M. T. Vaughn: COLLIDING BEAM REACTIONS. -

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COLLIDING BEAM REACTIONS[x]

I. - INTRODUCTION. -

Let me begin with a brief survey of the types of reactions which can be measured in e^+e^- colliding beam experiments, and the possible theoretical importance of such measurements.

I.1. - QED processes. -

Reactions such as

$$e^+ e^- \rightarrow e^+ e^-$$
$$\mu^+ \mu^-$$
$$\gamma \gamma$$
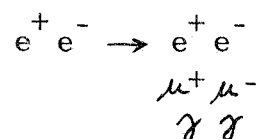
serve as a test of conventional quantum electrodynamics, and have already been measured[1]. Once the validity of QED has been checked, the elastic scattering reaction can serve as a beam monitor for expe riments involving other reactions.

Higher order processes, involving either hard bremsstrahlung or multiple e^+e^- pair production can also be significant as a background to hadronic processes[2]; they are also interesting in themselves as a further test of QED.

I.2. - Two-body and quasi-two-body hadronic reactions. -

These serve as a test of the vector dominance model, and of particular models (mainly derived from current algebra) for definite final states. Since these models are the main subject of the lectures, I will not discuss them at length now.

I.3. - Higher mass vector mesons. -

The Veneziano model predicts the existence of vector mesons with the same internal quantum numbers as ρ and ω with mass $1.6 \sim$ ~ 1.7 GeV, and with the same internal quantum numbers as φ with mass ~ 2.0 GeV, and an infinite chain of higher mass vector mesons as well. There is, however, no convincing estimate of the strength of the coupling of these vector mesons to the photon, so they may be difficult to discover, but it is important to find strong upper limits on these coupling strengths. An alternative indication of the existence of these vector mesons may be found in dips in particular reaction cross-sections about midway (on an m^2 scale) between two vector me sons (I will say more about this at the end); this is especially feasible

with the range of energies at Frascati.

I.4. - High-energy behaviour of total cross-sections.-

It is important to determine the high-energy behavior of total cross-sections, in order to test spectral function sum rules derived from different current algebra models (see below). Frascati energies are not really asymptotic (one does not expect asymptotic behavior to begin until well beyond thresholds for baryon-antibaryon production), but measurements in the Frascati energy region are important for extrapolation.

I.5. - Crossed electroproduction and structure functions.-

Experiments in which one (or two) particles in the final state are singled out for measurement, in analogy to the deep inelastic electron-scattering experiments at SLAC, are necessary to determine the behavior of structure functions in the time-like region. As mentioned above, Frascati energies are not really asymptotic, but the measurements are still useful.
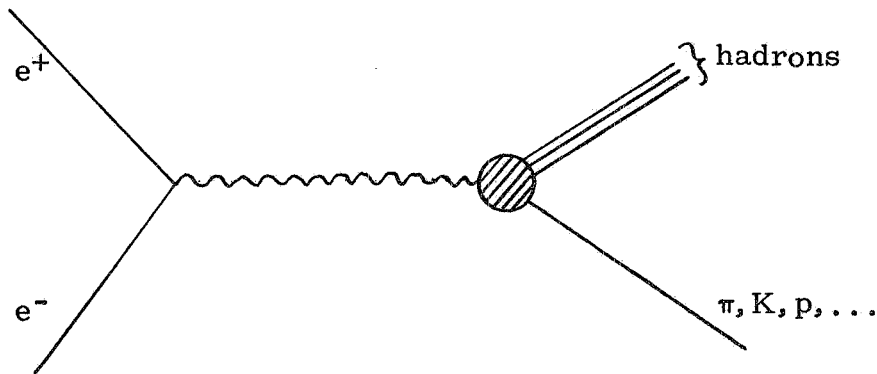
I.6. - γ - γ processes.-

