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E. Amaldi: THE ELECTROPRODUCTION OF PIONS. -

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Invited talk, given at the
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SUMMARY. -

The aim of the present report is to summarize the experimental and theoretical knowledge on electroproduction of positive and neutral pions at very low energy, in particular near threshold. After an introduction of kinematical nature, (Section 1), the general expression for the cross section - derived under the assumption of a single photon exchange - is given in Section 2, and the main lines of the various theoretical approaches are summarized in Section 3. Section 4 refers to the coincidence experiments made until now and to the result of their analysis, and Section 5 to a few concluding remarks.

1. - INTRODUCTION. -

In the frame of this Meeting, devoted to the discussion on the type of physics that can be made still usefully after 1970, with an electron accelerators of rather low energy, the term electroproduction should be taken in a restricted sense, namely to mean single pion production by inelastic scattering of electrons on nucleons. Then the only reactions involved are

$$e + p \rightarrow \begin{cases} e' + n + \pi^+ & \text{(1a)} \\ e' + p + \pi^o & \text{(1b)} \end{cases}$$

and possibly

$$e + n \rightarrow \begin{cases} e' + p + \pi^- & \text{(2a)} \\ e' + n + \pi^o. & \text{(2b)} \end{cases}$$

But even with such a drastic limitation, the number of theoretical and experimental papers relevant to the subject is so large that it is impossible to review all of them is a limited time. Being forced to apply some kind of selection criterion, I will include in the final discussion of the experimental results, only those obtained by means of set-ups in which two of the three final particles are detected in coincidence. Consequently, in the presentation of the essential points of the theoretical papers, I will mainly refers to those directly used for the analysis of experimental data of this type. Such a selection is justified by the fact that, at the energies considered here, only coincidence experiments can provide information with adequate detail, to fulfill the goal of showing that the subject is interesting and still deserves much attention by the experimentalists and, even more, by the theoreticians.

Let us start by recalling very rapidly the connections between electroproduction and other phenomena such as photoproduction and elastic scattering.

In the case of photoproduction (Fig. 1) the differential cross section, in the frame of the center of mass, is given by the expression

$$\frac{d\sigma}{d\Omega^{*}} = F_{ph} \frac{1}{4} \Sigma \left| \langle N' \mid A_\mu J_\mu \mid N \rangle \right|^2 \qquad (3)$$

where F_{ph} is a kinematical factor that we do not need to write explicitely at this moment, A_μ the fourpotential of the photon and J_μ the hadrons current. By performing the summ over the spin of the nucleons and the polarization of the photon before contracting the fourvectors, one obtains

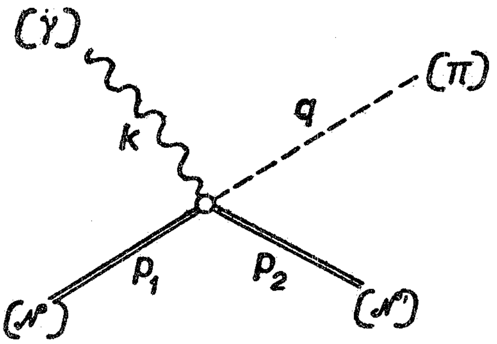

FIG. 1 - Graphical representation of photoproduction.

(4a) $$\frac{d\sigma}{d\Omega^x} = F_{ph}\left[\frac{1}{2}\Sigma A_\mu A_\nu^x\right]\left[\frac{1}{2}\Sigma\langle N'\pi|J_\mu|N\rangle\langle N'\pi|J_\nu|N\rangle^*\right]$$

which can be written in the abbreviated form

(4b) $$\frac{d\sigma}{d\Omega^x} = F_{ph}\, e_{\mu\nu}\, T_{\mu\nu}$$

where $e_{\mu\nu}$, called the polarization density matrix of the photon, is well known, while the physics that one tries to understand is contained in $T_{\mu\nu}$.

When we pass to write a similar expression for electroproduction, one has to recall two well known points. The first, of kinematical nature, derives from the fact that in this case three particles are present in the final state. Therefore, the values of five kinematical variables are necessary for specifying completely an event. From the experimental point of view the most convenient are: the energies r_{o1}, r_{o2} of the electron in the initial and final state, the angle of scattering of the electron ϑ_e and, for example, the angles ϑ_π and φ_π which specify the direction of motion of the emitted pion (or recoiling nucleon), all taken in the laboratory frame of reference.

The second point is a physical assumption, very reasonable, but essential for the simplicity of the interpretation of the experimental results. One assumes that the process takes place by the exchange of a single virtual photon between the electron and hadrons currents. This is equivalent to say that the cross section computed in the first Born approximation with respect to the fine structure constant α is adequate.

Extensive tests of this assumption, made in the case of electron-nucleon elastic scattering(1), have allowed the determination of an upper limit for the two-photon exchange amplitude of about one percent of the one-photon amplitude up to fourmomentum transfers around 5 $(GeV/c)^2$. In the case of electroproduction do not dispose of direct experimental tests of comparable precision but through various indirect arguments, it is very reasonable to expect that it should hold, inside the same limits of accuracy, at least in the case of very low values of fourmomentum transfer and total energy of the hadrons present in the final state, as are those considered here.

Under this assumption the electroproduction process can be visualized as shown in Fig. 2, which indicates also the notations I use.

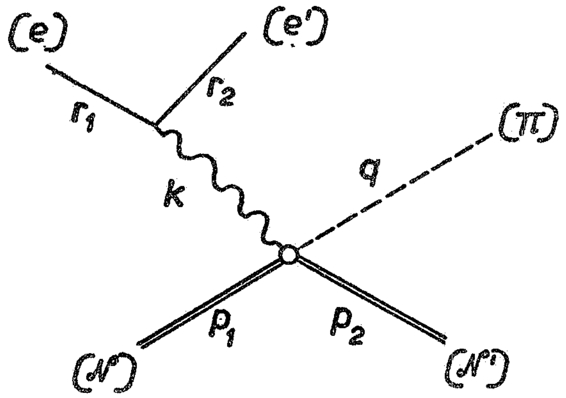

FIG. 2 - Grafical representation of electroproduction under the assumption of one-photon exchange between the electron and the hadron currents.

An expression for the differential cross section for electroproduction, similar to that given before for photoproduction, can be easily given:

(5a) $$\frac{d\sigma}{dr_{02}\, d\omega_e\, d\Omega_\pi^x} = F_e \frac{1}{4}\Sigma \left| \langle eN'\pi \left| \frac{e^2}{k^2} j_\mu J_\mu \right| eN \rangle \right|^2$$

where F_e is a factor similar to that appearing in Eq. (3), j_μ is the electron current operator and k^2 the propagator of the virtual photon.

Proceeding as in the case of photoproduction, this expression can be written in the form

(5b) $$\frac{d\sigma}{dr_{02}\, d\omega_e\, d\Omega_\pi^x} = F_e \frac{e^2}{k^4} \left[\frac{1}{2} \Sigma (\bar{u}_f \gamma_\mu u_i)(\bar{u}_f \gamma_\nu u_i)^x \right] \cdot \left[\frac{1}{2} \Sigma \langle \pi N' | J_\mu | N \rangle \langle \pi N' | J_\nu | N \rangle^x \right]$$

or, in abbrevied notations:

$$\frac{d\sigma}{dr_{02}\, d\omega_e\, d\Omega_\pi^x} = F_e \frac{e^2}{k^4} \bar{e}_{\mu\nu} T_{\mu\nu}.$$

Here, again, $\bar{e}_{\mu\nu}$ is a matrix originating from the electron current, the various elements of which can be easily written explicitly. The physics in which we are interested is contained again in $T_{\mu\nu}$.

It may be recalled at this point that the indexes of the two matrices appearing in Eq. (5c) run from 1 to 3 since, by current conservation, the longitudinal and scalar components of both currents j_μ and J_μ are not independent, so that one of the two can be forgotten provided the other is multiplied by a convenient kinematical factor(2).

Before giving the complete expression of the cross section (5c) it is necessary to add a few remarks on the virtual photon exchanged between the electron and the hadrons currents.

