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G. Paternò, M. V. Ricci and N. Sacchetti : EXPERIMENTAL TEST OF
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TIVITY OF TUNNEL JUNCTIONS

Experimental Test of the Microscopic Theory of a Current-Carrying Superconductor by Measurement of the Zero-Bias Conductivity of Tunnel Junctions

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The zero-bias conductivity of metal–insulator–superconducting-tin tunneling junctions has been measured as a function of the current flowing through the superconductor at various temperatures. Good agreement has been found with the microscopic theory by simply adjusting one parameter (the critical current) at an arbitrary temperature. Also the 0 °K critical current density is found to be of the expected order of magnitude.

I. INTRODUCTION

It is well known that the superconductive order can be reduced or even destroyed when certain physical factors, the so-called "depairing factors," are introduced in the superconductor. Within certain limits the effect of these factors can be studied in a general way by taking into account the fact that they provide a mechanism for breaking the time-reversal symmetry of the Cooper pairs, giving a finite lifetime to the pairs and thus decreasing the order of the system. Among these factors we recall the paramagnetic impurities, a magnetic field parallel to a thin film, and a uniform current. The theoretical aspects of the whole argument have been recently reviewed by Maki,[1] while experimental evidence of these effects has been reported by several researchers.[2-5]

In the present work we report measurements of the influence on the superconducting state of a supercurrent flowing through a thin film. Namely, we have determined the zero-bias conductivity $\sigma(0)$ of metal–insulator–superconductor tunnel junctions as a function of the current flowing through the superconducting film (tin in this case).

The experimental results are in good agreement with the theoretical calculations of $\sigma(0)$ in the whole range of temperatures and currents examined. Analogous measurements have been reported in the literature[6,7] but they are confined in a small range of temperatures near the critical temperature T_c. Moreover, they deduce the energy gap of the superconductor as a function of the current i from the I-V characteristics of the tunnel junctions. However, when we look at the tunneling characteristics in the presence of depairing factors, the correlation between these characteristics and the parameters describing the superconducting state becomes much more complicated owing to the effects on the density of states.[8] It is better then to measure quantities such as the zero-bias conductivity $\sigma(0)$, which can be directly compared with the theoretical values deduced from the microscopic theory in which the effects on the density of states are included.

Finally, a particular geometry for the junction has been adopted in order to avoid edge effects which can lead to doubtful results.

In Sec. II we review briefly the theoretical calculation of $\sigma(0)$ as a function of i. In Sec. III we give a detailed description of the tunnel junctions and of the detection appatatus, while Sec. IV is devoted to the experimental results and their discussion.

II. THEORY

In the theory of the tunnel effect, the differential conductance of a metal–insulator–superconductor junction is given by the following expression[9]:

$$\frac{dI}{dV} = \left(\frac{dI}{dV}\right)_N \int_{-\infty}^{\infty} \frac{N_s(E)}{N(0)} \left(\frac{1}{kT} \frac{e^x}{(1+e^x)^2}\right) dE , \qquad (1)$$

where $x = (E - eV)/kT$, $N_s(E)$ is the density of states in the superconductor, $N(0)$ is the density of states in the normal state at the Fermi level, E is the quasiparticle energy measured from the Fermi level, $(dI/dV)_N$ is the conductance when both metals are in the normal state, and V is the voltage applied across the junction.

In the absence of depairing factors a good description of $N_s(E)$ is given by the BCS theory, according to which

$$\frac{N_s(E)}{N(0)} = \begin{cases} 0 & \text{if } |E| < \Delta_0 \\ E/(E^2 - \Delta_0^2)^{1/2} & \text{if } |E| > \Delta_0 . \end{cases} \qquad (2)$$

We recall that $2\Delta_0$ is the so-called "energy gap" or "excitation gap" of the superconductor.

The presence of a uniform supercurrent gives rise to a modification in the quasiparticle excitation spectrum which no longer has the simple form given by Eq. (2). However, by using the Green's-function formalism[8] at $T = 0$ °K, it is possible to take into account of the effects of the current. The expression of the density of states then becomes

$$\frac{N_s(E)}{N(0)} = \frac{E}{\Delta}\left(\frac{1}{Z^{1/2}}\right)\text{Im}\left(\frac{1}{X-Z}\right) \quad . \qquad (3)$$

In the preceding equation, Δ is the order parameter in the presence of current. It differs from Δ_0 the energy gap in the absence of current.