A number of authors[3] have emphasized the importance of the "γ - γ" processes illustrated in the diagram below. Although these processes are formally of higher order in the fine structure constant, they are important in some kinematical regions, and increa singly at higher energies. They are of interest in themselves, and provide a background to the one-photon processes which must be eliminated for a clean measurement of the latter.

e^+ e^+

} hadrons

e^- e^-
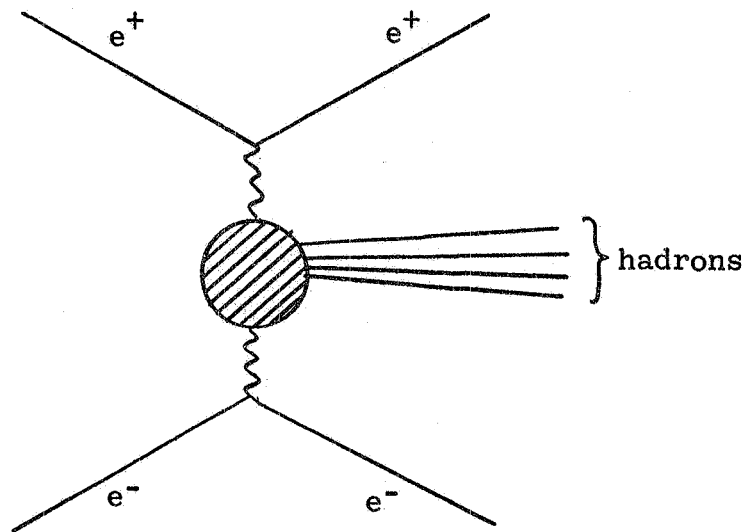
II. - STANDARD VECTOR DOMINANCE MODEL. -

To lowest order in the fine structure constant, the matrix element for the reaction

(1) $$e^+(p) + e^-(p') \longrightarrow (\text{hadrons})$$

is given in the form

(2) $$M = e^2 l_\mu(p, p') \frac{1}{t} \langle n | J_\mu^{e\text{-}m}(o) | vac \rangle$$

where l_μ is the standard matrix element of γ_μ between electron and positron spinors, $t = (p+p')^2$, $J_\mu^{e\text{-}m}(o)$ is the electromagnetic current operator, and $\langle n |$ is the state vector of the hadronic final state. This matrix element corresponds to the conventional picture

e^+

γ } hadrons

e^-
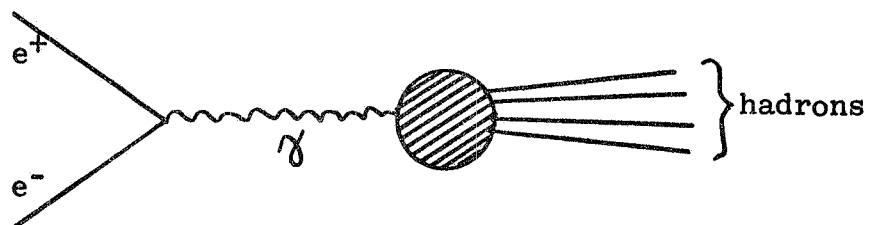
6.

It is then straight forward to derive formal expressions for any particular reaction; of importance here is the formula for the total hadronic cross-section (to lowest order in α)

(3)

$$\sigma \, (e^+ e^- \rightarrow \text{hadrons}) =$$

$$= \frac{2\alpha^2}{3 t^2} \sum_n \left| \langle n | J_\mu^{e-m}(0) | 0 \rangle \right|^2 \delta \, (P_n - p - p')$$

The summation on the right-hand side is precisely the spectral function appearing in the standard (Lehmann) representation of the propagator of the electromagnetic current, for which varions sum rules exist[4].