Since under the experimental conditions considered here, the energy of the incident and scattered electrons r_{01} and r_{02} and the angle of scattering ϑ_e are kept constants, the fourmomentum transfer from the electron to the hadrons currents

$$
(6) \qquad \begin{aligned} k^2 &= (r_1 - r_2)^2 = k_0^2 - \vec{k}^2 = (r_{01} - r_{02})^2 - (\vec{r}_1 - \vec{r}_2)^2 = \\ &= 2m_e^2 - 2\,(r_{01}\, r_{02} - \vec{r}_1 \cdot \vec{r}_2) \simeq -2 r_{01}\, r_{02}\,(1 - \cos \vartheta_e) \end{aligned}
$$

has a well defined negative value. This means that the virtual photons differ from the real photons of a bremsstrahlung beam for three properties:

1) they are space-like, off the mass shell by the amount $k^2 (< 0)$;
2) they are monoenergetic;
3) they are polarized with transversal and longitudinal components.

The first property is obvious since k^2 represents the square of the mass of the photon. The second properties is also evident since both k_0 and $\vec{k}$ have well defined values. As variable for expressing the energy of the virtual photon it is convenient to use the energy W of the pion-nucleon system in the frame of their center of mass which, like k^2, is a Lorentz invariant.

By attaching a star to the simbols of the various quantities when taken in the frame of the c. m. of the hadrons in the final state, one has, by definition,

$$
(7) \qquad W = (\vec{k}^{*2} + k^2)^{1/2} + (\vec{k}^{*2} + M_1^2)^{1/2},
$$

expressing that the photon and the initial nucleon move one against the other with opposite momenta. Once W is fixed, we know also the three momentum $\vec{q}^{*}$ with which, in this same frame, the pion and the final nucleon fly away:

$$
(8) \qquad W = (\vec{q}^{*2} + m^2)^{1/2} + (\vec{q}^{*2} + M_2^2)^{1/2}
$$

The threshold of electroproduction corresponds to

(9a) $$\vec{q}^{\,x} = 0$$

which, according to (8), gives

(9b) $$W = m_\pi + M_2 .$$

The relationship between W and the energy of the photon in the laboratory frame, k_0, is easily found:

(10a) $$s = W^2 = M_1^2 + k^2 + 2M_1 k_0 \qquad (k_0 = r_{01} - r_{02})$$

where s is the usual Mandelstam invariant variable. This equation can be written in the form

(10b) $$-k^2 = M_1^2 - W^2 + 2M_1\, k_0$$

which, for a given value of W^2, is represented by a straight line in the $-k^2$ vs k_0 plane (Fig. 3). Such a graph is very useful for undertanding the relationship between elastic scattering, electroproduction and photoproduction.

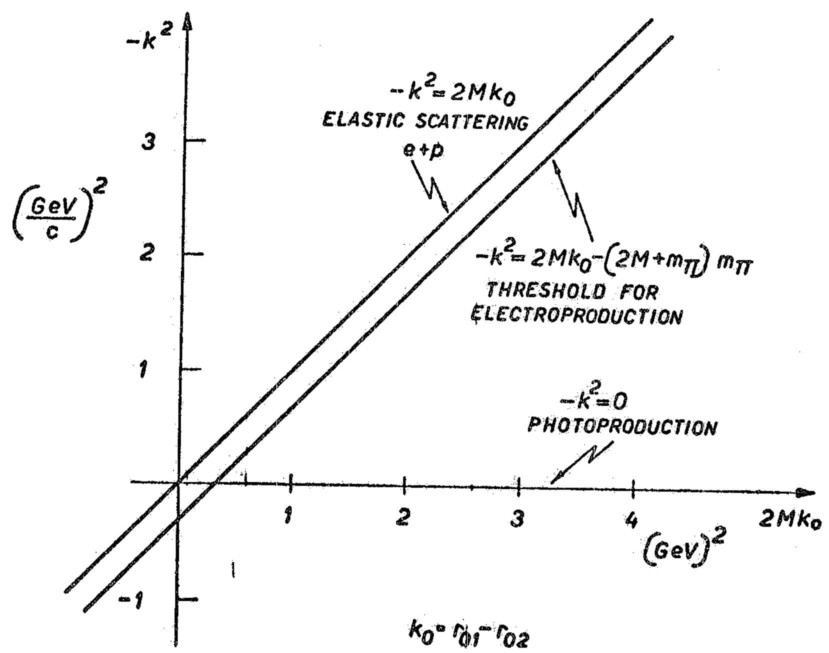

FIG. 3 - Kinematical relationship between elastic scattering, electroproduction and photoproduction.

In the case of elastic scattering one has

(11a) $$W = M_1$$

so that the straight line (10b) reduces to

(11b) $$-k^2 = 2M_1 k_0.$$

As we said before the threshold for electroproduction is given by the condition

(12a) $$W = m + M_2.$$

By introducing this value into (10b) one obtains

(12b) $$-k^2 = 2M_1 k_0 - (2M_2 + m_\pi) m_\pi ,$$

which is a straight line, parallel to the previous one. Similar straight lines represent the threshold for the electroproduction of heavier particles (or groups of particles). Finally the axis of abscissae, defined in Fig. 3 by

$$-k^2 = 0,$$

corresponds to real photon i. e. represent the conditions met in photoproduction. Thus, Fig. 3 shows clearly the relationships of pure kinematical nature, existing between elastic scattering, electroproduction and photoproduction.

We come now to the last of the three peculiar properties of the virtual photon, namely that of being polarized. This property is conveniently expressed by introducing the polarization parameter defined in the c.m. frame, which is specified by taking the direction of the three axis as shown in Fig. 4: the Z axis is taken in the direction of the threemomentum of the virtual photon

$$\hat{Z} = \hat{k}^x ;$$

the Y axis perpendicular to the scattering plane:

$$\hat{Y} = \hat{r}_1 \times \hat{r}_2$$

and the X axis in the scattering plane

$$\hat{X} = \hat{Y} \times \hat{Z}.$$

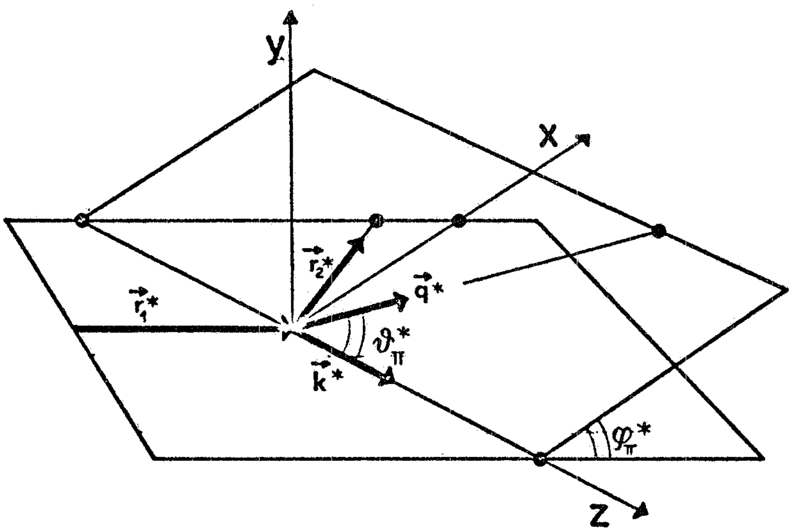

FIG. 4 - Diagram illustrating the definition of the various quantities in the frame of the c. m. of the hadrons in the final state.

Then one can define the polarization parameter by means of the equation

(13a)
$$\varepsilon = \frac{|A_X^x|^2 - |A_Y^x|^2}{|A_X^x|^2 + |A_Y^x|^2}$$

where A_X^x and A_Y^x are the X and Y components of the vector potential $\vec{A}^x$ of the photon (taken, of course, in the c. m. frame); $\varepsilon = 0$ means unpolarized photons, $\varepsilon = +1$ fully polarized photons in the X direction.

This parameter can be easily expressed in terms of quantities measured in the laboratory frame

(13b)
$$\varepsilon = \left[1 + 2\,\frac{|\vec{k}|^2}{-k^2}\,\mathrm{tg}^2\,\frac{\vartheta_c}{2}\right]^{-1}.$$

It can be easily recognized that, for fixed values of k^2 and W, the parameter ε is a function of ϑ_e which decreases by increasing the angle of scattering ϑ_e; this is clearly seen in Fig. 5 which shows the behaviour of ε for $-k^2 = 3\,F^{-2}$ and two values of W (1. 2 and 1. 3 GeV).