Z is a parameter directly connected with the depairing factor, i.e., the current in this case. It depends on i, in the limit $Z < 1$, according to the following expression:

$$i/i_{max} = 0.947\, Z^{3/4}(\pi - \tfrac{4}{3}Z^{3/2})\exp(-\tfrac{3}{8}\pi Z^{3/2}) \quad (4)$$

and, in the same limit, the following relation between Z and Δ holds:

$$\Delta/\Delta_0 = \exp(\tfrac{1}{4}\pi Z^{3/2}) \quad . \qquad (5)$$

Moreover, the relation between X and Z can be found by resolving the quartic equation

$$X^4 - 2ZX^3 + \left[\frac{1}{Z}\left(\frac{E}{\Delta}\right)^2 - \frac{1}{Z} + Z^2\right]X^2 + 2X - Z = 0 \quad . \quad (6)$$

It follows then, once the value of i/i_{max} is fixed, it is possible to deduce Z from Eq. (4), Δ from Eq. (5), and [by solving Eq. (6)] $N_s(E)$ from Eq. (3). We point out that $N_s(E)$ is different from zero when Eq. (6) has an imaginary root. It is possible to show that such a root exists for $|E|$ greater than a certain value $\mathscr{E}_M$, while for smaller values all the roots are real and therefore $N_s(E)$ is zero. It is then clear that $\mathscr{E}_M$ (excitation gap) has a meaning analogous to Δ_0 in the BCS theory. It is related to Z and Δ by the equation

$$\mathscr{E}_M/\Delta = (1-Z)^{3/2} \quad .$$

When $Z = 1$, $\mathscr{E}_M = 0$, while Δ is different from zero, that is, the metal is still in the superconducting state with zero excitation gap (gapless superconductivity).

The above calculations are relative to the case of short mean free path in which limit Eqs. (4) and (5) are valid.

In our experimental conditions the short mean-free-path requirement is satisfied (film thickness of the order of 500 Å). Moreover, Z is confined to be smaller than 1, i.e., i/i_{max} is small.

By numerical integration of Eq. (1) we have calculated the current dependence of the zero-bias conductivity

$$\sigma(0) = \left[\frac{dI}{dV}\bigg/\left(\frac{dI}{dV}\right)_N\right]_{V=0}$$

and the results are shown in Fig. 4 of Sec. IV together with the experimental data.

The calculation of $N_s(E)$ is made in the case $T = 0\,°K$, that is, without making use of the temperature-dependent Green's functions. However, the good agreement between the calculated $\sigma(0)$ and the experimental data seems to indicate that the substantial contribution to the temperature dependence of $\sigma(0)$ arises from the derivative of the Fermi function which appears in Eq. (1).

