COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 70/57
Dicembre 1970

D. B. Isabelle: ELECTRON SCATTERING ON NUCLEI: A BRIEF REVIEW. -

"Frascati Meeting on Electronsynchrotron"
Frascati, November 5-7, 1970

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

Invited talk, given at the
"Frascati Meeting on Electronsynchrotron"
Frascati, November 5-7, 1970

LNF-70/57
Dicembre 1970

D. B. Isabelle(x): ELECTRON SCATTERING ON NUCLEI: A BRIEF REVIEW.

I have been asked to present you with a review of the important features about electron scattering. Due to my personnal background I will limit myself to nuclear physics information and will not speak of the nucleon form factors which are well known in the energy range under consideration here.

As an introduction to this lecture, we will comment Figure 1 which represents a schematic spectrum of scattered electrons. It can be divided in different regions depending on the energy ω lost by the electrons during it interaction with the nucleus; namely:

- elastic scattering
- inelastic scattering with excitation of nuclear states
- quasi-elastic scattering i.e. electron elastic scattering on the individual nucleons inside the nucleus
- quasi-inelastic scattering i.e. electron scattering on the nucleons with pion production (not seen on Figure 1).

Experimental set-up -

Up to now all electron scattering facilities followed the same general design (Figure 2): an achromatic beam handling systems provides

(x) - Laboratoire de Physique Nucléaire II, Université de Clermont, CLERMONT-FERRAND.

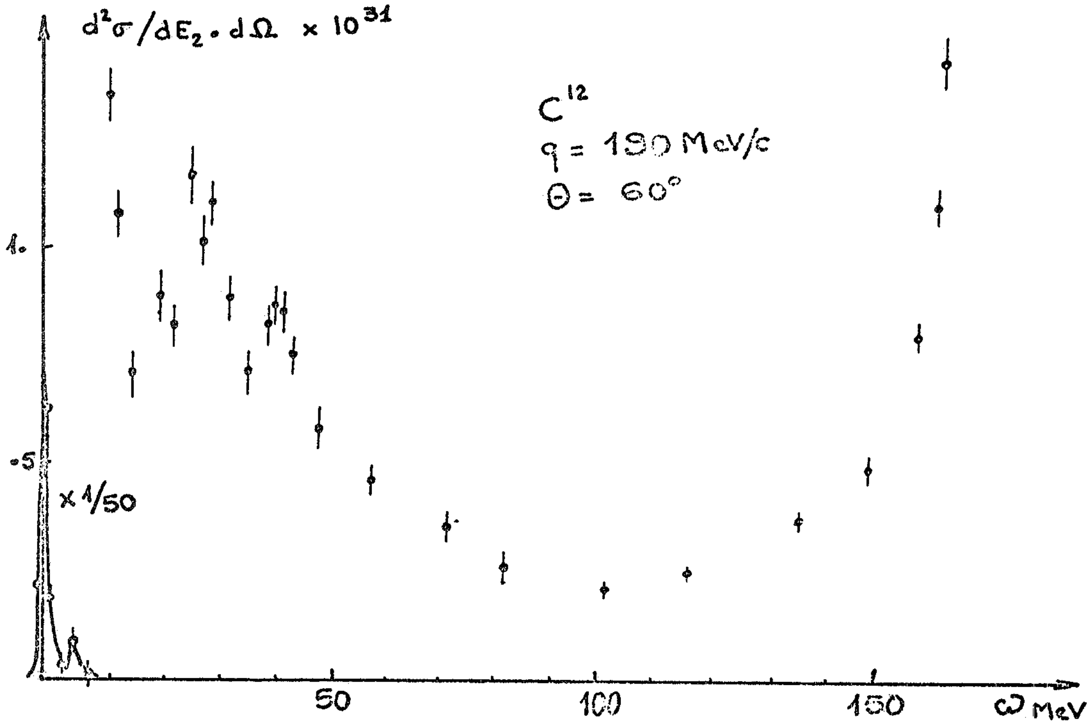

FIG. 1 - Typical electron scattering spectrum.

