COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-71/34
23 Giugno 1971

A. Bramon and M. Greco : PHOTON-PHOTON SCATTERING INTO HADRONS FROM FINITE ENERGY SUM RULES. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-71/34
23 Giugno 1971

A. Bramòn[(x)] and M. Greco: PHOTON-PHOTON SCATTERING INTO HADRONS FROM FINITE ENERGY SUM RULES. -

The production of hadrons in high energy $e^{\pm} e^{-}$ collisions by photon-photon scattering has been recently investigated by several authors[(1 ÷ 5)]. Hadrons are produced in a C = +1 state with logarithmically increasing cross sections which, at high energies, dominate over the more familiar one-photon processes. When the final electrons are detected within small angles θ_{1max} and θ_{2max} with respect to the initial beam direction ($\theta^2_{1,2max} \approx m/E$) the cross section for production of a final state F is

$$d\sigma_{ee\to eeF} = \left(\frac{2\alpha}{\pi}\right)^2 \int \frac{dk_1}{k_1}\frac{dk_2}{k_2} \frac{\left[E^2+(E-k_1)^2\right]\left[E^2+(E-k_2)^2\right]}{4E^2} \times$$

$$\times \left\{ \ln\frac{E(E-k_1)\theta_{1max}}{m k_1} - \frac{E(E-k_1)}{E^2+(E-k_1)^2} \right\} \left\{ \ln\frac{E(E-k_2)\theta_{2max}}{m k_2} - \right.$$

$$\left. - \frac{E(E-k_2)}{E^2+(E-k_2)^2} \right\} d\sigma_{\gamma\gamma\to F} ,$$

which is approximately given by

$$(1) \qquad \sigma_{ee\to eeF} \simeq \left(\frac{\alpha}{\pi}\right)^2 \ln^2\left(\frac{E}{m}\right) \int_{t_o}^{4E^2} \frac{dt}{t}\, \sigma^F_{\gamma\gamma}(t)\, f\left(\frac{t}{4E^2}\right)$$

with $f(y) = -(2+y)^2 \ln y - 2(1-y)(3+y)$. Here m, E are the mass and energy of the colliding electrons and $\sigma_{\gamma\gamma}^F(t)$ is the cross section for production of the state F by two real photons of total C.M. energy squared t with threshold t_o.

(x) - On leave of absence from Departamento de Fìsica Teòrica, Barcelona..

The contribution to eq. (1) of low energy processes, like π^0 and η production, and creation of pion and kaon pairs according to pure QED have been studied in refs. (1 ÷ 5). Some strong interaction corrections to $\gamma\gamma \to \pi\pi$ have also been considered. A complete theoretical analysis of $\sigma^F_{\gamma\gamma}(t)$, for any final state F, has not been done. However, due to the factor dt/t in the right hand side of eq. (1), low mass final states are expected to dominate the production cross section. Once $\sigma^F_{\gamma\gamma}(t)$ is expressed in terms of the lower lying resonances one expects therefore to get a fairly good description of the production mechanism. Remembering, in addition, that a two-photon system does not couple to a $J^P = 1^-$ state, the final states F therefore have the quantum numbers $J^P = 0^-$, 0^+ and 2^+.

The aim of this paper is to present an evaluation of the different contributions to the total hadronic cross section from the just mentioned states. We note that in the case of F being a pseudoscalar meson the situation seems rather firmly established[1, 2]. On the contrary, no definite predictions are available for the remaining cases, since the coupling of the two photon system to a scalar or tensor meson is unknown. In order to estimate such quantities duality and finite-energy sum rules are used in the way described by Aviv and Nussinov[6], who first applied these ideas to compute the $\omega \to \pi^0\pi^0\gamma$ decay rate, and more recently, by Gounaris and Verganelakis[7], who succesfully studied $\eta \to \pi^0\gamma\gamma$.

Briefly the procedure is as follows; consider Compton scattering from pseudoscalar mesons

$$P(q_1) + \gamma(-k_1) \to P(q_2) + \gamma(k_2) \tag{2}$$

and with the kinamatical invariants $s = (q_1 - k_1)^2$, $t = (k_1 + k_2)^2$, $u = (q_1 - k_2)^2$, $\nu = (1/4)(s-u)$ and $Q = 1/2\,(q_1 + q_2)$, write the Feynman - invariant amplitude as

$$F = \varepsilon_1^\mu \varepsilon_2^\nu T_{\mu\nu} \tag{3}$$

with the gauge-invariant tensor given by

$$\begin{aligned} T_{\mu\nu} = {} & A(\nu,t)\left\{ g_{\mu\nu}(k_1k_2) - k_{1\nu}k_{2\mu}\right\} + \\ & + B(\nu,t)\left\{ (Qk_1)Q_\nu k_{2\mu} + (Qk_2)Q_\mu k_{1\nu} - (Qk_1)(Qk_2)g_{\mu\nu} - (k_1k_2)Q_\mu Q_\nu \right\}. \end{aligned} \tag{4}$$