As I said before, the vector potential $\vec{A}^x$ has also a longitudinal or Z component. If one defines a longitudinal polarization parameter by the equation

(14a)
$$\varepsilon_\ell = \frac{|A_Z^x|^2}{|A_X^x|^2 + |A_Y^x|^2},$$

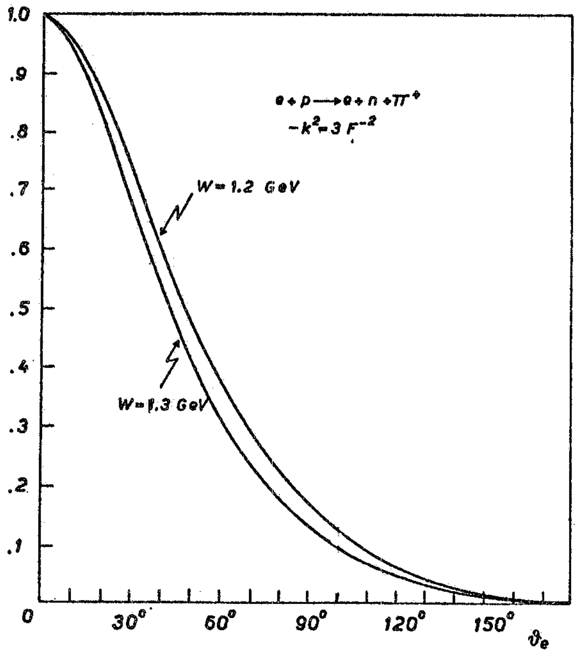

FIG. 5 - Dependence on the scattering angle ϑ_e (in the lab.) of the polarization parameter defined by Eq. (13a).

one finds that it can be expressed in terms of ε as follows

(14b)
$$\varepsilon_\ell = \frac{-k^2}{k_0^{x2}}\,\varepsilon$$

For fixed values of k^2 and W also ε_ℓ decreases by increasing ϑ_e.

One can easily recognize that the two invariants k^2 and W and the parameter ε, each of which is in some way related to one of the three peculiar properties of the virtual photons, constitute three independent variables which can replace, very conveniently from the point of view of the theoretical description of the process, the variables

$$r_{01},\ r_{02} \text{ and } \vartheta_e .$$

For the remaining two variables necessary for specifying completely an event, one uses the two angles ϑ_π^x and φ_π^x, which define the direction of the emitted pion in the frame of the c.m. (Fig. 4). In conclusion the five variables which will appear in the differential cross sections are

(15)
$$k^2,\ W,\ \varepsilon,\ \vartheta_\pi^x \text{ and } \varphi_\pi^x .$$

2. - THE GENERAL EXPRESSION OF THE CROSS SECTION. -

Eq. (5c) given above shows the general structure of the differential cross section for electroproduction.

Following the Cornell group, lead by Berkelman[3], one can now write down its explicit expression in terms of the five variables defined above: one has

(16a)
$$\frac{d\sigma}{dr_{02}\, d\omega_e\, d\Omega_\pi^x} = n_{ph}\, \frac{d\sigma_v}{d\Omega_\pi^x}$$

where the first factor

(16b)
$$n_{ph} = \frac{\alpha}{2\pi^2} \frac{r_{02}}{r_{01}} \frac{|\vec{k}|}{-k^2} \frac{1}{1-\varepsilon}$$

is of electrodynamic origin and contains the effect of the electron-photon vertex and the photon propagator. It can be interpreted as the number of virtual photons per electron scattered into dr_{02} and $d\omega_e$.

The second factor $d\sigma/d\Omega_\pi^x$ is the c. m. differential cross section for pion photoproduction by virtual photons, polarized as described above. One has

(16c)
$$\frac{d\sigma_v}{d\Omega_\pi^x} = \frac{1}{4(2\pi)^2} \frac{M^2|\vec{q}^x|}{W^2|\vec{k}^x|} \bar{e}_{\mu\nu} T_{\mu\nu}$$

where the factor $|\vec{q}^x|$ is zero at threshold and all the elements of the 3x3 matrix $\bar{e}_{\mu\nu}$ are simple function of the parameter ε. Since $\bar{e}_{\mu\nu}$ is symmetric, has two zero elements and two others equal, the product of the two matrixes $\bar{e}_{\mu\nu}$ and $T_{\mu\nu}$ reduces to four terms, so that one finally has

(17)
$$\frac{d\sigma_v}{d\Omega_\pi^x}(W, k^2, \varepsilon, \vartheta_\pi^x, \varphi_\pi^x) = A + \varepsilon B + \varepsilon C \sin^2\vartheta_\pi^x \cos^2\varphi_\pi^x + \left[\varepsilon(1+\varepsilon)\right]^{1/2} D \sin\vartheta_\pi^x \cos\varphi_\pi^x$$

where the coefficients A, B, C and D are simple linear combinations of the elements of the matrix $T_{\mu\nu}$ which depend only on the variables

$$W,\ k^2 \text{ and } \vartheta_\pi^x$$

but not on ε and φ_π^x.

The four terms appearing in Eq. (17) have a very simple meaning: the term A is the differential cross section for unpolarized transverse virtual photons. In the limit $k^2 = 0$ it approaches the photoproduction cross section by real photons. The third term is the modification of the cross section due to transverse linear polarization, as shown by the factor ε and its dependence on φ_π^x. In the $k^2 = 0$ limit it approaches the corresponding term in the crossection for photoproduction by real linearly polarized photons. The second term is the longitudinal contribution; it vanishes for real photons. The last term originates from the interference between the transverse and longitudinal amplitudes.

In principle the four terms appearing in Eq. (17) can be obtained separately from experiments: C and D show characteristic azimuthal dependence, while A and B can be separated by measurements made at different values of ε, which is mainly a function of ϑ_e.

If the pions are observed at $\vartheta^x_\pi = 0$, i. e. in the direction of the virtual photon, the third and fourth terms are zero and the cross section reduces only to two terms

$$\frac{d\sigma_v}{d\Omega^x_\pi} = A + \varepsilon B. \tag{18}$$

A similar properties holds for the cross section integrated with respect to the direction of emission of the pion which has the form(4)

$$\sigma_v = \sigma_T\,(W,k^2) + \varepsilon\sigma_L\,(W,k^2) \tag{19a}$$

where

$$\sigma_T\,(W,k^2) = \int A\,(W,k^2,\vartheta^x_\pi)\,d\Omega^x_\pi$$

(19b)

$$\sigma_L\,(W,k^2) = \int B\,(W,k^2,\vartheta^x_\pi)\,d\Omega^x_\pi$$

Eq. (16) and (17) summarize all our knowledge of the electroproduction cross section based on quantum electrodynamics, under the assumption of a single-photon exchange. It may be pointed out incidentally that the explicit dependence of Eq. (17) on ε and φ^x_π is a consequence of this assumption and the spin and parity of the photon.

3. - THE THEORY OF ELECTROPRODUCTION. -

The information about the structure of the hadrons involved in the process is contained in the four functions A, B, C and D. That these functions should depend on the form factors $(\gamma\pi\pi)$ (γNN) $(\gamma N\Delta)$ (Δ = 1236 resonance), is obvious from the fact that the diagrams of Fig. 6, plus, eventually, diagrams with higher resonances in the s and u channels and and exchanges in the t channel, should be adeguate for the description of electroproduction in the kinematical region considered here.

It is clear, however, that in order to isolate the contribution of the graphs of Fig. 6 to the cross sections and to obtain information on the three form factors mentioned above, a detailed theory of electroproduction is needed.

In a different approach, based on PCAC and current algebra, the value of the weak axial vector nucleon form factor $G_A(t)$ where

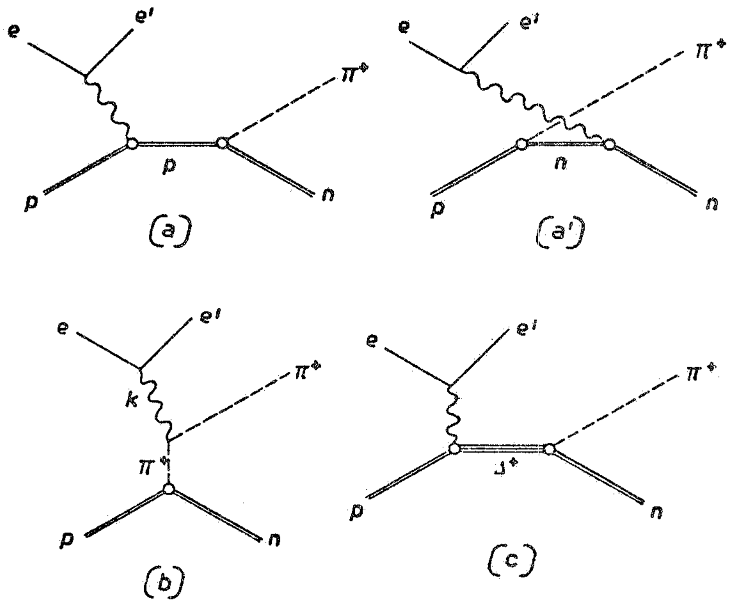

FIG. 6 - A few graphs expected to be among the more important in the vicinity of the electroproduction threshold.