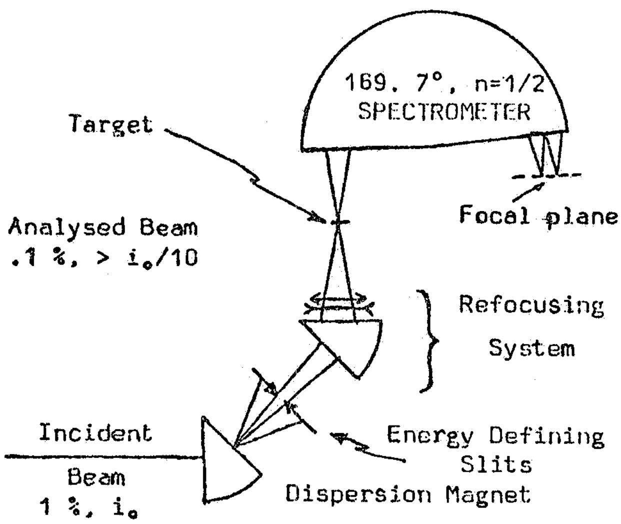

FIG. 2 - Classical electron scattering Set-up.

an analysed beam onto the target, then the scattered electrons are momentum analyzed by means of a double focusing spectrometer. The best energy resolution is achieved if the spectrometer mean bending angle is 169.7^o (the so-called Sakai-Penner magic angle): a resolution $\Delta p/p = 4 \times 10^{-4}$ FWHM has been obtained for the NBS spectrometer (radius: 0.75 m) while the Saclay group hope to reach 2×10^{-4} FWHM (radius : 1.80 m). Actually, an overall energy resolution of 10^{-3} can be reached for thin target (< 50 mg/cm^2) and a 5×10^{-4} spread in momentum for the incident beam for a solid angle of 5×10^{-3} Sr.

This system does not allow to make a full use of intense beam available with modern linacs as the use of such spectrometers implies a very small beam spot and a small solid angle. This is why the MIT group suggested about three years ago to extend to electron scattering a technique already used at some cylotron facilities and for pion experiment at Frascati. Such a system generally called an "energy loss spectrometer" or the "dispersed beam technique" is described by Kowalsky et al. "as a typical non-dispersive beam handling system with the spectrometer taking the place of the second magnet" (Fig. 3). It allows the use of the total beam even with a total incertainty of 1% in momentum while an energy resolution of less than 2×10^{-4} FWHM can be achieved. It will provide a mean of measuring very small cross-sections (10^{-35} cm^2/Sr) in a reasonable time and then to use fruitfully the large beam power available with the new accelerators.

As we will see later electron scattering at 180^o provides a direct measurement of the magnetic properties of nuclei. The set-up for such experiments was originally suggested by Barber. It is shown on Fig. 4, which is self explanatory if one realizes that the axis of rotation of the magnetic spectrometer is also the axis of symmetry of the auxilliary magnet.

I would like to mention a new set-up described by Bergstrom from MIT. It is based on a theorem by L.A.C. Koers according to which a particle passing through a cylindrically symmetric magnetic field which satisfies the relation:

$$\int_0^{\infty} B_z \, r \, dr = 0$$

will have the same z component of angular momentum in a field free region near the symmetry axis as it has in a field free region outside the magnet. Therefore a particle approaching radially will pass through the

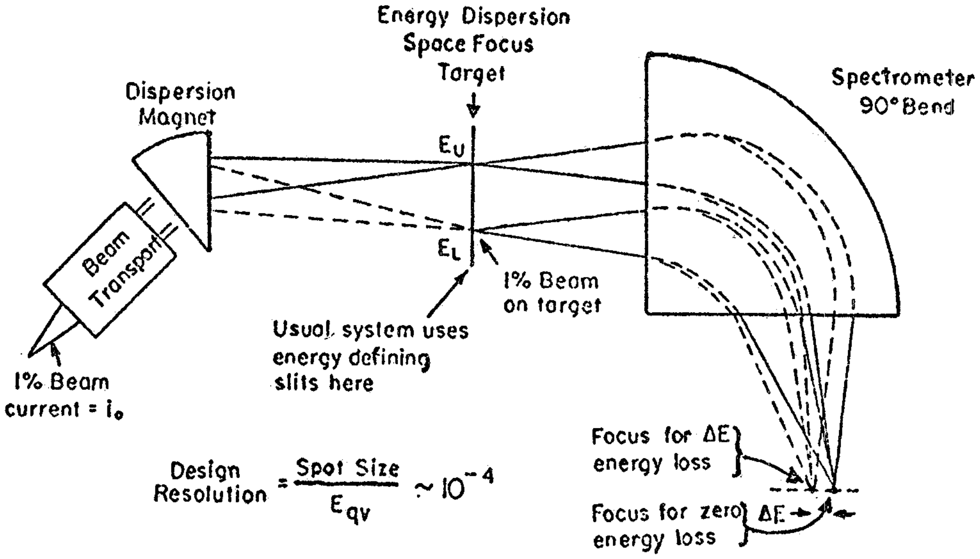

FIG. 3 - Schematic diagram of the energy loss spectrometer.