For low values of ν the amplitudes $A(\nu,t)$ and $B(\nu,t)$ are described in terms of contributions from nearby singularities in the s and u channels, namely the ρ, ω and B mesons. The high energy behaviour on the otherhand is parametrized by the Regge trajectories echanged in the t channel which, according to the two isospin possibilities, are P' and ε trajectories or A_2 and δ. The usual lowest-moment FESR is then applied to calculate the Regge residues. The details of this calculation are accurately discussed in refs.(6) and (7). Once the on-shell Regge residues are known it is easy to obtain the representation of the A and B amplitudes in the t channel, where the $\gamma\gamma$ system is coupled to the scalar and tensor mesons.

For the process $\gamma\gamma \to \delta, A_2 \to \eta\pi$ the result is

(5a) $$A^{\delta}(\nu, t) = \frac{2\beta_A^{\delta}}{t - m_{\delta}^2 + i m_{\delta}\Gamma_{\delta}}$$

(5b) $$A^{A_2}(\nu, t) = \frac{2\beta_A^{A_2}}{t - m_{A_2}^2 + i m_{A_2}\Gamma_{A_2}} \frac{1}{24} \lambda(m_{A_2}^2, m_{\eta}^2, m_{\pi}^2) P_2(\cos\theta_t)$$

(5c) $$B^{A_2}(\nu, t) = \frac{2\beta_B^{A_2}}{t - m_{A_2}^2 + i m_{A_2}\Gamma_{A_2}}$$

where

$$\lambda(m_1^2, m_2^2, m_3^2) \equiv \left[m_1^2 - (m_2 + m_3)^2\right]\left[m_1^2 - (m_2 - m_3)^2\right],$$

$$\beta_A^{\delta} = 0.118, \quad \beta_A^{A_2} = -0.128, \quad \beta_B^{A_2} = 0.0154, \quad \nu = \frac{1}{4}\left\{(t - m_{\eta}^2 - m_{\pi}^2)^2 - 4 m_{\eta}^2 m_{\pi}^2\right\}^{1/2} \cos\theta_t$$

and θ_t is the production angle of the $\eta\pi$ system in the C.M. frame of the two photons. The masses appearing into $\lambda(m_1^2, m_2^2, m_3^2)$ are expressed in GeV. In order to take account of the finite width of the δ and A_2 resonances which occur at the physical region in the t channel, we have phenomenologically added an imaginary part to $\alpha_{\delta}(t)$ and $\alpha_{A_2}(t)$. Notice also that the δ pole contributes only to $A(\nu, t)$.

Consider now the reaction $\gamma\gamma \to \pi^0\pi^0$ with the ε meson as intermediate state. From the smallness of the $\varphi \to \pi^+\pi^-\gamma$ decay rate ($\Gamma_{\varphi\to\pi^+\pi^-\gamma} < 0.16$ MeV)(8) and considering the ε meson as a member of an octet, one gets from VMD $g_{\varepsilon\gamma\gamma} = (-2/3)(e/f_3)\, g_{\varepsilon\omega\gamma}$. This relation together with the results from ref. (6) yields

(6) $$A^{\varepsilon}(\nu, t) = \frac{2\beta_A^{\varepsilon}}{t - m_{\varepsilon}^2 + i m_{\varepsilon}\Gamma_{\varepsilon}}$$

where $\beta_A^{\varepsilon} = 1.12\,(e/f_{\varsigma})$. It is worth noting that SU(3) symmetry gives for the ratio $R = g^2_{\varepsilon\omega\gamma}\, g^2_{\varepsilon\pi^0\pi^0} / g^2_{\delta\gamma\gamma}\, g^2_{\delta\eta\pi}$ the value $3f_{\varsigma}^2/4e^2$ which agrees well with R = 210 as can be easily deduced from (5a) and (6). This fact provides a good check for the above calculations.