The content of the standard vector dominance model is expressed by the field-current identity of Kroll, Lee and Zumino[5, 6]

(4)

$$J_\mu^{e-m} = g_\rho \, \rho_\mu + g_\omega \, \omega_\mu + g_\varphi \, \varphi_\mu$$

in which ρ_μ, ω_μ, φ_μ denote renormalized field operators for ρ, ω and φ mesons, and parameters g_ρ, g_ω, g_φ are directly related to the corresponding partial widths for decay into lepton pairs. An $\omega - \varphi$ mixing angle θ_Y can be defined by

(5)

$$\frac{g_\omega}{g_\varphi} \simeq \sin \theta_Y$$

(the current-mixing angle of Kroll, Lee and Zumino[5]).

There are two interpretations of the field-current identity Eq.(4), analogous to the two interpretations of PCAC (partial conservation of the axial-vector current $\partial_\mu A_\mu = F_\pi \, m_\pi^2 \, \phi_\pi$),

(i) Eq. (4) defines interpolating fields for ρ, ω, and φ, in the sense of Lehrmann, Symanzik and Zimmermann (we overlook the finite widths of the vector mesons here),

or (ii) Eq. (4) is true with fields which are essentially canonical feelds.

The second interpretation makes it more plausible that when the poles in the vector meson propagators are removed from the matrix elements of Eq. (4), the remaining factors are smooth functions

of the momenta[7], an assumption which is implicit in all vector meson dominance models.

There are formal difficulties with the second interpretation, since it is not true in a careful treatment of the Lagrangian model of Kroll, Lee and Zumino[8]. However, the practical impact of these difficulties is not large, and I will continue to use the language of the conventional model.

The matrix elements of the electromagnetic current operator, then, are replaced by the product of vector meson propagators and vertex functions appropriate to the final states in question, the vertex functions being assumed to be smooth functions of the particle momenta. The theory of hadron production in colliding beam reactions is then a theory of the vector meson propagators, and the vertex func-tions
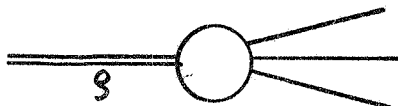
leading to I = 1 final states, and
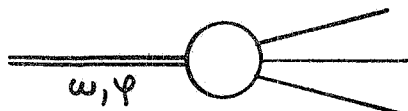
leading to I = 0 final states.

One further generality should be noted. The formula of Gell--Mann and Nishijima,

$$(6) \qquad Q = T_3 + \frac{1}{2} Y$$

(Q = electric charge, T_3 = isotopic spin component, Y = hypercharge); is generalized to a relation between currents

$$(7) \qquad J_\mu^{e-m} = j_\mu^3 + \frac{1}{\sqrt{3}} j_\mu^8$$

where j_μ^3, j_μ^8 are members of an octet of currents which satisfy the Gell-Mann current algebra[9]. From Weinberg's first sum rule[4] for the spectral functions of these currents, it follows that

8.

(8)
$$\int \left[\sigma^{I=1} (e^+ e^- \to \text{hadrons}) - 3\, \sigma^{I=0} (e^+ e^- \to \text{hadrons}) \right] t\, dt = 0$$

This sum rule should be tested experimentally (eliminating background due to γ-γ processes, etc., of course). If we tentatively accept it, then we expect that, on the average, $I = 0$ final states are produced only $1/3$ as frequently as $I = 1$ final states.

III. - VECTOR MESON PROPAGATORS. -

The further development of the theory requires a model for the vector meson propagators. Neglecting spin indices, we require a propagator $\Delta_\rho (t)$ for the ρ-meson, and a 2×2 matrix propagator $\underset{\sim}{\Delta}(t)$ for the ω and φ mesons.

The simplest approximation for these propagators consists of simple poles, neglecting the widths of the particles. (This corresponds to the "three approximation" of the phenomenological Lagrangian method). This approximation obviously fails in the timelike region, where finite width effects are important both in the resonance region itself, and to extrapolate form factors through the resonance region and further into the timelike region. We thus try to construct, models which respect the known analyticity properties of the propagators, and which reproduce the general resonant structure near the vector meson masses.