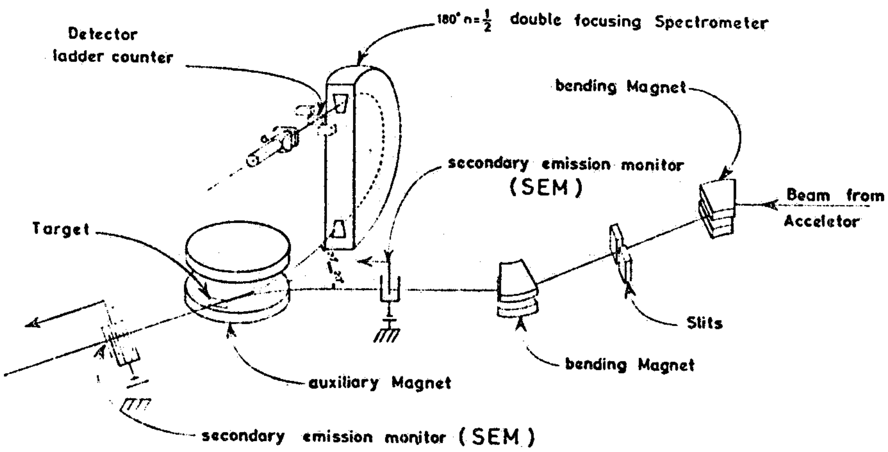

FIG. 4 - Barber's set-up for 180° electron scattering.

axis of symmetry and reciprocally (see Fig. 5). Such a device has been studied by the MIT group and is now under construction. It will also allow scattering at angles smaller than 180° which are difficult to reach with a classical set-up.

To show you the type of energy resolution obtained to-day in electron scattering experiments, we present on Figure 6 a spectrum of electrons scattered on ^{48}Ti obtained by Blum et al. at the National Bureau of Standards. Such spectra compare very favorably with those obtained for the scattering of heavy charged particles such as alphas.

Theoretical background -

It is well known that the advantage of electrons as nuclear probes is due to the fact that they can interact only electromagnetically. Within the framework of the first Born approximation, the cross-section for the interaction of an electron on a nucleus of mass M_T with a momentum transfer q_μ is given by:

$$\frac{d^2\sigma}{d\Omega_2 d\varepsilon_2} = \frac{4Z^2\alpha^2}{q_\mu^4}\frac{\varepsilon_2^2}{M_T} \times \cos^2\frac{\theta}{2} \times \left[W_2(q^2,\omega) + 2W_1(q^2,\omega)\mathrm{tg}^2\,\theta/2\right] \tag{1}$$

where θ is the scattering angle and:

$$q_\mu^2 = 4E_1E_2\sin^2\theta/2 = q^2 - \omega^2$$

while $W_{1,2}(q^2,\omega)$ are two form factors which contain all the nuclear information.

If we consider only the electron scattering with excitation of discrete levels the cross-section can be written:

$$\frac{d\sigma}{d\Omega} = \frac{8\alpha^2}{q_\mu^4}\left(\frac{\varepsilon_2}{1}\right)\left\{V_L(\theta)\sum_{J=0}^{\infty}\frac{|\langle J_f\|\hat{M}_J^{coul}(q)\|J_i\rangle|^2}{2J_i+1} + V_T(\theta)\sum_{J=1}^{\infty}\left[\frac{|\langle J_f\|\hat{T}_J^{el}(q)\|J_i\rangle|^2}{2J_i+1} + \frac{|\langle J_f\|\hat{T}_J^{mag}(q)\|J_i\rangle|^2}{2J_i+1}\right]\right\} \tag{2}$$