For the reaction $\gamma\gamma \to f \to \pi^0\pi^0$ we obtain from ref. (6) and SU(3) symmetry the following results for the invariant amplitudes $A(\nu, t)$ and $B(\nu, t)$

(7a) $$A^f(\nu, t) = \frac{2\beta_A^f}{t - m_f^2 + i m_f \Gamma_f}\,\frac{1}{24}\,\lambda(m_f^2, m_\pi^2, m_\pi^2)\, P_2(\cos\theta_t)$$

(7b) $$B^f(\nu, t) = \frac{2\beta_B^f}{t - m_f^2 + i m_f \Gamma_f}$$

where $\beta_A^f = \sqrt{3}\,\beta_A^{A_2} - \frac{4}{3}\frac{e}{f_{\varsigma}}\,4.7$, $\beta_B^f \cong \sqrt{3}\,\beta_B^{A_2}$ and

$$\nu = \frac{1}{4}\, t\left(1 - \frac{4 m_\pi^2}{t}\right)^{1/2} \cos\theta_t .$$

The evaluation of the cross sections for two-body processes is straightforward. From equation (3) we have

(8) $$\sigma_{\gamma\gamma\to PP'}(t) = \frac{1}{32\pi^2}\,\frac{1}{t}\int \frac{d^3q_1}{2q_{10}}\,\frac{d^3q_2}{2q_{20}}\,\delta(k_1 + k_2 - q_1 - q_2)\, T_{\mu\nu}\, T^{\mu\nu}$$

where the photon polarizations have been explicitly averaged and $T_{\mu\nu}$ is determined through eqs. (5), (6) and (7). An equivalent expression for $\sigma^F_{\gamma\gamma}(t)$, valid near a resonance R of angular momentum J, is

(9) $$\sigma_{\gamma\gamma\to R\to PP'}(t) = 8\pi(2J+1)\,\frac{\Gamma(R\to\gamma\gamma)\,\Gamma(R\to PP')}{(t - m_R^2)^2 + m_R^2\Gamma_R^2}$$

Comparing eqs. (8) and (9) we find

(10a) $$\Gamma(\delta \to \gamma\gamma)\,\Gamma(\delta \to \eta\pi) = 3.7\ \mathrm{MeV}^2$$

(10b) $$\Gamma(\varepsilon \to \gamma\gamma)\,\Gamma(\varepsilon \to \pi\pi) = 1.5\ \mathrm{MeV}^2$$

(10c) $$\Gamma(A_2 \to \gamma\gamma)\,\Gamma(A_2 \to \eta\pi) = 5 \times 10^{-3}\ \mathrm{MeV}^2$$

(10d) $$\Gamma(f \to \gamma\gamma)\,\Gamma(f \to \pi\pi) \simeq 0.1\ \mathrm{MeV}^2$$

where in the $\varepsilon, f \to \pi\pi$ decays all the charged pion states are included. Taking $\Gamma(\delta \to \eta\pi) \simeq 70$ MeV, $\Gamma(\varepsilon \to \pi\pi) \simeq 250$ MeV, $\Gamma(A_2 \to \eta\pi) \simeq$ $\simeq 15$ MeV and $\Gamma(f \to \pi\pi) \simeq 120$ MeV[8] we also obtain

(11) $$\Gamma(\delta \to \gamma\gamma) = 50\ \mathrm{keV} \qquad \Gamma(\varepsilon \to \gamma\gamma) = 6\ \mathrm{keV}$$
$$\Gamma(A_2 \to \gamma\gamma) = 0.3\ \mathrm{keV} \qquad \Gamma(f \to \gamma\gamma) \cong 0.8\ \mathrm{keV}.$$

The production cross sections are easily evaluated by substituting eq. (8) into (1). They are shown in Fig. 1 in the case of δ and ε mesons. The contribution coming from A_2 and f turns out to be negligeable. For $\pi^+\pi^-$ production we have shown the pure QED predictions and the complete cross section which is deduced from:

(12) $$\sigma_{\gamma\gamma}^{\pi^+\pi^-}(t) = \frac{1}{32\pi}\frac{\beta}{t}\Bigg\{ 4e^4\left[1 + \frac{4m_\pi^2}{t}\left(1 - \frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right) + \frac{8}{\beta}\frac{m_\pi^4}{t^2}\ln\frac{1+\beta}{1-\beta}\right] + $$
$$+ \frac{t^2}{(t - m_\varepsilon^2)^2 + m_\varepsilon^2\Gamma_\varepsilon^2}\,\beta_A^{\varepsilon\,2} +$$
$$+ \frac{8m_\pi^2}{\beta}e^2\beta_A^\varepsilon\frac{m_\varepsilon^2 - t}{(t - m_\varepsilon^2)^2 + m_\varepsilon^2\Gamma_\varepsilon^2}\ln\frac{1+\beta}{1-\beta}\Bigg\}$$

where $\beta = (1 - 4m_\pi^2/t)^{1/2}$.