(20)
$$t = \Delta^2 = (p_2 - p_1)^2 = (k - q)^2,$$

can be deduced from the electroproduction matrix element; but also in this case a detailed model of photoproduction by virtual photons is required.

The possibility of determining the pion form factor $F_\pi(k^2)$ (i. e. the $(\gamma\pi\pi)$ vertex function) from electroproduction data, was first suggested by Fraser in 1959[5]. The dependence on F_π comes in from the one-pion-exchange pole diagram (Fig. 6b), which can be described as the elastic scattering of an electron by a virtual pion emitted by the target proton.

To isolate this amplitude from the others, which mask its effect, Fraser proposed an extrapolation of the pion angular distribution data to the pole

$$(p_1 - p_2)^2 = m_\pi^2.$$

Such an approach, however, is impractical because it would require an accuracy of the measured points beyond present experimental capabilities.

Later a number of authors have developed different approaches which schematically can be grouped in a few cathegories:

1a) The fixed-t dispersion relations approach was first used by Fubini-Nambu and Wataghin[6] who extended to electroproduction the photoproduction theory made by Chew, Golberger, Low and Nambu[6]. In this approach, rather successful in the region of the first resonance, one assumes that the most important effects can be taken into account by Born terms and the resonant magnetic transition to the 3/2, 3/2 final state. The two original papers mentioned above are made within the framework of the static model.

1b) Theories based on the Mandelstam representation for the scattering amplitude were worked out by Ball[7] for photoproduction and by Dennery[8] and others[9] for electroproduction.
Along this line is also the paper by De Tollis and Nicolò[10], that I will mention later in the discussion of some of the experimental data. These authors have derived a very simple expression for the total cross section at threshold ($\vec{q}^{x} = 0$) in which only the Born terms appears. Since such an approximation had been found to be satisfactory in the case of charged pions photoproduction, it was reasonable to expect that it would be adequate also for electroproduction, as long as k^2 is small.

1c) This approach, closely related to the two previous ones, consists in the use of partial wave dispersion relations; it has been developed by a number of authors[11 ÷ 18] but only a few of them will be quoted specifically for discussing the experimental results.

The theories by Zagury[12] and by Adler[15], offer a few improvements with respect to the original Fubini et al. paper, the most outstanding of which is the fully-relativistic treatment of the nucleon recoil.

In these theories the pion production amplitude consists of the Born terms, dispersion integral estimates of the most important partial amplitudes, and an estimate of the final state interaction involving the rescattering of the pion by the outgoing nucleon.

In von Gehlen approach[16 ÷ 18] a system of coupled integral equations for the electroproduction partial waves amplitudes is derived from fixed-t dispersion relations: these integral equations have the form

$$\begin{aligned} \mathrm{Re}\,\mathcal{E}_i^{J,I}(W,k^2) &= \mathcal{E}_{i\,\mathrm{Born}}^{J,I}(W,k^2) + \frac{1}{\pi} P \int_{M+m_\pi}^{\infty} dW' \, \frac{\mathrm{Im}\,\mathcal{E}_i^{J,I}(W',k^2)}{W' - W} \\ &+ \frac{1}{\pi} \int_{M+m_\pi}^{\infty} dW' \sum_{J'I'i'} K_{ii'}^{JJ'II'}(W,W',k^2)\, \mathrm{Im}\,\mathcal{E}_{i'}^{J',I'}(W',k^2) \end{aligned} \tag{21}$$

where $\mathcal{E}_i^{J,I}$ are suitably normalized multipoles amplitude with total angular momentum J and isospin I. The kernels $K_{ii'}^{JJ'II'}$ are complicated kinematical functions the explicit form of which has been derived by von Gehlen. Using Watson theorem[19] at low energies and definite assumptions for the multipole phases in the inelastic region, von Gehlen obtained a solution of the system (21) by means of numerical iteration. I will come back later to some of the results obtained by von Gehlen[17, 18].

2) The use of isobaric models made by Salin[20], Loubaton[21] Kessler[22] and, more recently, by Walecka et al.[23].

3) We come now to the third and last approach to the theory of electroproduction, based of PCAC assumption and current algebra techniques. Theories of this type have been made by Gleeson, Gundzik and Kuriyan[24], by Fubini, Furlan[25] and Furlan and coworkers[26 ÷ 28] and by Nambu and Yoshimura[29].

As is well known the condition of partial conservation of axial vector current and current algebra predict that the amplitudes for electroproduction are expressible in terms of vector and axial-vector form factors of the nucleon in the ideal soft pion limit of zero fourmomentum $q_\mu = 0$[30]. This enables, in principle, to determine the unknown (isovector) axial form factor $G_A(t)$ which could be otherwise obtained only from neutrino-nucleon collision experiments. The PCAC assumption is used to connect the amplitude defined by the divergence of the axial vector current at zero mass, with the corresponding physical amplitude.

The extrapolation from the current algebra point ($q^2 = 0$, $q_\mu = 0$) to the physical region is a delicate analytic problem which involves the answer to two questions: the first concern the nature and importance of the next terms in the pion mass expansion. The second regards the choice of the physical point in which the current algebra prediction and the experiment have to be compared.

The problem of the mass extrapolation has been tackled, in general, by Fubini and Furlan[25] from a purely dispersion theoretical point of view based on the use of the equations obtained by Low by applying completeness to the standard time-ordered products of operators.

As variable they use u , ν and w defined as usual

$$u = q^2 \qquad \Delta = p_2 - p_1 = k - q, \quad t = \Delta^2$$

$$\nu = q \cdot P \qquad P = \frac{p_1 + p_2}{2} \tag{22}$$

$$w = Q \cdot \Delta = \frac{1}{2}(k^2 - q^2) \qquad Q = \frac{k + q}{2}$$

$$P \cdot \Delta = 0.$$

They write the amplitude for pions on the mass shell in the form of a subtracted dispersion relation

(23) $$F(m_\pi^2) = F(0) + \frac{m_\pi^2}{2} \int_\gamma \frac{\mathrm{Im}\, F(u')\, du'}{u'(u' - m_\pi^2)} \quad ,$$

where the equal time commutator enters in F(0), together with the term due to the nucleon pole, and the dispersion integral is made along a conveniently chosen line γ in the (u, ν, w) space, which will depend on the frame of reference adopted, as a consequence of the fact that the three momentum

$$\vec{Q} = \frac{\vec{k} + \vec{q}}{2}$$

is kept fixed performing the dispersion integral.

The crucial problem of finding the most appropriate line is solved by Fubini and Furlan by imposing the requirement that the dispersion corrections to the first approximation

$$F(m_\pi^2) = F(0)$$

are small, or at least easy to compute.

They specify these conditions in a few points which include, besides the necessity for some kind of general justification for the validity of the dispersion relation, the requirement of avoiding the presence of anomalous singularities near the point $u = \nu = 0$ $(q_\mu = 0)$, the use of physical selection rules due to angular momentum and parity, which rule out many of the contributions from low lying states, and the need for a convenient high energy behaviour of the amplitude.

The result of a long but simple calculation, is that, in any frame of reference, the line γ (corresponding to fixed values of $\vec{Q}$ and t in the space (u, ν, w) is a parabola

$$u = a\nu^2 + b\nu + c$$

in the plane

$$w = h\nu + \ell,$$

where the coefficients a, b, c, h and ℓ are simple expressions, the values of which depend on the adopted frame.

The next step for determining the most convenient line γ, consists in introducing the kinematical condition that q and P should remain collinear when moving along γ. Fubini and Furlan write

(24a) $$q = xP$$

where x is an adimensional variable in terms of which all other variables, except t, can be expressed:

(24b)
$$\nu = xP^2 \qquad k = xP + t$$
$$q^2 = x^2P^2 \qquad P^2 = M^2 - \frac{t}{4}$$
$$k^2 = x^2P^2 + t \qquad s = W^2 = (1+x^2)P^2 + \frac{t}{4}$$

and finally

(24c)
$$u = P^2 x^2$$

while the equation of the plane reduces to $w = \frac{1}{2}t$ = constant.

One has now to chose the reference frame is such a way that the point where the line γ enters the physical region, lies as close as possible to the point $q_\mu = 0$ ($\nu = 0$, $u = 0$). The best that one can do is to choose the Breit frame defined by

(25a)
$$\vec{P} = \frac{\vec{p}_1 + \vec{p}_2}{2} = 0$$

since it gives the parabola with the largest possible curvature.

