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M. Greco: MULTIPLE e.m. PROCESSES IN HIGH ENERGY $e^- e^{\pm}$ COLLISIONS.

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SUMMARY. - The production of electron and muon pairs is studied in high energy $e^- e^{\mp}$ collisions. Total and differential cross sections are discussed with special regard to experimental implications. Hadron production via photon-photon scattering is also studied, and suggestions are made for present and next future experiments.

1. - INTRODUCTION. -

Very recently some theoretical work has been done[1-4] to study the electromagnetic processes produced in both e^+e^- and e^-e^- colliding beams, via the photon-photon scattering. These processes include reactions in which hadrons are involved, or more simply pure electrodynamical reactions in which one or more pairs of electrons and muons are produced. The total cross sections relative to both groups of processes increase at least as the third power of the logarithm of the energy. Hadron production via the $\gamma\gamma$ scattering is expected therefore to dominate over one photon processes at sufficiently high energies, in spite of higher order in α. The experimental study of this kind of processes should provide informations on $\gamma\gamma$ total cross sections, C = 1 meson resonances and more generally, on the electromagnetic properties of hadrons.

On the other hand, the production of electron or muon pairs according to pure QED, in reactions like $e^+e^- \rightarrow e^+e^- + e^+e^-$, etc., of much less interest from a general point of view, has large total cross sections, increasing at any considerably high energy as $(\ln s)^{n+2}$, for n pairs produced. For that reason this kind of processes can provide large backgrounds and an appreciable degree of contamination with regard to pure hadronic events, unless special kinematical constraints are imposed on the particles produced. The QED production cross sections in fact attain large values corresponding to very low energy pairs emitted in the forward-backward direction. At large angles and for quite large effective masses produced the cross sections decrease considerably and are reduced of some orders of magnitude.

In the present paper a careful study is done of the main electrodynamical processes in order to make easier the interpretation of the experimental results obtained from colliding electron-positron beams. Total and differential cross sections are discussed with special regards to the experimental implications, so to estimate the order of magnitude of the effects in actual experiments. In Sec. 2 we study the energy behaviour of the total cross sections with production of electron and muon pairs, according to pure QED. The cross sections are evaluated in different ways, in order to show the degree of confidence of the methods of approximation currently used. In Sec. 3 we discuss the production of any hadronic or pure leptonic state via the photon-photon scattering. We derive the differential cross sections with respect the angles and the energies of the final particles. The relevance of the motion of the center of mass of the two photons on the detection of the particles produced is also discussed. In Sec. 4 we study the production of e. m. pairs of high masses at large angles. Particularly we discuss the production of electrons and muons which might be confused with hadrons, for experiments which do not single out univocally the particles detected. Some typical examples are given. In Sec. 5 finally hadron production via $\gamma\gamma$ scattering is discussed in the assumption of pure QED particles, and of meson resonances which can be produced by the two photons and in turn can decay into a multipion state. Suggestions are also made for present and next future experiments.

2. - QED PAIR PRODUCTION: TOTAL CROSS SECTIONS. -

The total cross sections for production of electron and muon pairs are derived by using different techniques. The results, which differ in the approximation used, agree within a factor 2-3 and give the order of magnitude of the effect. Numerical values for the cross sections are given in Table I.

$e^{\mp} e^- \to e^{\mp} e^- e^+ e^-$

The first derivation of the total cross section is due to Landau and Lifshitz[5]

$$(1) \qquad \sigma = \frac{28}{27} \frac{\alpha^2}{\pi} (z_1 z_2)^2 r_o^2 \lg^3 \left(\frac{s}{2 m_1 m_2} \right) .$$

Eq. (1) refers to e^+e^- pair production in a collision of two fast charged particles, of charges $z_1 e$ and $z_2 e$, masses m_1 and m_2, and total energy s. Eq. (1) is calculated by replacing the effect of the colliding particles by an external field. Exchange terms must be taken into account when the colliding particles are electrons or positrons, so that eq. (1) holds only as an order of magnitude.