For the sake of completeness we have also plotted in Fig. 2 the cross section for production of π^0, η and η'. They have been obtained using $\Gamma(\pi^0 \to \gamma\gamma) = 7.8$ eV, $\Gamma(\eta \to \gamma\gamma) = 1$ keV[8] and $\Gamma(\eta' \to \gamma\gamma) =$ $= 55$ keV. The latter partial width has been deduced from SU(3) symmetry assuming a quadratic mass formula for the $\eta - \eta'$ mixing[9, 10].

Let us briefly discuss our results. In the reaction $ee \to ee\pi^+\pi^-$ the Born terms practically dominate over all other exchanges. Similar conclusions have been drawn by Lyth[5] using dispersion relations and unitarity. On the contrary our result is smaller by a factor of about 8 than

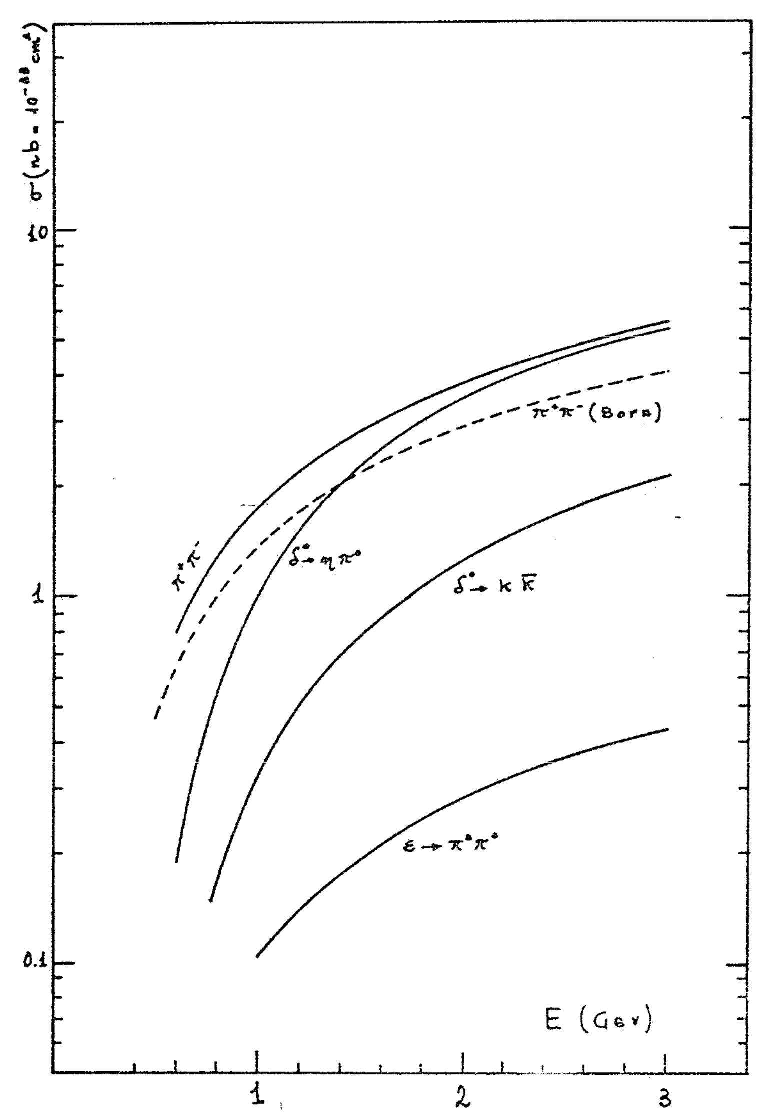

FIG. 1 - Total cross sections for δ and ε production. The values m_{ε} = 0.8 GeV and m_{δ} = = 1 GeV have been used.