In this frame one has

$$\vec{p}_1 = -\vec{p}_2 = -\frac{\vec{\Delta}}{2} \qquad p_{01} = p_{02} \qquad \Delta_0 = 0$$
$$\vec{P} = 0 \qquad P_0 = p_{01} \qquad P^2 = P_0^2$$

and the parabola (24c) becomes (Fig. 7)

(26)
$$u = \frac{\nu^2}{P_0^2}$$

The dispersion integral becomes a simple integral in tha variable x

(27)
$$\frac{2}{\pi}\frac{m_\pi^2}{P_0^2}\int_0^\infty \frac{\mathrm{Im}\, F(x)\, dx}{x(x^2 - x_0^2)}$$

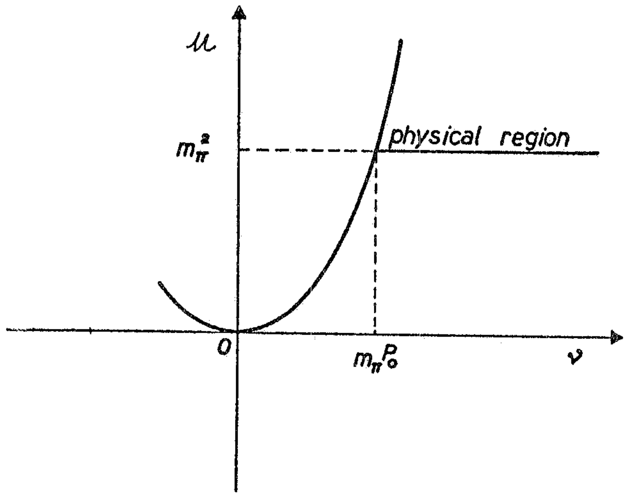

FIG. 7 - The line γ along which the dispersion integral (21) is computed by Fubini and Furlan.

with x = 0 corresponding to pions of zero mass, and

$$x = x_0 = \frac{m_\pi}{\sqrt{P_0^2}} \tag{28}$$

to the Breit threshold, i. e. to the kineamtical configuration for which, in the Breit frame, the final physical pion is produced at rest.

Since all variables, except t, are expressed in terms of x, when one varies x from zero to infinity (keeping t = constant) one passes through a number of resonances in the various channels. Among the contributions originating from the variable q the most important is the pion pole, which, however, has already been taken into account explicitely from the beginning. A contribution from the bipion should be excluded on parity and angular momentum conservation. The three pion is so far away from the Breit threshold that its contribution is expected to be small.

The pole in x, originating from its relationship with k, are those due to isovector (ρ, ...) and isoscalar mesons (ω, ...) the most important of which is the ρ.

Finally the poles originating from the variable ν (direct channel) have been considered. These are:

a) S-waves resonances, the first of which, however, is at 1660 MeV;
b) P-wave resonances, the first of which is the Δ;
c) the continuum of S and P waves which are evaluated by means of Watson theorem in terms of the scattering phases.

The final expression derived by Fubini and Furlan are as follows

$$T_1^{(-)} = \frac{1}{2f_\pi}\left[G_A(t) + \frac{g_A}{4P^2}\, t\, G_M^V(t)\right] + \delta T_1^{(-)}$$

$$T_3^{(-)} = \frac{1}{2f_\pi}\left[G_p(t) + \frac{Mg_A}{2P^2}\, G_M^V(t)\right] + \delta T_3^{(-)} \tag{29}$$

$$T_1^{(+)} = -\frac{m_\pi}{4\sqrt{P^2}}\,\frac{g_A}{f_\pi}\, G_E^V(t) + \frac{t}{2M}\, T_3^{(+)} + \delta T_1^{(+)}$$

$$T_3^{(+)} = -\frac{\sqrt{P^2}m_\pi}{f_\pi}\frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}\,F_3^{+}}{m_\pi^2 - x^2P^2}$$

(29)
$$T_1^{(0)} = -\frac{m_\pi}{4\sqrt{P^2}}\frac{g_A G_E^S(t)}{f_\pi} + \frac{t}{2M}T_3^{(0)} + \delta T_1^{(0)}$$

$$T_3^{(0)} = -\frac{\sqrt{P^2}\,m_\pi}{f_\pi}\frac{2}{\pi}\int_0^\infty \frac{m F_3^{(0)}}{m_\pi^2 - x^2P^2}\,dx$$

where

$$G_M = F_1 + F_2,\quad G_E = F_1 + \frac{t}{2M}F_2,\quad g_A = G_A(o)$$

and G_P is the pseudoscalar from factor; the three values of the upper isospin index have the usual meaning: 0 isoscalar, ± isovectors with isospin 0 and 1 in the t-channel, while the three values 1, 2, 3 of the lower index correspond to the various terms of the decomposition of the amplitude in Lorentz invariants, which reduce only to three because of the collinearity condition (24a). For example, in the first and second eq. (29), the first term originates from the equal time commutator, the second from the nucleon pole, while the third has the form

$$\delta T_{1,3}^{(-)} = \frac{2\,m_\pi^2}{f_\pi P^2}\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\,F_{1,3}^{(-)}(x)}{x(x^2 - x_0^2)}\,dx.$$

The same expression (29) have been obtained by Furlan, Paver and Verzegnassi(26) who have recast essentially the same concepts in a formulation involving directly the properties of the equal time commutators. The problem has been further studied by Furlan, Paver and Verzegnassi(27) who succeded in computing the contribution of the P-waves that are different from zero at $\vartheta_\pi^{*} = \pi/2$, so that an extrapolation can be made at physical points close to the physical threshold; also the interference term between an S and a D wave has been computed in ref. (28).

The theory of Nambu and Yoshimura(29) has been published only in a very short form: they have given the final formula and a few qualitative hints about the procedure followed in its derivation. They claim that their theory is simpler than that of Fubini et al. and that it should be better for large values of k^2; but of course, it also is valid only at threshold ($W = M_2 + m_\pi$).

4. - COINCIDENCE EXPERIMENTS. -

As stated at the beginning, only coincidence experiments will be considered here, since they are superior to experiments in which a single particle is detected. If the single particle detected is one of the hadrons present in the final state[31], the values of the variables k^2, W and ε remain unknown. If the single particle detected is the inelastically scattered electron[4, 32 ÷ 37] one knows the value of k^2, W and ε but the measured cross section is the sum of the π^+ and π^o cross sections, integrated with respect to ϑ_π^x and φ_π^x. Each of these is already a complicated expression containing a few unknown functions. A comparison of experimental results of this type with the predictions of different models, turns out to be rather insensitive to the details of the models.

Furthermore such data are subject to uncertainties arising from the large background due to the process

$$e^- + p \rightarrow e' + p + \gamma \qquad (30)$$

the computed value of which should be subtracted. A correction due to this effect is applied also to the π^o production coincicence data, but its value is usually smaller by a very large factor.

All coincidence experiments, that, to my knowledge, have been made on electroproduction of π^o are listed in Table 1.

TABLE 1

Coincidence experiments on π^o electroproduction

	$W^{(1)}$ (MeV)	$-k^2$ (F^{-2})	Comments or results
Orsay[38] (1964)	1200	1.3	isobar model
Cornell[39] (1966 - 1967)	around 1230	1;2;6;8	$G_M^*(k^2)$
Tokyo[40] (1966 - 1967)	1190	2.29	agrees with Cornell
CEA[41] (1968)	1166 1279	1.3;3.4;6.5;10.4	S and P waves $G_M^*(k^2)$
Desy[42] (1970)	1136 1276	15	S and P waves
Frascati[43] (1969)	$M+m_\pi = 1080$	5.2	$\lim_{q^x \to 0} \left(\frac{1}{q^x} \frac{d\sigma}{dr_{02}\, d\omega_e}\right) \leq 0.27\times10^{-31} cm^2/sr(\frac{GeV}{c})$

(1) - $\Delta = W = 1236$ MeV.

The results of the first one, made at Orsay[38], were in agreement within the experimental error of about 50%, with the theoretical prediction made by Salin[20] on the basis of an isobaric model.

The work of the Cornell group lead by Berkelman on both π^o and π^+ production, is particularly important because it represents the first attempt to analyze in detail, experimental data taken around the first resonance. Furthermore Berkelman himself contributed to the clarification of various aspects of the physical problem[3].

The values of k^2 and W explored by the different authors also are shown in Table 1. The more extensive investigations are those made by the CEA group[41] and by the Hamburg-Paris-Marburg group at Desy[42].