III. EXPERIMENTAL

A. Sample Preparation

In Fig. 1 a schematic drawing of the junctions we have used is shown. The successive steps of fabrication are the following. First, a thick film of lead (ground plane) is vacuum evaporated on a standard microscope glass slide and then covered by a thick film of silicon monoxide. The aluminum film (0.5 mm wide) is then evaporated and subsequently oxidized in a low-pressure oxygen atmosphere (about 100 Torr) for a few minutes. The two silicon monoxide spots A and A' (200 Å thick) are then deposited before the evaporation of the tin
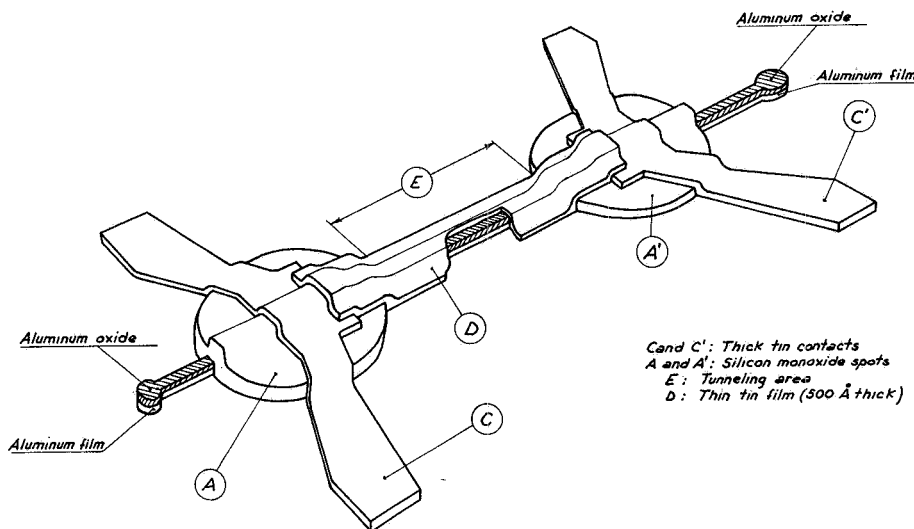

FIG. 1. Schematic drawing of a tunnel junction.

C and C': Thick tin contacts
A and A': Silicon monoxide spots
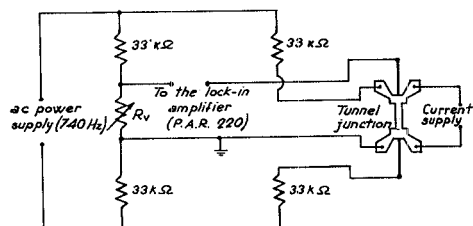E: Tunneling area
D: Thin tin film (500 Å thick)

FIG. 2. Measuring apparatus.

contacts C and C' which is made with the substrate cooled at liquid-nitrogen temperature. At this point the tin film D (500 Å thick and 2 mm wide) is evaporated, also at 77 °K.

The thickness of this last film has been determined within 10% by an oscillating quartz monitor.

As it is well known[10] the lead ground plane makes the current distribution in the tin film more uniform with the consequence of increasing the experimental critical current. The tunnel area is reduced to that marked as E in Fig. 1 because of the presence of the spots A and A' which prevent any substantial contribution to the tunnel current arising from the contact region. Moreover, the fact that the tin film is wider than the aluminum film makes negligible the edge effects.

The junction resistance when both metals are in the normal state ranged from a few ohms to some hundreds.

B. Measuring Apparatus

The zero-bias conductivity $\sigma(0)$ is measured by inserting the junction in the bridge whose electric circuit is shown in Fig. 2. A detailed description of this bridge is given in Ref. 11. The bridge is balanced with no current flowing through the tin film. The variations of the zero-bias resistance $\Delta R_g(0)$ induced by the current i produce a signal which is proportional to $\Delta R_g(0)$ as long as the deviations from balance are small. This signal is fed, through a lock-in amplifier, to the y axis of an x-y recorder, while the x axis is driven by a voltage proportional to i. The ac voltage across the junction (473 Hz) has been carefully controlled to be small enough (few tens of microvolts) to avoid any error in the measurement of $R_g(0)$ due to the strong nonlinearities of the tunnel characteristics. In Fig. 3 we show an example of a typical x-y recording.

C. Temperature Control and Measurement

Above the λ point the temperature has been controlled by means of a Cartesian manostate which gives a stability better than 10^{-3} °K. Below the λ point an electronic regulator has been employed, the temperature being controlled with the same stability as before. The temperatures have been obtained

by measuring the corresponding values of the liquid-helium vapor pressure and using the 1958 International Scale.