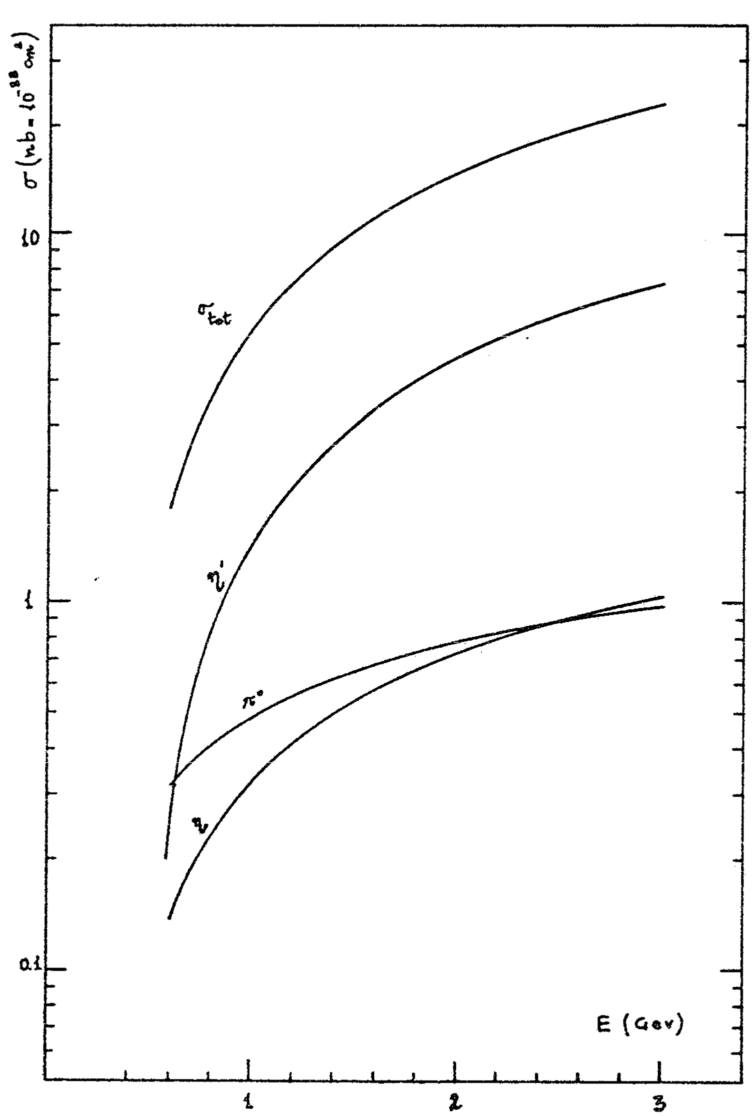

FIG. 2 - Total cross section for production of π^0, η and η'. σ_{tot} refers to the total hadronic two photon cross section.

that obtained by Brodsky, Kinoshita and Terazawa(2). A reason for such a discrepancy can be traced to the fact that in the superconvergent sum rule used by these authors only the ε resonance was taken into account. If we had used only the ε and δ poles to explain the $\omega \to \pi^o \pi^o \gamma$ and $\eta \to \pi^o \gamma\gamma$ decays, our results (10a, b) would have increased by more than a factor 10.

For high multiplicity productions we find that the δ and η' intermediate states dominate over all others. Adding together the different $\sigma_{ee \to eeF}$ for any state F we show in Fig. 2 the total cross section for hadron production by $\gamma\gamma$ scattering. It can be seen that the predicted rate for hadron production by two-photon mechanism would exceed 10^{-32} cm^2 at E = 1.5 GeV and is therefore comparable with the e^+e^- annihilation cross sections. If follows that the use of tagging systems is necessary to separate the one-photon from the two photon processes.

After this work was completed we have received the paper "Finite-Energy Sum Rules and the Reactions ee $\to$ ee ε(750) and ee $\to$ $\to$ eef(1260)" by B. Schrempp-Otto, F. Schrempp and T. F. Walsh, DESY 71/20, where some of the arguments discussed above are studied in a similar way.

One of us (A. B.) acknowledges G. I. F. T. for financial support.

REFERENCES. -

(1) - N. Arteaga-Romero, A. Jaccarini, J. Parisi and P. Kessler, Lett. Nuovo Cimento 4, 933 (1970) and 1, 935 (1971).

(2) - S. J. Brodsky, T. Kinoshita and H. Terazawa, Phys. Rev. Letters 25, 972 (1970) and preprint 1971.

(3) - V. N. Budnev and I. F. Ginzburg, Novosibirsk preprint TP-55 (1970).

(4) - M. Greco, Frascati Internal Report LNF-71/1, to be published on Nuovo Cimento.

(5) - D. H. Lyth, University of Lancaster preprint (1970).

(6) - R. Aviv and S. Nussinov, Phys. Rev. D 2, 209 (1970).

(7) - G. J. Gounaris and A. Verganelakis, CERN preprint TH-1310 - March 1971.

(8) - Particle Date Group, Revs. Modern Phys., Vol. 43, n. 2, (1971).

(9) - H. Harari, Proceedings of the Vienna Conference, 1968.

(10) - A. Baracca and A. Bramón, Nuovo Cimento 69A, 613 (1970).