A perhaps better estimate for σ is obtained by regarding the above process as pair production in the collision of one electron with the equivalent distribution of photons of the other electron:

(2) $$d\sigma = \int \sigma(k)\, n(k)\, dk$$

where:

(3) $$n(k)\,dk = \frac{2\alpha}{\pi}\, \lg\left[\frac{2E(E-k)}{mk}\right] \frac{E^2+(E-k)^2}{2E^2} \frac{dk}{k}$$

and

(4) $$\sigma(k) = \frac{28}{9}\, \alpha\, r_o^2 \lg \frac{2k}{m}\, .$$

All the energies are expressed in the rest frame of one electron. By approximating $\lg \frac{2E(E-k)}{mk} \simeq \lg \frac{E}{m}$ one gets:

(5a) $$\sigma = \frac{28}{9} \frac{\alpha^2}{\pi} r_o^2 \lg^3 \left(\frac{s}{2m^2}\right)$$

while in the case of a "classical" spectrum of photons $n(k) \simeq \frac{2\alpha}{\pi} \lg\left(\frac{E}{k}\right) \frac{dk}{k}$:

(5b) $$\sigma = \frac{28}{27} \frac{\alpha^2}{\pi} r_o^2 \lg^3 \left(\frac{s}{2m^2}\right) .$$

The method of impact diagrams by Cheng and Wu[(6)] gives a further and simple way to compute the total cross section for any production process. The e^+e^- pair production cross section in particular is related to the forward-scattering amplitude for e^+e^- elastic scattering, via one electron pair, by the optical theorem. With the notations of ref. (6) the forward-scattering amplitude corresponding to the diagrams of Fig. 1,

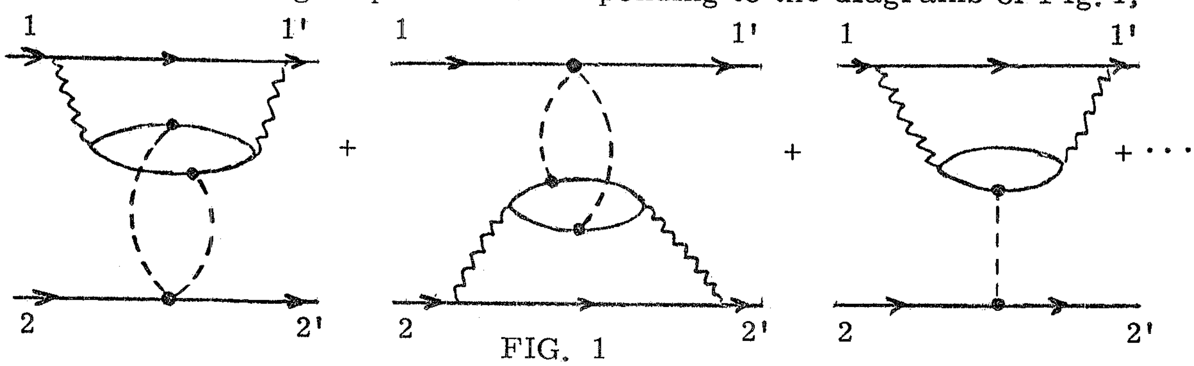


FIG. 1

in the lowest order of e, is given by:

$$\mathcal{M}(\vec{r}_1=0) = 4\,is\,\lg\gamma\,\frac{\alpha^4}{\pi^3}\,\frac{1}{m^2}\,\delta_{12}\,\delta_{1'2'}\int\frac{d\vec{q}_\perp}{\vec{q}_\perp^{\,2}}\,\frac{d\vec{q}_\perp^{\,\prime}}{\vec{q}_\perp^{\,\prime 2}}\ \times$$

(6)
$$\times\left\{\int_0^1 dx\int_0^1 dy\,\frac{x(1-x)+y(1-y)-5x(1-x)\,y(1-y)}{x(1-x)q_\perp^2+y(1-y)q_\perp^{\prime 2}+m^2}\right\}\simeq$$

$$\simeq\frac{28}{9}\,is\,\frac{\alpha^4}{\pi}\,\frac{1}{m^4}\,\lg^3\gamma\,\delta_{12}\,\delta_{1'2'}$$

where r_1 is the total momentum transfer, $\gamma = E/m$, E being the energy of the initial electrons in their center of mass system, and δ_{12}, $\delta_{1'2'}$ are the Kronecker δ's in spin. By the optical theorem one gets:

(7)
$$\sigma \simeq \frac{4\times 28}{9}\,\frac{\alpha^2}{\pi}\,r_o^2\,\lg^3\gamma\,.$$

The eqs. (1), (5a), (5b) and (7) are in a quite good agreement within a factor 2-3. As one can see from Table I, the cross sections are of order 10^{-26} - 10^{-27} cm^2 for C. M. energies ranging between 1 - 10 GeV.