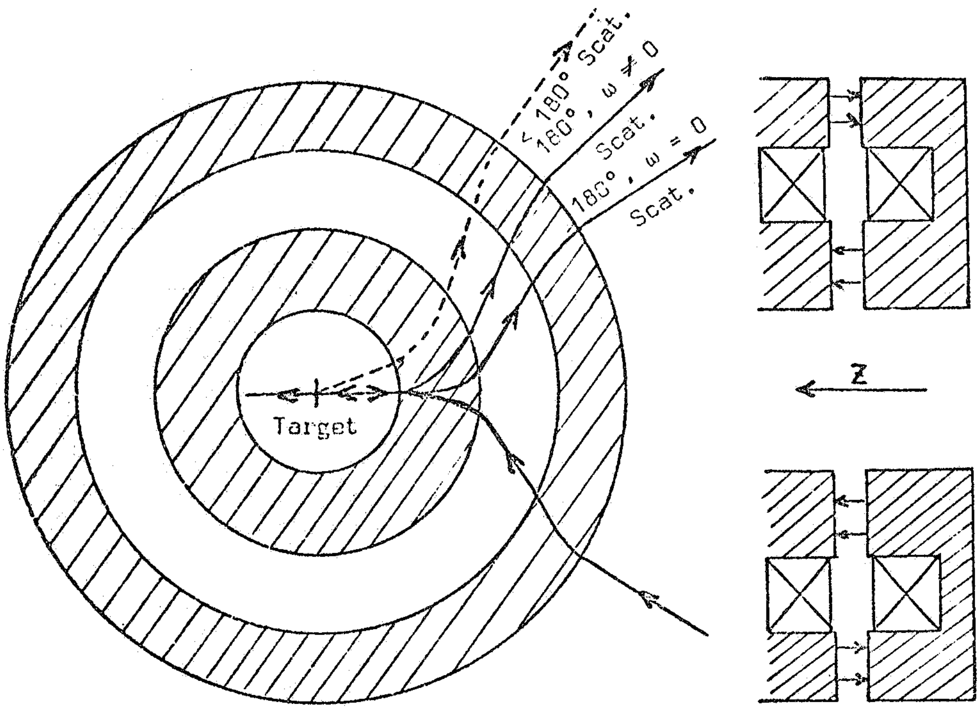

FIG. 5 - Principle of the MIT proposal for 180° scattering.

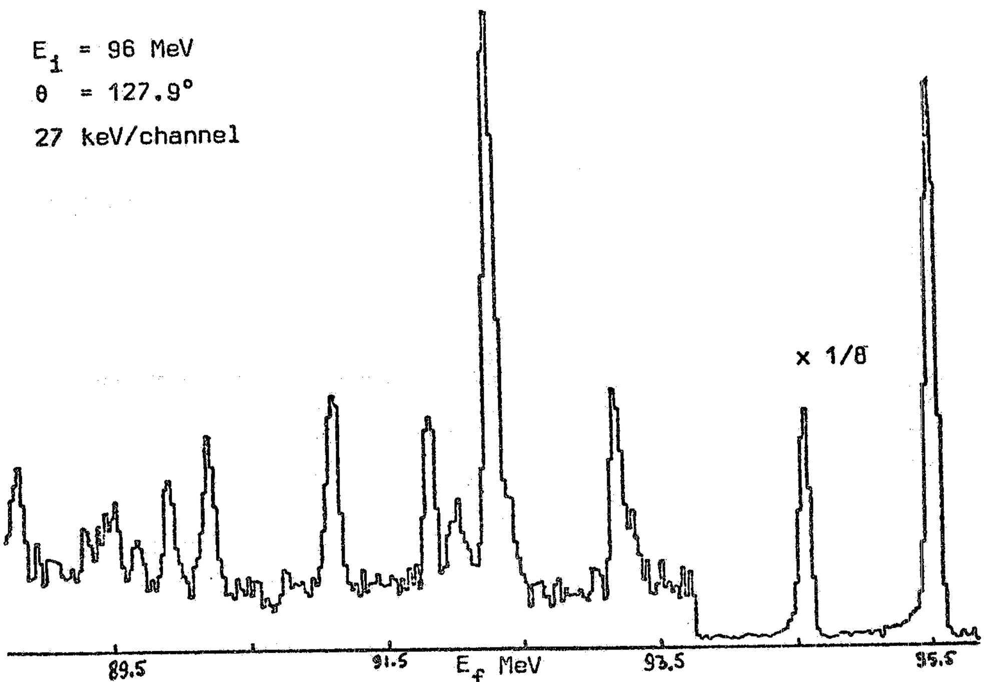

FIG. 6 - Spectrum of electrons scattered on ^{48}Ti

$$V_L(\theta) = \frac{q_\mu^4}{q^4}\, 2\,\varepsilon_1 \varepsilon_2 \cos^2\frac{\theta}{2} \xrightarrow[\varepsilon_2\to\varepsilon_1]{} 2\varepsilon_1^2 \cos^2\theta/2.$$