The critical temperature T_c of the tin films has been determined by recording directly the variations of $R_g(0)$ as a function of T. Starting from 4.2 °K and lowering the temperature, $R_g(0)$ is quite constant until the critical temperature T_c is reached; at this point there is a sudden increase of $R_g(0)$ corresponding to the onset of the superconducting state in the tin film. We have also used an indirect method to get T_c. Namely, we have deduced its value from measurements of the tin energy gap (at zero current) at temperatures very close to T_c. Using the expression for the temperature dependence of Δ_0 valid in this region,[12]

$$\Delta_0(T) = 1.81\Delta_0(0)[1 - (T/T_c)]^{1/2} ,$$

and the BCS relation $2\Delta_0(0) = 3.52KT_c$, the values of T_c calculated in this way are found to be in good agreement (within a few millidegrees) with those directly measured.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The critical temperatures of the tin films in the junctions we have used are very close to 4 °K, that is, about 0.3 °K higher than the bulk critical temperature. This is consistent with the fact that these films are quite thin (500 Å) and have been evaporated on cooled substrates. However, the dependence of the energy gap (at zero current) on the reduced temperature $t = T/T_c$ is in very good agreement with the BCS prediction.

As it can be seen from Eq. (4), in order to compare the experimental results with the theoretical calculations it is necessary to determine the critical current i_c of the tin film at the various tempera-
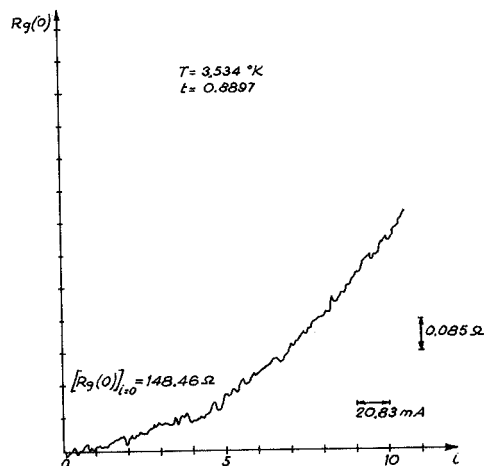
FIG. 3. Example of an x-y recording of the variation of $R_g(0)$ as a function of the current.

O $T = 3.714\,°K$; $t = 0.9350$
▲ $T = 3.534\,°K$; $t = 0.8897$
□ $T = 3.357\,°K$; $t = 0.8452$
■ $T = 2.320\,°K$; $t = 0.5841$
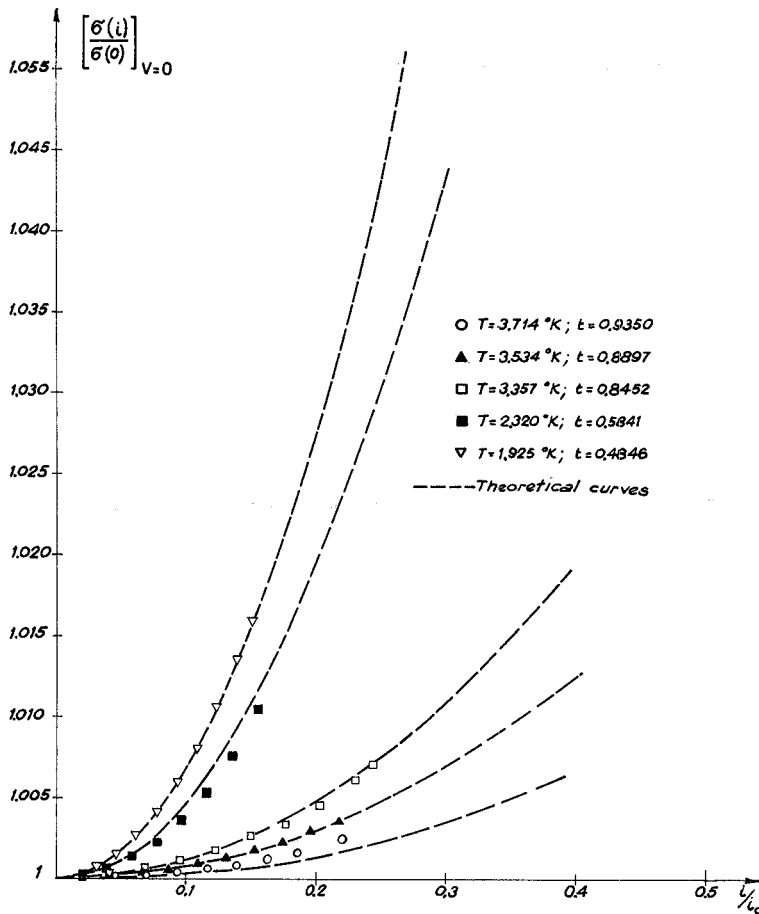▽ $T = 1.925\,°K$; $t = 0.4846$
— — —Theoretical curves