TABLE I

	E=1 GeV	E=3 GeV	E=5 GeV	E=10 GeV
$\sigma(e^\mp e^- \to e^\mp e^-\ e^+e^-)$ (7)	7×10^{-27} cm^2	1×10^{-26} cm^2	1.24×10^{-26} cm^2	1.6×10^{-26} cm^2
$\sigma(e^\mp e^- \to e^\mp e^-\ \mu^+\mu^-)$ (9)	4.8×10^{-32} cm^2	0.9×10^{-31} cm^2	1.1×10^{-31} cm^2	1.7×10^{-31} cm^2
$\sigma(e^\mp e^- \to e^\mp e^-\ e^+e^-\ e^+e^-)$ (17)	1.3×10^{-30} cm^2	1.9×10^{-30} cm^2	2.3×10^{-30} cm^2	3.5×10^{-30} cm^2
$\sigma(e^\mp e^- \to e^\mp e^-\ e^+e^-\ \mu^+\mu^-)$ (19)	2.2×10^{-34} cm^2	3.2×10^{-34} cm^2	5.8×10^{-34} cm^2	5.8×10^{-34} cm^2

Total cross sections for e. m. pair production. The numerical values are computed from eqs. (7), (9), (17) and (19) respectively.

Such large cross sections reduce however by many orders of magnitude in the region of large angles and high masses of final electrons, as will be shown in the Section 4.

$e^\mp e^- \to e^\mp e^- \mu^+ \mu^-$

The total cross section for this process can be computed by the method of impact diagrams together with the optical theorem, as above, calculating the forward-scattering amplitude for e^+e^- elastic scattering, via one muon pair.

One gets:

(8) $$\mathcal{M}(\vec{r}_1=0) \simeq \frac{28}{9}\, i s \frac{\alpha^4}{\pi} \frac{1}{m^2 \mu^2} \lg\frac{E}{\mu} \lg^2\frac{E}{m}\, \delta_{12}\delta_{1'2'}$$

and

(9) $$\sigma \simeq 4 \times \frac{28}{9} \frac{\alpha^4}{\pi} \frac{1}{\mu^2} \lg\frac{E}{\mu} \lg^2\frac{E}{m}$$

where m and μ are the electron and muon masses respectively.

A different approach can also be used, which is related to the high energy photon-photon scattering. This approach will be discussed in the next Section, where we derive the following formula for the total cross section :

(10) $$\sigma \simeq \left(\frac{\alpha}{\pi}\right)^2 \lg^2\frac{E}{m} \int_{4\mu^2}^{4E^2} \frac{ds}{s}\, \sigma_{\gamma\gamma\to\mu^+\mu^-}(s) \quad \times$$
$$\times \left\{ \left(2 + \frac{s}{4E^2}\right)^2 \lg\frac{4E^2}{s} - 2\left(1 - \frac{s}{4E^2}\right)\left(3 + \frac{s}{4E^2}\right) \right\}$$

where :

(11) $$\sigma_{\gamma\gamma\to\mu^+\mu^-}(s) = \frac{4\pi\alpha^2}{s} \left[\left(1 + \frac{4\mu^2}{s} - \frac{8\mu^4}{s^2}\right) \lg\frac{1+\sqrt{1-4\mu^2/s}}{1-\sqrt{1-4\mu^2/s}} - \right.$$
$$\left. - \sqrt{1 - \frac{4\mu^2}{s}}\left(1 + \frac{4\mu^2}{s}\right)\right] \simeq \frac{4\pi\alpha^2}{s}\sqrt{1-4\mu^2/s}\,.$$

The last approximate equality holds because the integral in the r.h.s. of (10) is dominated by the region $s \sim 4\mu^2$. One finds :

(12) $$\sigma \simeq \frac{2}{3} \frac{\alpha^4}{\pi\mu^2} \lg^2\left(\frac{E}{m}\right)\left(8 \lg\frac{E}{\mu} - 6\right).$$

Eqs. (9) and (12) at high energies differ of about a factor 2. Numerical values for σ are given in Table I.

<u>$e^+e^- \to e^+e^- + 2(e^+e^-)$</u>

The total cross section for this process has been computed by Serbo[7, 8], using the method of equivalent photons :

(13) $$\sigma\left(e^+e^- \to e^+e^- + 2(e^+e^-)\right) = \frac{1}{6}\left(\frac{\alpha}{\pi}\right)^2 \sigma(\gamma\gamma \to 2e^+e^-) \lg^4\left(\frac{s}{m_e^2}\right)$$

vhere

$$\sigma(\gamma\gamma \to 2e^+e^-) = \frac{\alpha^4}{9\pi m_e^2}\left(\frac{175}{4}\zeta(3) - \frac{19}{2}\right) \simeq 6 \times 10^{-30}\ \mathrm{cm}^2 . \tag{14}$$

Eq. (13) however should be taken only as an order of magnitude, because of the identity of the target with the particles produced. We give therefore a second derivation for the total cross section, using again the method of impact diagrams. From ref(6), the amplitude relative to the diagrams of Fig. 2 is given by:

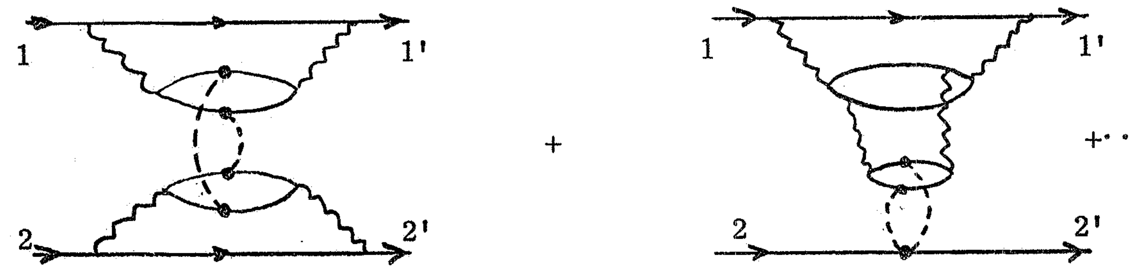


FIG. 2

$$\mathcal{M}(\vec{r}_1=0) = 2is\,\lg^2\gamma\,(2\pi)^{-6}\int\frac{dq_{1\perp}}{q_{1\perp}^4}\int\frac{dq_{2\perp}}{q_{2\perp}^4}\int\frac{dq_{3\perp}}{q_{3\perp}^4}\ \times$$

(15)

$$\times\ K(0, q_{1\perp}, q_{2\perp})\,K(0, q_{2\perp}, q_{3\perp})\,\frac{1}{4}e^4 m^{-2}\,\delta_{11'}\,\delta_{22'}$$

where :

$$K(0, q_{1\perp}, q_{2\perp}) = \frac{4e^4}{(2\pi)^3}\,q_{1\perp}^2\,q_{2\perp}^2\ \times$$

(16)

$$\times\int_0^1 dx\int_0^1 dy\ \frac{x(1-x) + y(1-y) - 5xy(1-x)(1-y)}{x(1-x)q_{1\perp}^2 + y(1-y)q_{2\perp}^2 + m^2} .$$

We have approximately:

$$\sigma \simeq \frac{6\,\alpha^4 r_o^2}{\pi^3}\,\lg^4\left(\frac{s}{m^2}\right) \tag{17}$$

which is a factor 2.8 higher than (14). In the Section 4 we shall discuss over the factor $1/m_e^2$ which fixes the scale of the cross section for this process.

$e^{\mp}e^{-} \to e^{\mp}e^{-} + e^{+}e^{-}\mu^{+}\mu^{-}$

An extimate of the cross section for this process can be obtained using the method of equivalent photons, from the knowledge of $\sigma(\gamma\gamma \to e^{+}e^{-}\mu^{+}\mu^{-})$. This cross section has been obtained by Masujima[9], using the Cheng and Wu's techniques :

$$\sigma(\gamma\gamma \to e^{+}e^{-}\mu^{+}\mu^{-}) = \frac{2\alpha^4}{27\pi}\,\frac{1}{m_\mu^2}\left\{7\,\lg^2\left(\frac{m_\mu^2}{m_e^2}\right) + \frac{103}{3}\lg\frac{m_\mu^2}{m_e^2} + \frac{985}{9}\right\} \simeq \tag{18}$$

$$\simeq 2.82 \times 10^{-33}\ \mathrm{cm}^2 .$$

In analogy with (13) one gets :

$$\sigma(e^{+}e^{-} \to e^{+}e^{-} + e^{+}e^{-}\mu^{+}\mu^{-}) \simeq \frac{2.82}{6}\,\frac{\alpha^2}{\pi^2}\,\lg^4\left(\frac{s}{m_e^2}\right) \times 10^{-33}\ \mathrm{cm}^2 \tag{19}$$

which is more than a thousand smaller than $\sigma(e^{+}e^{-} \to e^{+}e^{-} + 2(e^{+}e^{-}))$.

All the cross sections for the processes studied increase with the energy with some power of lns. More generally the cross section for n pairs production in $e^{(\mp)}e^{-}$ collisions goes as $(\ln s)^{n+2}$ in the lowest order of perturbation theory. If further diagrams with fermion loops are included, the Froissart bound has been shown[10] to be saturated at infinite energy. More explicity the perturbation series loses its meaning when $\alpha^2 \ln\frac{s}{m_e^2} \sim 1$, which is however for enough from any conceavable high energy future experiment.

3. - PRODUCTION OF ANY STATE f VIA $\gamma\gamma$ SCATTERING. -

Let's discuss the production of any state f via the photon-photon scattering, according to the diagram of Fig. 3.