The ρ-meson propagator has the general structure

(9)
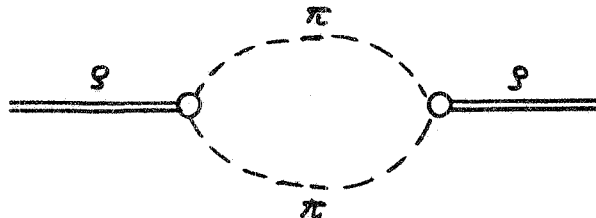$$\Delta_\rho(t) = \frac{1}{t - m_\rho^2 - \pi_\rho(t)}$$

where $\pi_\rho(t)$ is expressed as an integral over the intermediate states which couple to the ρ, and renormalization conditions are imposed so that

(10)
$$\text{Re } \pi_\rho (m_\rho^2) = 0$$

(11)
$$\frac{\text{Im } \pi_\rho (m_\rho^2)}{1 + \text{Re } \pi_\rho' (m_\rho^2)} = m_\rho \Gamma_\rho$$

where Γ_ς is the width of the ς. (An analogous structure is expected for the matrix propagator of ω and φ ; see below).

The simplest model for π_ς (t) is to consider the self-energy diagram

computed according to the usual rules of quantum field theory. The integral is divergent, but the renormalization procedure implicit in Eqs. (10) and (11) leads to a finite result[10].

Other intermediate states, as well as corrections to the ς - π -π vertex, should be included to extend this propagator to Frascati energies. For K-$\bar{K}$, N-$\bar{N}$ (in general, for intermediate states with particles of spin 0, 1/2), this is straight forward, but a satisfactory model for other states of importance (ω-π , A_1- π, A_2- π, ς -ς , etc) is lacking[11]. In view of this, the propagator of Eq. (9), with π (t) calcula ted from the π-π diagram, has been used extensively in model calculations; it is presumably better than a simple pole, but it could well contain errors of 10 $\sim$ 20% at Frascati energies.

To describe the pion form factor for $t \lesssim 1.0$ (GeV)^2, including the ς region in particular, the propagator in this form should be adeguate, the pion electromagnetic form factor is then given by

$$(12) \qquad F_\pi (t) = \frac{\Delta_\varsigma (t)}{\Delta_\varsigma(0)} \Gamma_{\varsigma\pi\pi}(t)$$

where $\Gamma_{\varsigma\pi\pi}(t)$ is a ς-π-π vertex function normalized to unity at $t = 0$.

The general philosophy of the vector dominance model, as well as particular current algebra models[12], suggest a form

$$(13) \qquad \Gamma_{\varsigma\pi\pi}(t) = 1 - \frac{\lambda t}{4 m_\varsigma^2}$$

where λ is a dimensionless parameters; there exist theoretical arguments supporting both $\lambda = 0$ and $\lambda = 1$. An analysis[13] of the Orsay and Novosibirsk data, using such a form factor[14], shows that

$$\lambda \simeq 0 \ (\pm 0.3)$$

Although the data from the two laboratories is somewhat inconsistent.

The isovector form factor of K-mesons can in principle be ana lyzed similarly; to be formally consistent the K-$\overline{\text{K}}$ intermediate state should be included in the expression for $\pi_\rho(t)$. The best measurements of the K-meson form factor at present are in the ψ-region, where of course the isoscalar form factor is dominant, but it would be useful to examine the effect of the ρ tail. To my knowledge, this has not yet been done.

There is no simple model for the 2 x 2 matrix propagator for ω and ρ. The difficult is that the most important (at low t) intermediate states are K-$\overline{\text{K}}$ and $\rho - \pi$, and the $\rho - \pi$ self-energy integral
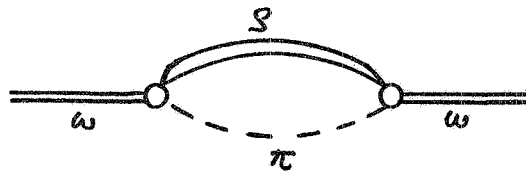
computed using the model of Gell-Mann, Sharp and Wagner[15] for the $\omega - \rho - \pi$ vertex, is too badly divergent. K. C. Wali and I made an estimate[16] of the effects of this integral; using a strong cutoff; we found effects on the order of 20%, but the cutoff energy was so small that we could not draw reliable quantitative conclusions. The situation may be improved using the alternative model for the $\omega - \rho - \pi$ vertex described in the next section, but the calculation has not yet been done.

