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G. Parisi: A DUALITY SUM RULE FOR DEEP INELASTIC SCATTERING. -

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ABSTRACT. -

Duality is used to derive a sum rule for the difference between proton and neutron structure functions.

Recently Bloom and Gilman[1] have proposed to extend duality to deep inelastic scattering.

Their hypothesis is that $F_2(\omega) = \lim_{q^2 \to \infty} \nu W_2(\nu, q^2)$ can be decomposed into two terms, resonances and background: the background is only $I = 0$ in the t channel and the resonance contribution is not zero.

In this letter we want to show that from the assumptions of duality, SU(3) simmetry, absence of s-channel and t-channel exotic contributons and validity of Fubini - Dashen - Gell Mann[2] sum rule, we are able to derive a sum rule[3] for the difference between proton and neutron structure functions:

$$(1) \qquad \int_0^1 \left[F_{2P}(\omega) - F_{2N}(\omega) \right] \frac{d\omega}{\omega} = \frac{1}{3} .$$

This sum rule is nearly satisfied : the experimental value for the left hand side is 0. 19 with an error of 40%[3]. Of course we do not expect our sum rule to be exactly true in the real wordl : it may have a 20% error, due to SU(3) breaking. This sum rule was also obtained by P. V. Landshoff and J. C. Polkingorne[4] in the framework of the quark-parton model.

The derivation of eq. (1) is very simple. We denote by $F_{2B}^{ik}(\omega)$ the structure functions relative to $\langle B \,|\, J_i^{\mu}(x)\, J_k^{\nu}(0) \,|\, B\rangle$ where B is or the one proton or the one neutron state and $J_{\mu}^{i}(x)$ are the eight SU(3) currents.

Duality allows us to decompose $F_{2B}^{ik}(\omega)$ into resonances and background, and the background is only a t-channel SU(3) singlet.

Under the hypothesis of absence of exotic t channel exchange $(10, \overline{10}, 27)$, we can write the resonance contribution in the following way :

$$F_{2B}^{ik}(\omega) = \mathcal{E}(\omega)\left[\rho(\omega)\,\delta_{ik} + D(\omega)\,d_{ik8} \pm d(\omega)\,d_{ik3}\right] + \tag{2}$$

$$+ \quad F(\omega)\,f_{ik8} \pm f(\omega)\,f_{ik3}$$

where we must take the sign + for the proton and - for the neutron.

The absence of s-channel exotic resonances $(\overline{10}, 27)$ fixes :

$$d(\omega) + f(\omega) = 0 . \tag{3}$$

From the Fubini - Dashen - Gell Mann[2] sum rules :

$$\int_0^1 f(\omega)\,\frac{d\omega}{\omega} = \frac{1}{4} \tag{4}$$

follows that :

$$\int_0^1 \left[F_{2P}(\omega) - F_{2N}(\omega)\right]\frac{d\omega}{\omega} = -\frac{4}{3}\int_0^1 \frac{d(\omega)}{\omega}\,d\omega = \frac{1}{3} \tag{5}$$

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