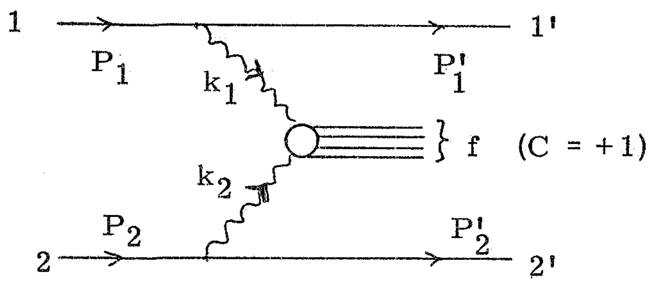


FIG. 3

For the validity of the formulae below, if the state f contains one or more pairs of electrons, they should be kinematically separable from the particles 1' and 2', which are strongly peaked in the forward-backward direction. The cross sections of processes of the type shown in

Fig. 3 in fact, are large only where the virtual photons k_1 and k_2 have small masses; this means that the initial electrons tend to proceed in the beam direction.

We have checked out the validity of the double Weizsäcker-Williams approximation by studying the photon-photon scattering in the representation of the helicity amplitudes. The main contribution to the cross sections comes from transverse photons, and in the case in which the final electrons are detected within angles θ_{1max} and θ_{2max} with respect to the initial beam direction the result is :

$$d\sigma(e^{(\stackrel{+}{-})}e^- \to e^{(\stackrel{+}{-})}e^- f) = \left(\frac{2\alpha}{\pi}\right)^2 \frac{dk_1}{k_1}\frac{dk_2}{k_2}\left\{ \lg\frac{2EE'}{mk_1} + \frac{1}{2}\lg\left(\frac{1-\cos\theta_{1max}}{2}\right) - \frac{EE'}{E^2+E'^2}\right\} \times$$

(20)

$$\times \left\{ \lg\frac{2EE''}{mk_2} + \frac{1}{2}\lg\left(\frac{1-\cos\theta_{2max}}{2}\right) - \frac{EE''}{E^2+E''^2}\right\} \times$$

$$\times \frac{E^2+E'^2}{2E^2}\,\frac{E^2+E''^2}{2E^2}\, d\sigma(\gamma\gamma \to f) .$$

where E, $E' = (E-k_1)$ and $E' = (E-k_2)$ are the energies of the initial and final electrons respectively in the C. M. system.

An approximate formula which gives the order of magnitude of the cross sections is obtained by taking only the leading term $\lg(E/m)$ from the full logarithmic dependence. In these approximations we get for the total cross section :

$$\sigma(e^{(\mp)}e^- \to e^{(\mp)}e^- f) \simeq \left(\frac{\alpha}{\pi}\right)^2 \lg^2\left(\frac{E}{m}\right)\int_{s_{th}}^{4E^2}\frac{ds}{s}\,\sigma^f_{\gamma\gamma}(s) \times$$

(21)

$$\times\left\{\left(2+\frac{s}{4E^2}\right)^2\lg\frac{4E^2}{s} - 2\left(1-\frac{s}{4E^2}\right)\left(3+\frac{s}{4E^2}\right)\right\}$$

where $\sigma^f_{\gamma\gamma}(s)$ is the cross section for production of f by the two photons of C. M. energy squared s[(x)]. Eq. (21) is valid if the experimental apparatus does not impose cuts on the velocity of the center of mass of the two photons. If this is the case eq. (21) becomes :

(x) - In ref. (2), formula (5) should be read :

$$f(x) = (2+x^2)^2\lg\left(\frac{1}{x}\right) - (1-x^2)(3+x^2) .$$

$$\sigma(e^{(\mp)}e^- \to e^{(\mp)}e^- f) \simeq \left(\frac{\alpha}{\pi}\right)^2 \lg^2\left(\frac{E}{m}\right) \int_{s_{th}}^{4E^2} \frac{ds}{s}\, \sigma^f_{\gamma\gamma}(s) \, \times$$

(22) $$\times \left\{ (2+x_o)^2 \lg \sqrt{x_o}\sqrt{\gamma} - 2\sqrt{x_o}\sqrt{\gamma}\,(2+x_o) \, + \right.$$

$$\left. + \, x_o\gamma + 2x_o(2+x_o)\frac{1}{\sqrt{x_o}\sqrt{\gamma}} - \frac{x_o}{\gamma} \right\}_{\gamma_{min}}^{\gamma_{max}}$$

where: $\left\{ F(\gamma) \right\}_{\gamma_{min}}^{\gamma_{max}} \equiv \left\{ F(\gamma_{max}) - F(\gamma_{min}) \right\}$, $x_o = \frac{s}{4E^2}$, and

$\gamma \equiv \frac{1+\beta}{1-\beta} = \frac{4k_1^2}{s}$, with $\beta = \frac{k_1 - k_2}{k_1 + k_2}$. In absence of cuts, the range of variability of γ is given by $\gamma_{max} = 1/\gamma_{min} = s/4E^2$.