IV. - SOME IMPORTANT VERTEX FUNCTIONS AND REACTIONS. -

At Frascati energies, there are many quasi-two-body reactions which lead to final states with three or more hadrons. Kramer, Uretshy and Walsh[17], and Layssac and Renard[18], have constructed phenomenological models for many of the relevant vertices, and esti mated cross-sections in the Frascati energy region. From these mo dels, it is reasonable to expect a total cross-section for hadron production on the order of 20 nanobarns (1 nb = 10^{-33} cm^2) at a total energy of 2 GeV[19], which is consistent with the early experimental results.

I will concentrate here on two types of reaction which are of particular interest, both from a theoretical point of view, and, because

of the relatively large cross-sections and (or) unique signature, on experimental grounds. These are:

1 - reactions leading to $A + \pi$ final states ($A = A_1$, A_2,... denotes an $I = 1$, $G = -1$ meson);

2 - reactions involving the ω-φ-π (or ψ-φ-π) vertex, leading to $\varphi + \pi$, $\omega + \pi$, $\psi + \pi$ final states.

IV.1.- A + π final states.-

The reaction

$$e^+ + e^- \rightarrow A^{\pm} + \pi^{\mp}$$

(14)

$$\quad\quad \hookrightarrow \pi^{\pm} + \pi^{\pm} + \pi^{\mp}$$
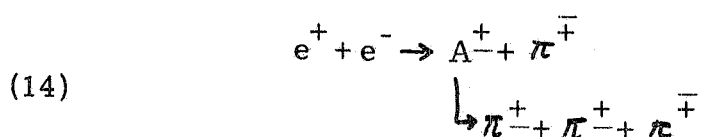
where A denotes an $I = 1$, $G = -1$ meson, has a unique signature, and is relatively uncontaminated by competing reactions (the final states $\omega + \pi$, $\varphi + \varphi$ must yield two neutral pions, while the state $\varphi + \sigma$ has a somewhat higher threshold). At 2 GeV total energy, the peak in the single $\pi^{\pm}$ energy spectrum associated with $A_1^{\pm}$ should be distinguishable from background, and at 2.4 GeV, the peak associated with A_2 should also be distinguishable.

The reaction (14) can be useful both as a spectroscopic tool for examining the decay properties of A_1 and A_2, and as a test of models for the production cross-section. Hard-pion current algebra models[12] lead to definite predictions for the $A_1 + \pi$ cross-section[20] (at 2 GeV this cross-section is expected to be ~ 6.6 nb, compared to ~ 3.0 nb in the phenomenological model of Kramer, Uretsky and Walsh[17] and angular distribution. The $A_2 + \pi$ angular distribution is unique $(1 + \cos^2 \theta)$, but the cross-section is a direct measure of the energy dependence in the A_2-φ-π vertex. With a minimum energy dependence in the vertex (analogous to the GMSW model of the ω-φ-π vertex[15]), a cross-section ~ 2.0 nb at 2 GeV is expected[17]. Further details can be found in the references.

IV.2.- ω - φ - π (ψ - φ - π) vertex.-

The ω-φ-π and ψ-φ-π vertices are important both for the vector meson propagators and for the corresponding colliding beam reactions. Moreover, these vertices require special treatment in phe nomenological Lagrangian models, as discussed below, and experimental information on them would be extremely useful.

