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LNF - 71/30  
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G. Parisi and F. Zirilli: HARD BREMSSTRAHLUNG IN  $e^+e^-$   
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It is well known that there are important radiative corrections to  $e^+e^-$  scattering due to bremsstrahlung. It is usually assumed that these corrections are important only for the emission of soft photons; in the case they are hard, the angle between them and the direction of at least one of the incoming or outgoing electrons should be of order  $m/E_\gamma$ .

The aim of this letter is to show that there is an important contribution to the total production cross section due to the process  $e^+e^- \rightarrow e^+e^-\gamma$  where the final photon is hard and emitted with a large angle. An approximate evaluation brings to the following typical results, we denote by  $\bar{\theta}$  the angle between the incoming and the outgoing  $e^-$  in the center of mass frame, by  $\theta_{in}$  and  $\theta_{out}$  the angles between the photon and the incoming and outgoing  $e^-$ , by  $\eta$  the ratio between the photon energy  $E_\gamma$  and the beam energy  $E$ . With this notation the cross section for the production of  $e^+e^-\gamma$  with  $135^\circ > \bar{\theta} > 45^\circ$ ,  $\theta_{in}, \theta_{out} > 20^\circ$  is  $\sim 0.6 \sigma_\mu$  where  $\sigma_\mu = \pi\alpha^2/3E^2$  is the total cross section for the production of a  $\mu^+\mu^-$  pair.

This result is brought out in the following way; using almost real approximation, one gets for the differential cross section, in the case of zero mass electrons:

$$(1) \quad d\sigma = \sigma(\bar{\theta}) d\Omega_{e^-} \frac{\alpha}{4\pi^2} \left[ \frac{1}{\sin^2\theta_{in}} + \frac{1}{\sin^2\theta_{out}} \right] \frac{1+\eta^2}{\eta} d\Omega_\gamma d\eta$$

where

$$(2) \quad \sigma(\bar{\theta}) = \frac{\alpha^2}{8E^2} \left[ \frac{1+\cos^4(\frac{\theta}{2})}{\sin^4(\frac{\theta}{2})} - \frac{2\cos^4(\frac{\theta}{2})}{\sin^2(\frac{\theta}{2})} + \frac{1+\cos^2\theta}{2} \right]$$

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2.

is the differential cross section for the Bhabha scattering.

This formula is obtained if one treats  $\gamma$ ,  $\theta_1$ ,  $\theta_2$  as small quantities. If we try to use this formula at all angles and energy, we find that the ratio between  $\sigma(\bar{\theta})$  and the cross section for the production of  $e^+e^-\gamma$  with the same  $\bar{\theta}$  and  $\gamma > 0.1$ ,  $\theta_1, \theta_2 > 20^\circ$  is  $\sim 40$ .

By a simple integration over  $\bar{\theta}$  we obtain the required cross section. In order to test the reliability of this result, we have to check the validity of formula (1) in the large angle region.

To do this we have performed the exact calculation of the differential cross section, not using the trace theorems, but calculating the helicity amplitudes and summing their squared absolute value<sup>(1)</sup>. The results are in perfect agreement with the trace calculations of ref. (2).

The ratio between the exact value and formula (2) is plotted in Figs. 1 and 2.

The cross section is calculated at  $\bar{\theta} = 90^\circ$ ,  $\omega$  and  $\psi$  are the latitude and longitude of a polar coordinate system where  $\omega = 0$ ,  $\psi = 0$  axis is in the beam direction and the  $\omega = 90^\circ$ ,  $\psi = 0$  axis is in the direction of the outgoing electron.

One can easily see that our approximation is not too bad: eq. (2) gives in the average the correct result. The most prominent departure is given by a destructive interference which causes a zero at  $\psi \sim 180^\circ$ ,  $\omega \sim 45^\circ$ .

We can conclude that our result is essentially correct and therefore the process  $e^+e^- \rightarrow e^+e^-\gamma$  gives a relevant contribution to the multiple production of non collinear particles at large angle in  $e^+e^-$  colliding beam.

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#### REFERENCES. -

- (1) - The details of this method can be found in: G. Parisi and F. Zirilli, to be published.
- (2) - S. M. Swanson, Phys. Rev. 154, 1601 (1967).