4. - LARGE ANGLE AND HIGH MASS PAIR PRODUCTION. -

In Sec. 2 we have studied the total cross sections for the main processes of electron and muon pairs production and we have shown that they attain values comparable and in some cases much larger than production of single pairs via the one photon channel. Such large cross sections however correspond to very low energy pairs emitted mainly in the forward-backward direction. In this section we estimate the order of magnitude of the effect of these processes on experiments which observe hadron production at large angles as those performed by the Frascati groups(12, 13). Let's consider first the production at large angles of two particles coplanar with the beam direction. We shall use the results of the preceding section.

For the reaction $e^{(\mp)}e^- \to e^{(\mp)}e^- + e^+e^-$, which has the largest total cross section, the angular distribution of the pair produced in the C.M. by the two virtual photons is given by:

(23) $$d\sigma = \frac{2\pi\alpha^2}{s}\,\beta\,\frac{1-\beta^4\cos^4\theta + 8(m^2/s)\,\beta^2\sin^2\theta}{(1-\beta^2\cos^2\theta)^2}\,\sin\theta\, d\theta$$

where $\beta^2 = 1 - 4\,m^2/s$. By integrating (23) over the range of interest, for example $60^o \leq \theta \leq 120^o$, and combining with (21), the result is:

(24) $$\sigma^{e^+e^-}_{exp} \simeq \frac{\alpha^4}{\pi}\lg^2\left(\frac{E}{m}\right)\frac{1}{s_{th}}\left(4\lg\frac{4E^2}{s_{th}} - 6\right).$$

An additional factor 1/4, due to the identity of the final particles, has been included in (21). At E = 1 GeV, and $s_{th} = 4 \times 10^{-2}$ GeV^2 one has $\sigma^{e^+e^-}_{exp} \simeq 0.6 \times 10^{-32}$ cm^2. In the case of an experimental cut on the velocity of the C. M. of the two photons, equation (24) has to be replaced by:

$$\text{(25)} \qquad \sigma^{e^+e^-}_{exp} \simeq \frac{\alpha^4}{\pi} \lg^2\left(\frac{E}{m}\right) \frac{1}{s_{th}} \, 4 \lg \gamma_{max} \, .$$

For example, for $|\beta| \leq 0.5$, at E = 1 GeV and $s_{th} = 4 \times 10^{-2}$ GeV^2 one gets $\sigma^{e^+e^-}_{exp} \simeq 0.2 \times 10^{-32}$ cm^2.

Quite singular arguments hold for the reaction $e^{(\mp)}e^- \longrightarrow e^{(\mp)}e^-\mu^+\mu^-$. For example, at E = 1 GeV, for $60^o \lesssim \theta \lesssim 120^o$, and 200 MeV $\lesssim E_\mu \lesssim$ 700 MeV, one has $\sigma^{\mu^+\mu^-}_{exp} \simeq 0.3 \times 10^{-32}$ cm^2. Furthermore for $|\beta| \lesssim 0.5$ and $|\beta| \lesssim 0.05$ one gets $\sigma^{\mu^+\mu^-}_{exp} = 0.2 \times 10^{-32}$ cm^2 and 0.2×10^{-33} cm^2 respectively. Last situation corresponds to almost collinear muons in the colliding beams frame.

We have only studied up to now the production of e. m. pairs coplanar with the beam direction. In order to estimate the effect of non-coplanar pairs we consider the process shown in Fig. 4, where

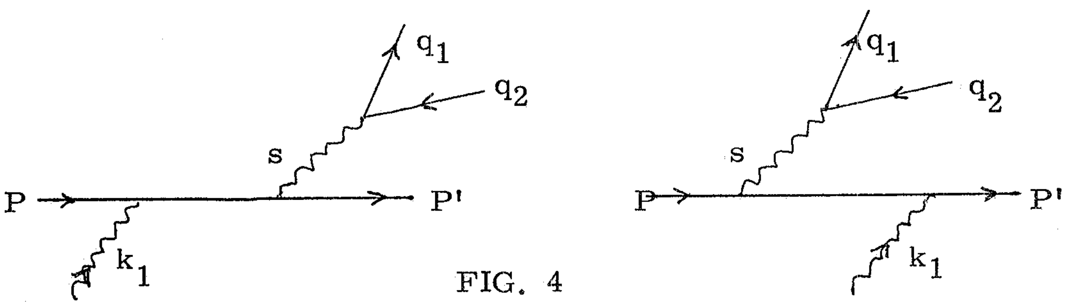


FIG. 4

the incoming photon belongs to the bremsstrahlung spectrum of the other electron. By integrating over the whole of the solid angle of the pair and the field electron, on has:

$$\text{(26)} \qquad d\sigma = \frac{\alpha^3}{12} \frac{ds}{s^2} \frac{1}{\omega^2} \frac{4\omega^2 - s}{4\omega^2} \lg \frac{4E'^2}{m^2} \left\{ \omega^2 (4\beta_q + 3 - \beta_q^2) + s\left(\frac{\beta_q}{2} - 2 - \frac{\beta_q^2}{2}\right) - \frac{s^2}{8\omega^2}\left(\frac{7}{2} + \beta_q + \frac{\beta_q^2}{2}\right) + \frac{4\omega^2 s}{4\omega^2 - s} \right\}$$

where: $E' = \frac{4\omega^2 - s}{4\omega}$, $\beta_q = \frac{4\omega^2 - s}{4\omega^2 - s}$ and ω is the energy of the photon in the electron-photon c. m. system. For fixed ω, the cross section will be maximized at small s, and therefore by neglecting s with respect to ω^2 we have:

$$(27) \qquad d\sigma \simeq \frac{\alpha^3}{2}\,\frac{ds}{s^2}\,\lg\frac{4\omega^2}{m^2} = \frac{\alpha^3}{2}\,\frac{ds}{s^2}\,\lg\frac{4Ek_1}{m^2}$$

where now E and k_1 are the energies of the electron and the photon in the electron-positron c. m. system. By integrating over the photon spectrum one gets :

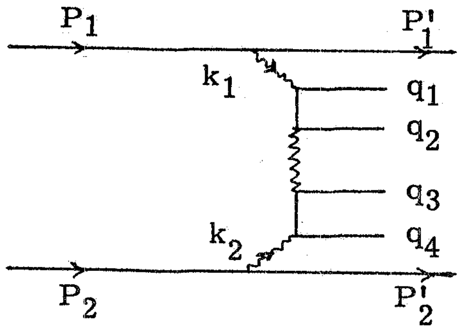
$$\sigma_{e^+e^-\rightarrow e^+e^-(e^+e^-)_{n.\,cop.}} \simeq \frac{2\alpha}{\pi}\lg\left(\frac{E}{m}\right)\frac{\alpha^3}{2} \quad \times$$

$$(28) \qquad \times \int_{s_{min}}^{4E^2}\frac{ds}{s^2}\int_{s/4E}^{E}\frac{E^2+(E-k)^2}{2E^2}\,\frac{dk}{k}\,\lg\frac{4Ek}{m^2} =$$

$$= \frac{2\alpha^4}{\pi}\lg^2\left(\frac{E}{m}\right)\frac{1}{s_{min}}\,\lg\left(\frac{4E^2}{s_{min}}\right) \simeq 3 \times 10^{-33},$$

where the last equality is obtained at $E = 1$ GeV and $s_{min} = 4 \times 10^{-2}$ GeV^2. Eq. (28) gives an upper limit to the large angle non-coplanar pair production, because of the integration over all the solid angle of the pair.

As far as the double electron pair production is concerned, this process is represented by Fig. 5, one can note the following.



+ (crossed terms)

FIG. 5

From eq. (17) follows that large values of the cross section, in spite of sixth powers of α, come from a factor m_e^{-2} together with a fourth power of lg(E/m). The factor m_e^{-2} comes from the internal photon propagator. Now it is easily seen that k^2 can attain values comparable with m_e^2 only if the pairs are produced with small mass squared and along the beam direction. Both these conditions cannot be satisfied if one is interested at quite large masses produced at large angles. For the same reason is very reasonable to loose a factor $\lg^2(s/m^2)$, in favor of $\lg(s/s_1)\lg(s/s_2)$ in the best case, where s_1 and s_2 are the quadratic masses of the two pair. We espect therefore in the kinematical conditions of actual experiments, a contribution of the type :

$$(29) \qquad \sigma \sim \frac{\alpha^6}{\pi^3}\,\frac{1}{s_{min}}\,\lg^2\left(\frac{E}{m}\right)\lg\left(\frac{s}{s_1}\right)\lg\left(\frac{s}{s_2}\right)$$

which gives values 10^{-5} - 10^{-6} smaller than (17).

From all the above considerations it follows that the main contribution to large angles events come from an electron or muon pair with a rate

$$(30) \qquad \sigma \sim \frac{\alpha^4}{\pi} \frac{1}{s_{min}} \lg^2\left(\frac{E}{m}\right)$$

which is quite neglegeable for the Frascati experiments. The threshold dependence of (30), together with the analysis of radiative corrections to Bhabha scattering, can be useful to recognize and reduce this kind of effects.