The CEA group uses a magnetic spectrometer in the electron channel and two hodoscopes, one in the electron the other in the hadron channels; the latter detects the electroproduced π^+ as well as the protons recoiling from π^o electroproduction. The identification of π^+ and p is made by the energy loss in two scintillation counters.

The DESY group uses magnetic spectrometers in both the electron and the hadrons channels.

With these experimental set-ups, both groups measure the angular distributions of the recoiling protons for a few values of W around the Δ and analyze their data in terms of S and P waves only.

Under this assumption the coefficients appearing in Eq. (17) have the form

$$A = A_o + A_1 \cos\vartheta^*_\pi + A_2 \cos^2\vartheta^*_\pi$$

$$B = B_o + B_1 \cos\vartheta^*_\pi + B_2 \cos^2\vartheta^*_\pi$$

$$C = C_o$$

$$D = D_o + D_1 \cos\vartheta^*_\pi .$$

From the analysis of its data the Desy group, for example, obtains the values of the quantities

$$\bar{A}_o = A_o + \varepsilon B_o , \qquad \bar{A}_1 = A_1 + \varepsilon B_1$$

$$\bar{A}_2 = A_2 + \varepsilon B_2 , \qquad C_o, D_o \text{ and } D_1$$

which are then expressed in terms of multipoles: M_1^+, E_1^+ and S_1^+ (S means scalar) of the $J = 3/2^+$ state and a non resonant part $\bar{A}_{os}$ of $\bar{A}_o$.

The measurements and the analysis is repeated for various values of W, so that they determine the dependence of M_{1+}, E_{1+}, S_{1+} and $\bar{A}_{os}$ on this variable around the Δ.

While the DESY group has made a very careful work for a single value of $k^2(= -15F^{-2})$, the CEA group(41) has made the measurements for a few values of this variable.

The dependence of the M_1^+ amplitude upon k^2 can be interpreted as the form factor of the $(\gamma N\Delta)$ transition(41)

$$(31) \qquad G_M^x(k^2) = 2M\left[\frac{3}{2}\,\alpha\,\frac{|\vec{q}^{\,x}|}{\sin^2\delta_{33}}\;\Gamma\,\frac{|M_1^+|^2}{|\vec{k}|^2}\right]^{1/2}$$

where Γ = 120 MeV is the width of the Δ and δ_{33} is the $P_{3/2\ 3/2}$ phase shift.

The result of the CEA group is in agreement with previous analysis of the same type, in particular with that of the Cornell group(39):

a) the electroproduction values of $G_M^x(k^2)$ join well to the $G_M^x(o)$ value deduced from photoproduction, thus confirming the interpretation of the electroproduction in terms of virtual photoproduction;

b) up to momentum transfers $-k^2 \simeq 10\ F^{-2}$, $G_M^x(k^2)$ is proportional to $G_M^V(k^2)$ as expected, independent of uncertainties in the neutron charge formfactor;

c) the experimental results on G_M^x are, however, also consistent with the exponential formfactor dependence suggested by Dufner and Tsai(44).

Fig. 8 shows a comparison of the experimental determinations of $G_M^x(k^2)$ with various theoretical predictions based on models some of

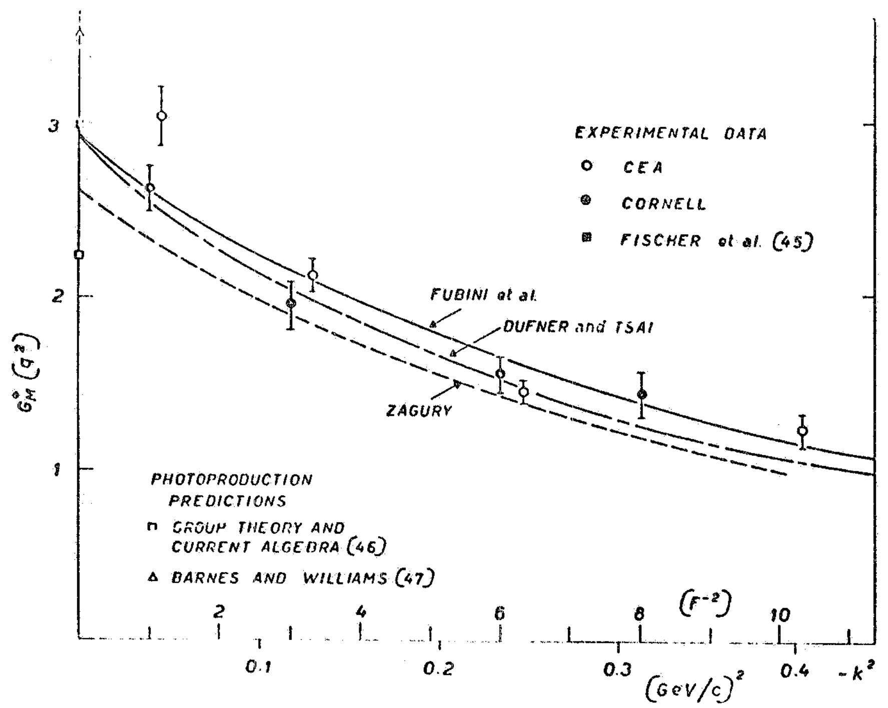

FIG. 8 - Transition form factor of the $(\gamma N\Delta)$ vertex.

which have been mentioned above(5, 12, 44), while others, not discussed here, involve group theory and current algebra(46, 47).

Finally the experimental result obtained at Frascati by the Pisa-Roma group(43), reported in the table, refers to an experiment made at threshold, by means of which an upper limit has been established for the quantity

$$\lim_{|\vec{q}^{x}| \to 0} \left(\frac{1}{|\vec{q}^{x}|} \; \frac{d\sigma}{dr_{02}\, d\omega_e} \right) \tag{32}$$

This type of information, clearly, is of a different nature from those deduced from the other experiments and will not be discussed here.

The experiments made until now on the π^+ electroproduction are listed in Table 2. The measurements of the Cornell group(48, 49) have been made by observing the pions emitted at $\vartheta_\pi^x = 0$ i.e. in the direction of the incident virtual photon. In this condition the pions are produced only by the longitudinal photons, because, as pointed out by Berkelman, they have no helicity to get rid of. A transverse photon has one unit of spin angular momentum, parallel or antiparallel to its motion (the Z axis), but a pion moving in the same direction has $J_z = L_z = 0$.

TABLE 2

Coincidence experiments on π^+ electroproduction.

	W (MeV)	$-k^2$ F^{-2}	Information deduced on:
Cornell(49) (1966 - 1967)	1175, 1200, 1300	1;3;6	F_π
CEA(41) (1968)	1166↔1279	1. 3;3. 4;6. 5;10. 4	F_π
Frascati(43) (1969)	$M+m_\pi = 1080$	5. 2	(F_π); $G_A(t)$

The reason for chosing these kinematical conditions is that the pion pole amplitude contains a factor

$$\left[(p_1 - p_2)^2 - m_\pi^2 \right]^{-1}$$

which, in the limit $k^2 = 0$, is proportional to

$$\frac{1}{1 - \beta_\pi \cos\vartheta_\pi^x}$$

The pole occurs at the unphysical angle

$$\cos \vartheta_{\pi}^{*} = \frac{1}{\beta_{\pi}}$$

and therefore the corresponding amplitude should reach its maximum, inside the physical region, at $\vartheta_{\pi}^{*} = 0$.

The measurements were taken by the Cornell group with magnetic spectrometers in both the electron and π^{+} channel, for W = 1175, 1200 and 1300 MeV and $-k^2 = 1, 3$ and $6\ F^{-2}$ and various values of ε ranging from 0.45 to 0.93.

Zagury's theory[12] was used for analyzing the experimental data: the theoretical cross section is evaluated for each of the experimental point, using the known nucleon from factors and bearing the pion form factor F_{π} as the only free parameter. For each data point the values of F_{π} is determined which gives agreement with the experiment.

The experiment of the CEA group[41] was made with the set-up mentioned in the discussion of the π^{0} electroproduction, which allowed the detection of the π^{+} emitted in a range of values of ϑ_{π}^{*} and φ_{π}^{*}, in coincidence with an electron scattered inelastically.