The simplest model, due to Gell-Mann, Sharp and Wagner[15], expresses this vertex as

$$= g_{\omega \rho \pi} \; \varepsilon_{\mu\nu\lambda\sigma} \; k_\lambda \, g_\sigma$$

where $g_{\omega \rho \pi}$ is a dimensional coupling constant. The value of $g_{\omega \rho \pi}$ determined from the $\omega \to 3\pi$ decay computed according to the diagram
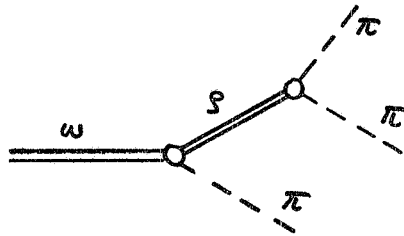
is given by

(15)
$$\frac{g_{\omega \rho \pi}}{4\pi} \;\approx\; \frac{0.5}{m_\pi^2}$$

The corresponding φ- ρ - π coupling constant is much smaller; from the same analysis of the $\varphi \to 3\pi$ decay we obtain

(16)
$$\frac{g_{\varphi \rho \pi}}{4\pi} \;\approx\; \frac{10^{-3}}{m_\pi^2}$$

for $\Gamma(\varphi \to 3\pi) = 0.8$ MeV.

The model can also be used to predict the decay rate for $\omega \to \pi^0 + \gamma$ and $\pi^0 \to \gamma\gamma$; the predictions are[22]

(17)
$$\frac{\Gamma(\omega \to \pi^0 \gamma)}{\Gamma(\omega \to 3\pi)} \approx .13 - .14 \quad (\Gamma_\rho = 120 \text{ MeV})$$

(18)
$$\Gamma(\pi^0 \to 2\gamma) \approx 10 \,(\pm 2) \text{ eV}$$

Experimentally, the ratio (17) is $.105$ ($\pm$ $.01$) according to the latest Particle Data Group tables; recent reports at Bologna suggest that it may be as small as $.06$. The π° width is slightly high, but perhaps satisfactory.

A straightforward application of the model to the colliding beam reaction

$$(19) \qquad e^{+} + e^{-} \longrightarrow \omega + \pi$$

predicts a cross-section of ~ 2.5 nb at total energy $2\,\text{GeV}$[23]. If the φ - ς - π vertex is neglected[24], then

$$(20) \qquad \frac{\mathbf{G}\,(e^{+}+e^{-} \rightarrow \varsigma + \pi)}{\mathbf{G}\,(e^{+}+e^{-} \rightarrow \omega + \pi)} \quad (1+\varepsilon)\,\sin^{2}\theta_{Y}$$

where ε is a small correction term due to interference effects in the $\varsigma + \pi \rightarrow 3\,\pi$ Dalitz plot (it is $\lesssim 5\%$ at $2\,\text{GeV}$), and θ_{Y} is the KLZ mixing angle[25].

This model has two major difficulties:

1 - it cannot be used to estimate corrections to the vector meson propagators from ς - π , ω - π and φ - π intermediate states without the introduction of strong cutoff[16], and

2 - the effective coupling cannot be introduced in a phenomenological Lagrangian model which incorporates SU(3) x SU(3) current algebra, the field-current identity, and partial conservation of current conditions, in addition to the possible discrepancy between Eq. (17) and experiment.

One way out of these difficulties is to abandon the attempt to express the ω - ς - π vertex in terms of a point coupling, but to consider diagrams
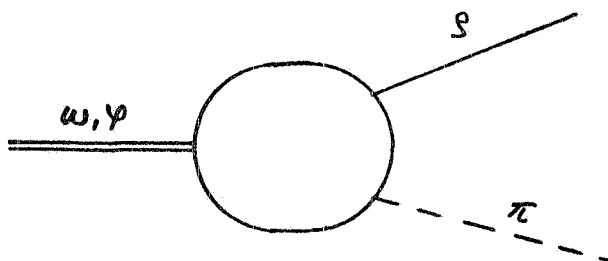
with can internal baryon loop[26]. When the complete baryon octet is included, the effective coupling constant $g_{\omega \rho \pi}$ seems to be adequately reproduced[27], and the violation of PCAC appears in a dynamical way[26]. A complete calculation has not been done, but it appears also that the branching ratio Eq. (17) is improved, since the diagram
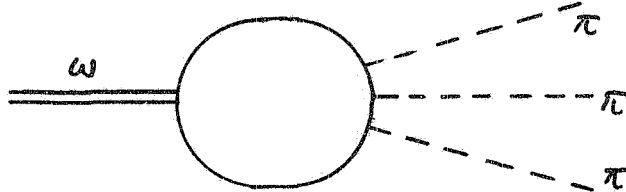
gives an additional contribution to the $\omega \to 3\pi$ decay.