4. - HADRON PRODUCTION IN $\gamma\gamma$ COLLISIONS.

Hadron production in $e^{(\pm)}e^{-}$ collision does occur via $\gamma\gamma$ interaction as discussed previously by several authors[1-4], with a rate given by (21) or (22), once $\sigma_{\gamma\gamma}^{hadrons}(s)$ is given. In the simplest case of $\pi^+\pi^-$ production, assuming pointlike pions one gets :

$$(31) \quad \sigma_{tot}(e^+e^- \to e^+e^-\pi^+\pi^-) \simeq \frac{1}{3} \frac{\alpha^4}{\pi m_\pi^2} \lg^2\left(\frac{E}{m}\right)\left(8 \lg\left(\frac{E}{m_\pi}\right) - 6\right)$$

which is 3×10^{-33} cm^2 at E = 1 GeV. This value becomes $\sim 2 \times 10^{-34}$ cm^2 when: $E_\pi \gtrsim 250$ MeV, $60 \leq \theta_{CM} \leq 120$ and $|\beta| \lesssim 0.5$.

The possibility that a strong resonance with J = 0 or 2 and C = +1 can be produced by the two almost-real photons and which could subsequently decay, gives a source of multipion final states and offers a simple way to measure partial and total decay rates[1]. This can be seen from simple considerations based on a Breit-Wigner description in the vicinity of the resonance :

$$(32) \qquad \sigma_{\gamma\gamma}^{R}(s) = 2\pi(2J+1) \frac{\Gamma_{2\gamma}\Gamma}{(s - M^2)^2 + \Gamma^2 M^2}$$

where $\Gamma_{2\gamma}$ is the decay rate of the resonance of mass M and width Γ into 2γ. Combining (21) and (32) one sees that at least in principle the width can be measured by detecting in coincidence the outgoing electrons scattered at very small angles, by measuring the energy loss and plotting the events against s. In any actual experiment however the measured quantity will be the integral of the cross section in a region determined by the experimental resolution and if the resonance is very narrow, with a width much smaller than the experimental energy resolution one gets from (21) and (32) :

$$\sigma(e^{(\mp)}e^- \rightarrow e^{\pm}e^- R) \simeq \frac{2\alpha^2}{M^3}(2J+1)\,\Gamma_{2\gamma}\,\lg^2\left(\frac{E}{m}\right) \times$$

(33)

$$\times \left\{ \left(2+\frac{M^2}{4E^2}\right)^2 \lg\frac{4E^2}{M^2} - 2\left(1-\frac{M^2}{4E^2}\right)\left(3+\frac{M^2}{4E^2}\right) \right\}$$

At $\frac{E}{2}$ = 1 GeV, for example, for the η meson one has $\sigma \simeq 0.8 \times 10^{-34}$ cm^2 and for the $X^o(958)$ or η' $\sigma \simeq 0.67 \times 10^{-35}\, cm^2 (keV)^{-1}\, \Gamma_{2\gamma}$. For the η' a direct measurement of the $\eta' \rightarrow \gamma\gamma$ branching ratio has been made and turns out to be ~ 0.1(14). It follows therefore that an observation of the above effect will lead to a direct measure of the total width, or at least to reduce the present upper limit of 4 MeV. Similar arguments hold for all mesonic resonances with the 2γ quantum numbers.

The $\gamma\gamma$ total cross section into hadrons can be obtained from (21) and (22) by detecting in coincidence the final electrons. Due to the form of the mass spectrum dM^2/M^2, the direction of the photon momenta, and the small transverse momentum distribution, which is a characteristics of all high energy hadronic reactions, the hadrons produced will more in general along the beam direction. A theoretical extimate of $\sigma^{total}_{\gamma\gamma}$ is highly model dependent. A pure Q. E. analogy will give asymptotically constant cross sections, in agreement with a crude factorization hypothesis of strong cross sections at high energies. A logarithmic dependence of the total cross sections $\sigma(e^{\mp}e^- \rightarrow e^{\mp}e^-$ hadrons) from the threshold energy s_{th} is the experimentally observable consequence of the constancy of $\sigma(\gamma\gamma \rightarrow$ hadrons). For production of high hadron masses at large angles however, is quite conceivable to expect a rate of the order of (30), as in pure e. m. pair production. It follows that at sufficiently high energies one espects in electron positron collision a large contamination between hadrons produced via the one-photon channel and the $\gamma\gamma$ mechanism. It would be therefore highly desirable to have both electrons-electrons and electrons-positrons running in the next future rings in order to separate the contribution of the $\gamma\gamma$ interaction and subtract it out in studying the deep e^+e^- annihilation into hadrons.

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