The data were analyzed with a procedure similar to that of the Cornell group, but using the theory of Zagury[12] as well as that of Adler[15], which become identical in the $k^2 = 0$ limit, but differ appreciably for value of k^2 around $10\ F^{-2}$;

The results of these two groups are shown in Fig. 9, where the

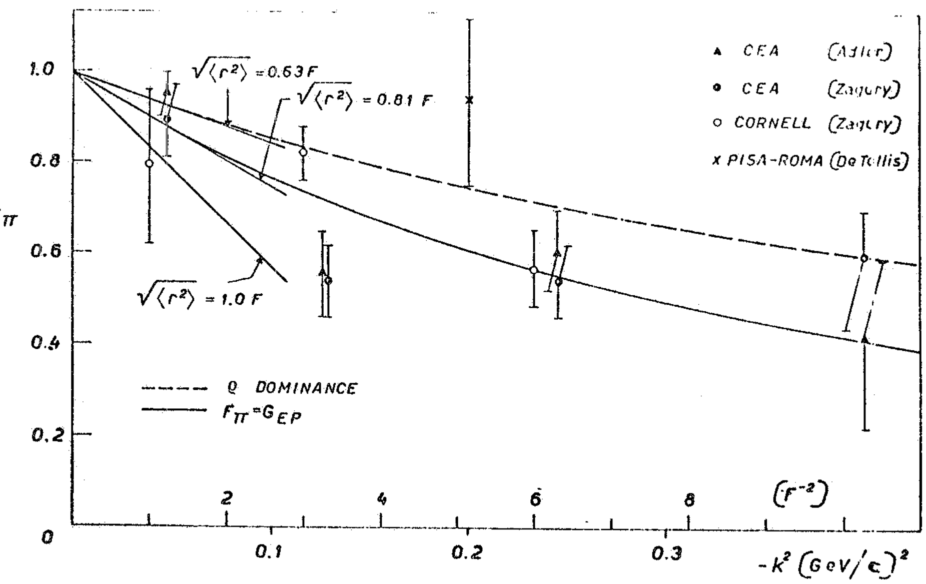

FIG. 9 - The values of the pion electromagnetic form factor deduced from electroproduction experiments and comparison with a few theoretical predictions.

broken line corresponds to ρ-dominance, while the full line to assume

$$F_\pi = G_{Ep} .$$

In Table 3 are given the values of the pion radius deduced from the Cornell and CEA groups and putting together all their data. They have been computed by assuming a fitting function of the form

$$F_\pi(k^2) = \frac{1}{1 + \frac{-k^2}{m^2}} \tag{33}$$

as suggested by the ρ-dominance.

TABLE 3

Pion radius from π^+ electroproduction.

		$\langle r_\pi^2 \rangle^{1/2}$ (F)	m (MeV)
Cornell (Zagury)	3 points	0.80 ± 0.10	
CEA (Zagury)	4 points	0.95 ± 0.11	507 ± 60
CEA (Adler)	4 points	0.84 ± 0.10	577 ± 70
Cornell + CEA (Zagury)	7 points	0.86 ± 0.09	560 ± 60
with model dependence error		0.86 ± 0.14	560 ± 80

If the differences in the values of $\langle r^2 \rangle^{1/2}$ and m are taken as an indication of the additional model dependent error, and are composed quadratically with the quoted errors, one gets the value written in the last line of the table.

These results should be compared with those obtained until now by other methods as shown in Table 4.

The first two experiments(50, 51) based on the comparison of the elastic scattering of positive and negative pions by ^{4}He, are in clear disagreement. Furthermore it has been pointed out that a correct analysis of data of this type would require a theoretical understanding of the strong interaction of pions with nuclei much more complete and detailed than available at present(54). The results of Allan, Ekspong et al. (52), are based on the observation in emulsion of knock-on electrons by pions of 16 GeV/c momentum. But even for a pion of p_π = 200 GeV/c the maximum fourmomentum transfer amounts to

$$-k^2 = 2m_e T_{emax} \lesssim 2m_e p_\pi = 0.2\,\left(\frac{GeV}{c}\right)^2 = 5\ F^{-2}\ .$$

TABLE 4

Pion radius from other method

	$\langle r_\pi^2 \rangle^{1/2}$ (F)	Method
Crow et al. (50)	2.96 ± 0.43	π^+ / π^- elast. scatt. on He
Block et al. (51)	0.9	
Allan, Ekspong et al. (52)	< 4.5	π -e elast. scatt.
Devons, Di Capua et al. (53)	< 1.9	radiative capture of π^- by protons.

Finally the experiment of Devons, Di Capua et al. (53) is based on the observation of the frequency of emission of electron-positron pairs in the radiative capture of negative pions by protons. As one can see from Fig. 10 the process involved is the same as in electroproduction (Fig. 2) apart from the fact that, in radiative capture, the momentum transfer k^2 is time like, and amounts to about 0.01 $(GeV/c)^2$.

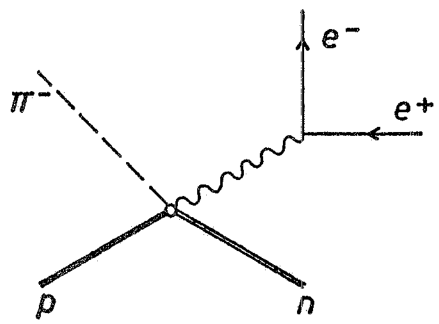

FIG. 10 - Graph describing the production of e^+-e^- pairs in the capture of negative pions by protons.

From a comparison of the data collected in Tables 3 and 4 one can conclude that, waiting for experiments on π -e elastic scattering made with high energy pions produced by 200-300 GeV machines, the study of electroproduction of π^+ appears to be the best approach at our disposal for determining the electromagnetic pion form factor; and even then it will be competitive provided the electroproduction theory attains an adeguate level of accuracy.

In the previous discussion I have omitted to talk of the experiment made at threshold by the Pisa-Rome group(43), which is based on a different principle.

Near threshold the cross section, integrated over the direction of emission of the pion (Eq. 19), can be written in the convenient form

$$(34)\qquad \frac{d\sigma}{dr_{02}\,d\omega_e} = \frac{\alpha^2}{2\pi^2}\,\frac{r_{02}}{r_{01}}\,\frac{M}{W}\,\frac{|\vec{q}^{\,*}|}{1-\varepsilon}\,\frac{1}{-k^2}\left[a_t + \varepsilon\, a_\ell\right]$$

where $\mathcal{A}_t$ and $\mathcal{A}_\ell$ are functions of k^2 and W only, or of k^2 and $|\vec{q}^x|$, since, according to Eq. (8), W depends only of $|\vec{q}^x|$.

On the basis of general arguments one can recognize that, the developments of $\mathcal{A}_t$ and $\mathcal{A}_\ell$ in powers of

$$y = \frac{|\vec{q}^x|}{m_\pi}$$

contain only even powers of this variable. Therefore, a plot, - made at k^2 = = constant - versus y^2 of the measured values of the quantity appearing inside the square bracketts in Eq. (34) should show, a linear behaviour for $y^2 \ll 1$. This model independent property allow the determination of the quantity

$$(35) \qquad \lim_{|\vec{q}^x| \to 0} \left(\frac{1}{|\vec{q}^x|} \frac{d\sigma}{dr_{02}\, d\omega_e} \right) = \frac{\alpha^2}{2\pi^2} \frac{r_{02}}{r_{01}} \frac{M}{W} \frac{1}{1-\varepsilon} \frac{1}{-k^2} \left\{ \left[E_0^+ \right]^2 - k^2 \varepsilon \left[\frac{L_0}{R_0^x} \right]^2 \right\}$$

which is very appealing for its semplicity.

The measurements of the Pisa-Rome group, made at $-k^2 = 5.2\, F^2 = 0.2\,(GeV/c)^2$, have been analyzed by means of the De Tollis and Nicolò formula valid at threshold and which takes into account only the Born terms[10]. Taking $G_{En} = 0$, as all other authors do in this type of analysis, one obtains a point rather high with respect to those of the Cornell and CEA groups (Fig. 9):

$$(36) \qquad F_\pi\, (0.2\, (\frac{GeV}{c})^2 = 0.95 \pm 0.20 \,.$$

If one takes

$$G_{En} = - G_{Mn} \frac{\tau}{1 + 4\tau}\,, \qquad \tau = \frac{|k^2|}{4M^2}\,,$$

one obtains

$$F_\pi\, (0.2\, (\frac{GeV}{c})^2 = 0.80 \pm 0.25 \,.$$

Considering the errors affecting the results of all the experiments, one can not state a clear disagreement between the Pisa-Rome point and the results of all other authors, also because the measured quantities are different, and their analysis are based on different models. In particular it should be pointed out that, while the value of F_π, deduced by threshold measurements, appears to be rather sensible to the value adopted for

$G_{En}(k^2)$, it is not known at present how large is the influence of a similar change of this quantity on the theoretical results of Zagury and Adler. Furthermore these theories are rather primitive in comparison with the approach of von Gehlen.