The implications of this improved model for the colliding beam reactions are the following (numerical calculations have not yet been done, but I intend to do them):

1 - A reasonable model for the ω and ρ propagators, and for the ω -π contribution to the ρ propagator, should result.

2 - The cross-sections for $\omega + \pi$ and $\rho + \pi$ should be larger than predicted the GMSW model up to baryon-antibaryon thresholds, and decrease more rapidly beyond them. I cannot give a quantitative estimate of the increase at present, although $25 \sim 50\%$ seems to be the right order of magnitude. We note again that Eq. (20) remains valid in this model.

3 - The ρ -ρ -π vertex should remain small (relative to the ω -ρ -π vertex). An obvious experimental check of this is to look hard for the $\rho + \pi$ final state; a cross-section of even 0.1 nb at 2 GeV would be difficult to explain with the present theory.

V. - CONCLUDING REMARKS. -

The vertices discussed in the previous sections are responsible for the largest quasi-two-body cross-sections below 2 GeV. Final states involving K, K^x, K_A mesons (as well as states like $\rho + \rho$, $\rho + \omega$, and so forth) will become increasingly important above 2 GeV, and SU(3) x SU(3) current algebra models will provide predictions for many of them. Until the calculations are done, it is not clear which states will be the most important, but experiments to measure single-particle energy spectra will be useful in any event.

I emphasize again the use of the reaction (14) as a spectroscopic

tool for measuring the decay properties of A_1, A_2 and higher mass A mesons. The problems of interpretation which plague strong interaction (and photoproduction) experiments are very much suppressed here.

I have neglected completely here the problem of interference between various quasi-two-body states. For example,

$$e^+ + e^- \nearrow^{\omega + \pi^0}\searrow \atop \searrow_{A_1^{\pm} + \pi^{\mp}}\nearrow \pi^+ \pi^- \pi^0 \pi^0$$
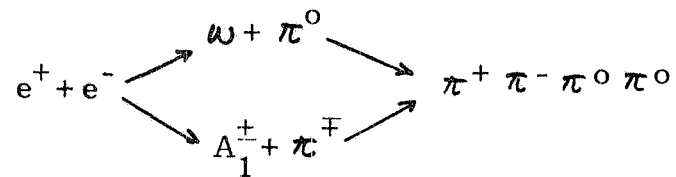
Also effects due to the finite widths of the resonances (except as they appear in the vector meson propagators). It seems premature to analyze them in detail now, but they must be considered at the 10 $\sim$ 20% level.

I have also not discussed the problem of higher mass vector mesons, if these couple to the electromagnetic current. One attempt to include them is to simply add CDD poles to the inverse vector meson propagators. If this procedure is correct, then the cross-sections for the reactions which couple to a particular propagator will vanish simultaneously at the position of the CDD pole. For the $I = 1$ states, the first CDD pole should occur at 1.2 $\sim$ 1.3 GeV, the second at 1.8 $\sim$ 1.9 GeV. For the $I = 0$ states, the CDD poles corresponding to the ω should occur at about the same position, while those corresponding to φ should occur at 1.4 $\sim$ 1.5 GeV and 2.0 $\sim$ 1.1 GeV. Unfortunately, the dips may be quite narrow.

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(2) - M. Greco, Frascati report LNF-71/1 (1971) (to be published on Nuovo Cimento).

