longer strictly valid; C. D. Graham and H. R. Hart, Proc. Int. Cryog. Eng. Conf., Kyoto, Japan, 1967, 101 (1968).

[7] P. J. Hart, *Universal Tables for Magnetic Fields of Filamentary and Distributed Circular Currents* (American Elsevier, New York, 1967).

[8] Many authors write $H_{0z}/a_1J\lambda$ as $G(\nu)^{\frac{1}{2}}$, where G is the Fabry G function

$$G=[(2\pi)^{\frac{1}{2}}/5][\beta/(\alpha^2-1)]^{\frac{1}{2}}\log_e\{[\alpha+(\alpha^2+\beta^2)^{\frac{1}{2}}]/[1+(1+\beta^2)^{\frac{1}{2}}]\}.$$

[9] J. H. P. Watson, Appl. Phys. Lett. **16**, 428 (1970); to be published in J. Appl. Phys.

[10] We note here that Day has made his calculations in emu cgs units, and therefore a factor of 10 has to be introduced in Eq. (7) if the currents are expressed in amperes.