On the other hand the analysis of the experimental data at threshold, made by the Pisa-Rome group is based on the De Tollis and Nicolò formula, which involves the assumption that, for small values of k^2, the consideration of the Born terms only should be adequate. According to the recent calculations of von Gehlen, the dispersive contributions coming from the first resonance and the S-wave, all tend to increase the value of E_{0+} at threshold. The effect is $\approx 6\%$ in the amplitude in the case of π^+ photoproduction and increases going to electroproduction, reaching, at $-k^2 = 5\ F^{-2}$, about 15%. The situation is even more complicated since, always according to von Gehlen, the multipole $E_{0+}^{\pi+}$ is the only multipole in which the influence of the second resonance is important: at threshold it may be as large as -20%, possibly compensating the other dispersion contributions. Such an unexpected large influence of the second resonance at threshold should be further investigated aiming to obtain a quantitative estimate of the overall correction to the Born terms.

The interest of measurements at threshold lies, however, mainly in the possibility of their analysis in terms of theories based on current algebra and PCAC. The experimental result of the Pisa-Rome group gives

$$G_A(t) = (0.65 \pm 0.08)\, g_A \qquad t = 0.2\, \left(\frac{GeV}{c}\right)^2 \tag{38}$$

when compared with the computations of Paver and Verzegnassi, and

$$G_A(t) = (0.63 \pm 0.05)\, g_A \tag{39}$$

when analyzed with the Nambu and Yoshimura theory, taking in both cases $G_{En} = 0$. The indicated accuracies are obtained by comining quadratically only statistical and systematic experimental errors.

If one assumes for $G_A(t)$ the dipole expression

$$G_A(t) = g_A \frac{1}{\left(1 + \frac{t}{M_A^2}\right)^2} \tag{40}$$

one gets

$$M_A = 7 m_\pi = (0.98 \pm 0.06)\ GeV \tag{41}$$

This result should be compared with those of two recent single arm experiments made at threshold, one by a russian group(55), the other by a group at Daresbury(56). The russian group analyses its data, taken as a function of $|\vec{q}^{*}|$ - at $-k^2 = 5\ F^{-2}$ - essentially by the same methods used by the Pisa-Rome group, but uses the De Tollis and Nicolò formula for deriving information on G_{En} (and not on F_π) arriving to establish the following limits

(42) $$-0.39 \leq G_{En} \leq -0.31 \qquad \text{at } -k^2 = 5\ F^{-2} .$$

The analysis of the same data, in the frame of current algebra and PCAC assumption, is based on the papers of Gleeson et al. (24) and of Vainshtein and Zakhazov(57). Assuming for G_A the dipole expression (40) they deduce

(43) $$M_A = 1.05 \text{ GeV}$$

without quoting the corresponding experimental error, which, however, is, probably, around 20%.

The Daresbury group has made a similar experiment but analyses its data by comparison with Paver and Verzegnassi calculations and gives

(44) $$M_A = (1.05 \pm 0.20) \text{ GeV} .$$

The large error is probably due, at least in part, to the necessity (in the case of single arm experiments) of subtracting the contribution of reaction (1b) from the experimental data.

The two values (43) and (44) of M_A are inside the experimental error in agreement with the value (41) which, for the moment, is the only one obtained by a coincidence experiment.

They are lower than the value deduced by Nambu and Yoshimura(29) by making the best fit of their theory (with M_A as free parameter) of the experimental results of a few single arm experiments(34,35,58,59,41,42) covering values of $-k^2$ up to 7.3 $(GeV/c)^2$. Taking $G_{En} = 0$ and for $G_A(t)$ the dipole expression (40) Nambu and Yoshimura find

(45) $$M_A = (1.34 \pm 0.05) \text{ GeV} .$$

A similar analysis of the same experimental data, based on the assumption

$$G_A(t) = \frac{1}{1 + \frac{t}{M_A^2}}$$

seems to be ruled out.

Finally these various determination of M_A, derived from electroproduction experiments, should be compared with the results of the latest analysis[60] of the neutrino experiments, which gives, assuming again for G_A the dipole expression (40)

$$M_A = (1.05 \pm 0.20) \text{ GeV} . \tag{46}$$

From all these data it appears reasonable to conclude that, waiting for new neutrino experiments to be made with accelerators in the range of 200-300 GeV, any information derived from electroproduction can be very valuable, especially in the case of coincidence experiments which do not involve the subtraction from the measured data of the cross section for π^0 electroproduction computed by adopting some more or less reliable model.

5. - A FEW FINAL REMARK. -

By considering the overall picture of our present knowledge of electroproduction at low energy, one arrives to a few conclusive remarks which appear worthwhile to be mentioned.

Some of these refer to the theoretical aspects to the problem which clearly need to be improved in general, and in particular, along two main lines: the one based on the solution of the set of coupled integral equation (21) derived from fixed - t dispersion relations, the other based on current algebra and PCAC assumption or equivalent assumptions.

A discussion of the relationship between these two approaches has been initiated from both sides[18, 27, 28] but, one has the impression that much more should be done for reaching a complete understanding of the problem.

Also very useful would be a clarification of the relationship between the two approaches based on current algebra: the theory of Nambu and Yoshimura and that of Fubini and Furlan and coworkers.

This point has recently been considered by Furlan, Paver and Verzegnassi[27] and Verzegnassi[28], but the situation is not jet sufficiently clear for providing some guidance to the experimentalist that faces the problem of adopting the more appropriate procedure for deriving from the measured quantities, reliable information on $G_A(t)$, and possibly $G_p(t)$[28].

It would also be very useful if any of these theories could be cast in such a form to allow easily the introduction of changes in the value of $G_{En}(k^2)$. As one has shown above, most calculations have been made by assuming $G_{En}(k^2) = 0$; only von Gehlen[18] has made two sets of computations, one with

(47a) $$G_{En}(k^2) = \frac{k^2}{4M^2} G_{Ep} ,$$

the other with

(47b) $$G_{En}(k^2) = 3 \frac{k^2}{4M^2} G_{Ep} .$$

But even in this case it does not appear clear the type of dependence of the final results neither on $G_{En}(k^2)$ nor on $F_\pi(k^2)$, the form and value of which is introduced from the beginning in the computations by some specific assumption: either

(48a) $$F_\pi(k^2) = G_{Ep}(k^2)$$

or

(48b) $$F_\pi(k^2) = F_1^V (k^2) .$$

A clear understanding of these points would allow an appropriate choice of the kinematical conditions to be chosen in experiments aiming to the determination of the different unknown (or badly known) measurable quantities. Such a dependence is clearly shown only in De Tollis and Nicolò formula which appears, however, to be not sufficiently accurate.

The paper of von Gehlen as well as those of Furlan, Paver and Verzegnassi contain a number of very interesting and useful suggestions for the experimentalists, a few of which will be mentioned here as a conclusion. The first one refers to the importance of measuring, by coincidence experiments (at threshold and in its vicinity), the ratio

(49) $$\frac{\sigma_L(W, k^2)}{\sigma_T(W, k^2)}$$

where σ_L and σ_T are defined by Eq. (19b). This ratio has been measured by many authors by means of single arm experiments so that their results refer to the sum of the π^o and π^+ cross sections. In the case of π^+ electroproduction, this ratio appears to be one of the more appropriate quantities for distinguishing among the two different approaches since they give different predictions.

Furthermore it should be noted that in both approaches σ_L is larger for reaction (2a) than for reaction (1a), so that, the electroproduction of negative pions is expected to show more clearly one of the most typical aspects of photoproduction by virtual protons.

Finally a knowledge of $\sigma_v(k^2, W)$ defined by Eq. (19), for a few small values of k^2 (and W at threshold) would allow the determination of this quantity in the limiting case $k^2 \rightarrow 0$; as pointed out by Schwela(61), this limit provides an information very useful for the analysis of experimental photoproduction data, on a quantity which can not be measured directly (at threshold).

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More accurate tests of the one-photon assumption are provided by the search for effects that do not exist in this approximation such as: (a) a difference $\sigma_+ - \sigma_-$ between the differential cross-sections of positron and electrons scattering against nucleons, which depend on the real part of the two-photon exchange amplitude; (b) a polarization of the recoiling protons in e-p elastic collisions which depend on the imaginary part of the same amplitude. A few references to experiments of these types are:

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