FIG. 4. Current dependence of the ratio $[\sigma(i)/\sigma(0)]_{V=0}$ at various temperatures. The temperature T^* at which the normalization has been made is 1.925 °K. The junction resistance at 4.2 °K is 113.05 Ω.

tures. On the other hand, it is well known that the experimentally determined transition current is not the "true" critical current of the film, but is much smaller.

The following procedure then has been adopted. As we have shown at the end of Sec. II, Eq. (1), when numerically integrated, allows one to calculate $[\sigma(i)]_{V=0}$ as a function of i/i_c. Then at a certain temperature T^* we have measured $[\sigma(i)]_{V=0}$ corresponding to a given value i_0 of the current that flows through the tin film. By comparison with the theoretical value, we have deduced $i_c(T^*)$. From Fig. 4 we see that this normalization, made at an arbitrary value of i, allows one to get a good agreement between the theoretical and experimental behavior in the whole range of currents examined, at the temperature T^*. For all the other temperatures the corresponding critical currents $i_c(T)$ have been calculated using the following expression[13]:

$$i_c(t) = i_c(0) \, (1 - t^2)^{3/2} \ . \tag{7}$$

Again from Fig. 4 we see that the agreement is satisfactory for all the temperatures reported.

We point out that the temperature dependence in Eq. (7) is the same as that of the critical current density j_c given in Ref. 13. Also we have made some direct measurements of critical currents defined by the onset of resistance in the tin film. However, the temperature dependence of such critical currents has been found to be in disagreement with any theory.[13] This can be understood if we keep in mind that the critical current corresponding to the onset of resistance depends on various factors, such as joule heating, edge effects, etc., and no particular care has been devoted to avoid these effects. The fact that the behavior of the critical currents obtained by the previously described normalization agrees with Eq. (7) can be explained by the presence of the superconducting lead ground plane and by the peculiar geometry adopted. In this way the tunneling area is in a region of uniform current and far from film edges.

An evaluation of the 0 °K critical current density $j_c(0)$ from the normalization critical current and the film dimensions relative to the junction of Fig. 4 gives the value $j_c(0) = 3.7 \times 10^7$ A/cm². For the same junction the theoretical value has been calculated using the expression[13]

$$j_c(0) = \tfrac{1}{2} H_c(0) \left(\frac{\Delta_0(0)}{\zeta \hbar} \right)^{1/2} \quad , \tag{8}$$

where $\Delta_0(0)$ is the zero-current energy gap at $0\,°K$ $[\Delta_0(0) = 3.52 KT_c]$, $H_c(0)$ is the bulk critical field of tin (304 G) and ζ is the residual resistivity experimentally measured. The obtained value of $j_c(0)$ is $2.2 \times 10^7 \, A/cm^2$.

These values can be compared with those reported by Glover and Coffey[14] for a 500-Å tin film evaporated on a cooled substrate, which were determined by quite a different technique: $3.2 \times 10^7 \, A/cm^2$ for the experimental value and $2.7 \times 10^7 \, A/cm^2$ for the theoretical one.

V. CONCLUSIONS

Our measurements constitute a good experimental confirmation of the microscopic theory of the effects of a uniform current in a superconducting film in the limit of short mean free path. The agreement shown by the data of Fig. 4 has been obtained by adjusting only one parameter: the critical current at a certain temperature T^*.

Moreover, the evaluated critical current density at $0\,°K$, $j_c(0)$, differs from the theoretical value by an amount which is of the same order of magnitude as that reported by other researchers.[14]

An inherent limitation of our measurements is due to the unavoidable thermal switching which, in spite of the presence of the ground plane, limits the current to values $< 0.3 i_c$.

To overcome this difficulty we are currently examining the possibility of extending these measurements to a multiply-connected geometry with the current controlled by an external magnetic field[15] in order to reach the gapless region.

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