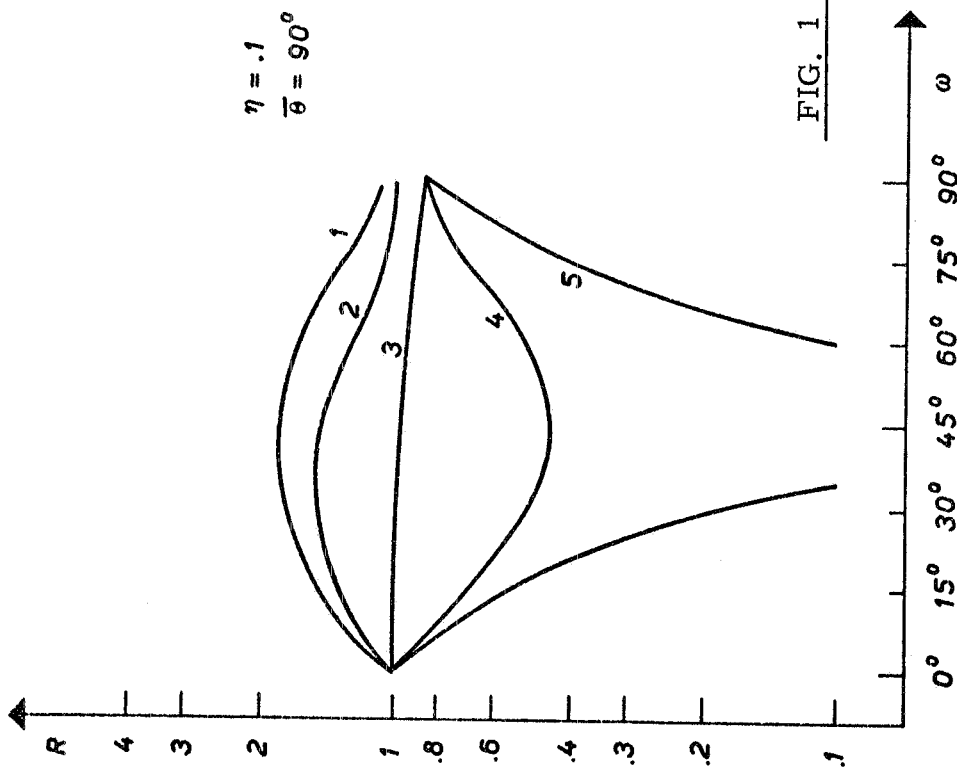


FIG. 1

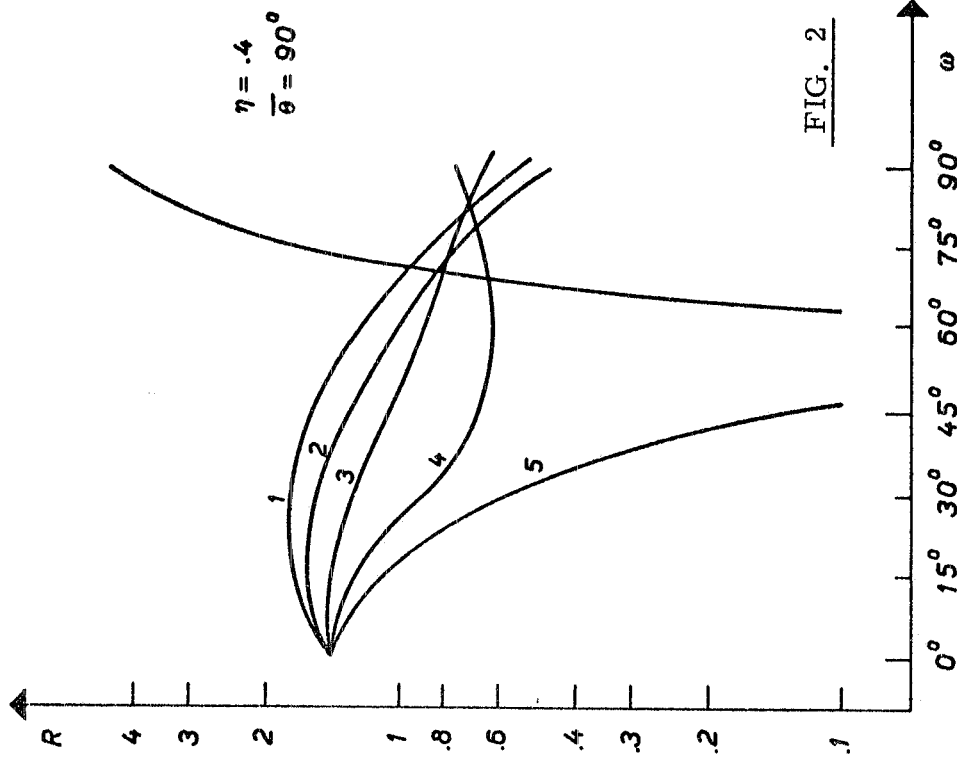


FIG. 2

FIG. 1-2 - R is the ratio between formula (1) and the exact cross sections. The angle  $\varphi$  has the following values: curve 1:  $\varphi = 0$ ; 2:  $\varphi = 45^\circ$ ; 3:  $\varphi = 90^\circ$ ; 4:  $\varphi = 135^\circ$ ; 5:  $\varphi = 180^\circ$ .