(3) - S. Brodsky, T. Kinoshita and N. Terazawa, Phys. Rev.. Letters 25, 972 (1970); V. Budnev and I. Ginsburg, XV Intern. Conf. on High Energy Phys., Kiev (1970).

(4) - The sum rules are based mainly on the spectral function sum rules of S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

(5) - N. Kroll, T. D. Lee and B. Zumino, Phys. Rev. 157, 1376 (1967).

(6) - The model itself is much older, of course, it can be traced back to the work of J. J. Sakurai, Ann. Phys. 11, 1 (1960) and M. Gell--Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).

(7) - This argument is due to M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

(8) - M. T. Vaughn,, to be published; are also Appendix B of Ref. (5).

(9) - M. Gell-Mann, Phys. 1, 63 (1964).

(10) - B. W. Lee and M. T. Vaughn,, Phys. Rev. Letters 4, 578 (1960).

(11) - The technical reason for this is that the simplest Lagrangians including these states are non-renormalizable, so that the simplest closed loop diagrams cannot be made finite simply by a mass and coupling constant renormalization.

(12) - H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Letters 19, 1085 (1967); Phys. Rev. 174, 1999, 2008 (1968). For a recent analysis, with references, see M. T. Vaughn, Phys. Rev. D 3 (to be published).

(13) - M. T. Vaughn,, Lettere al Nuovo Cimento 2, 851 (1969).

(14) - An historical note is in order here. The first model for the pion form factor was given by W. R. Frazer and J. Fulco [Phys. Rev. Letters 2, 365 (1959), Phys. Rev. 117, 1603 (1960)], who obtained a form factor equivalent to the present one with λ negative and very small, of order 10^{-6}). The propagator formula was given by B. W. Lee and M. T. Vaughn, (Ref. 10). The form was redisco vered by G. Gounaris and J. J. Sakurai, Phys. Rev. Letters 21, 244 (1968), and applied at the sametime to the colliding beam data by M. T. Vaughn and K. C. Wali, Phys. Rev. Letters 21, 938 (1968) and Ref. (13).

(15) - M. Gell-Mann, D. Sharp and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

(16) - M. T. Vaughn and K. C. Wali, Phys. Rev. 177, 2199 (1969).

(17) - G. Kramer, J. Uretsky and T. F. Walsh, DESY preprint (1970) (unpublished).

(18) - J. Layssac and F. M. Renard, Lettere al Nuovo Cimento 1, 197 (1971).

(19) - The details of these models are open to question, of course, but I am convinced that improved models will not predict smaller cross-sections. In fact, in two important channels (A_1- π , ω - π) discussed in detail here, improved models pre dict larger cross-sections than the models of Refs. (17) and (18).

(20) - M. T. Vaughn and P. J. Polito, Lettere al Nuovo Cimento 1, 74 (1971).

(21) - These cross-sections must be multiplied by a factor $\sim 1/2$ to obtain cross-sections for reaction (14), due to the alternative decay mode π^+ π^o π^o (for A_1, this factor assumes dominance of the $\mathcal{S}$ + π decay mode). In addition, there can be interferen ce effects of order 10% in the 4π final state (P. J. Polito, private communication).

(22) - See Reference (16). The coupling constants quoted there are too small by a factor 2/3.

(23) - M. T. Vaughn, unpublished. See also Refs. (17) and (18). A factor of 2 error in Ref. (17) has been corrected (T. Walsh, private communication).

(24) - Even though this appears reasonable in view of Eq. (16), there is an interference term which can be as large as + 10%.

(25) - Eq. (20) is more general than the model, although the γ - $\mathcal{S}$ - π coupling is not enterely negligible[24].

(26) - J. S. Bell and R. Jackiw, Nuovo Cimento 60 A, 47 (1969); S. L. Adler, Phys. Rev. 177, 2426 (1969).

(27) - B. Zumino, rapporteur's talk at XV Intern. Conf. on High Energy Phys., Kiev (1970).