(3)

$$V_T(\theta) = \frac{2\,\varepsilon_1\varepsilon_2}{q^2}\sin^2\frac{\theta}{2}\left[(\varepsilon_1 + \varepsilon_2) - 2\,\varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2}\right] \xrightarrow[\varepsilon_2\to\varepsilon_1]{} \varepsilon_1^2 \times$$

$$\times\,(1 + \sin^2\frac{\theta}{2}).$$

The matrix elements $\langle J_f \| \hat{M}_j^{coul}(q) \| J_i \rangle$ and $\langle J_f \| \hat{T}_j(q) \| J_i \rangle$ represent the interaction of the electron respectively with the charge density $\rho(r)$ and current densities. The exposants "el" et "mag" refer to the electric and magnetic current densities.

The formula obtained with the first Born approximation has the advantage to display clearly the nuclear part of the interaction. But we know that it does not represent the truth. In particular it predicts the excistence of zeroes in the form factor behaviour, where we observed experimentally diffraction minima only. One can compute numerically the exact cross-section solving the Dirac equation by a phase shift technique. Such programs have been written by various laboratories following the pionner work of the group at Duke University. Many theoreticians have also tried to compute the dispersive corrections i.e. the fact that during the interaction the nucleus could be in a state different from the initial or final state. Despite the crudeness of the calculation due to the complexity of the problem we can say that such dispersive corrections have a very small influence except eventually at the diffraction minima.

Elastic scattering -

For a spherical nucleus with a 0-spin ground state the elastic form factor is the Fourier transform of the charge distribution. The best fit to the experimental data has been obtained using a Fermi-type charge distribution.

$$\rho(r) = N\left\{1 + \exp\,(4\ln 3(r - c)/t)\right\}^{-1}$$

where c is the distance at which $\rho(r) = (1/2)\rho(0)$ and t is the distance over which ρ drop from $0.9\ \rho(0)$ to $0.1\ \rho(0)$.

From measurement performed at values of qR small compared to unity (R being the "average radius" of the nucleus) one obtains a model independant determination of the charge root-mean-square radius. A systematic work in this field was done beautifully by the Darmstadt group. The values obtained are in very good agreement with those derived from muonic atom experiment.

As one increases the momentum then one gains information about the surface thickness and the half-density radius. A general survey over the mass number A scale shows that the surface thickness t is relatively constant (t = 2.4 fm) while, on the average the half density radius varies as $A^{1/3}$ ($c = c_o A^{1/3}$ with $c_o \sim 1.2$ fm). Comparison between the charge distribution of isotopes and isotones has provided very valuable information about the influence of extra neutrons on the charge distribution.

Finally if the measurements are performed at very large momentum transfer there we reach information about the central part of the charge distribution. The beautiful experiments of the Hofstadter group at Stanford have shown that in order to fit the experimental data one must assume a non uniform charge density at the center of the nucleus. This results is in general agreement with predictions obtained using a shell-model with a wood-Saxon potential.

Up to now we discuss only the case of 0-spin nuclei. For a deformed nucleus the previous results are still valid, but the cross-section is not due only to the charge distribution but also to the electric and magnetic moments distribution. From electron scattering experiments one can determine the static values of those moments as well as their spatial distribution. It provides a mean of measurement which can be used for any type of nucleus while atomic methods can be applied only to a few species. Both techniques have the disadvantage to provide a model dependent result. Let us remind that the 180° electron scattering is sensitive only the magnetic terms (Figures 7 and 8).

Inelastic scattering -

We have already seen on Figure 6 that electron scattering experiments have reached an energy resolution such that it can be used to derive very accurate information about the energy of the excited states as well as about their caracteristics.

We do not wish to discuss here all the results obtained from inelastic electron scattering. Any one who wishes to study this problem will it clearly described in many recent pubblications. We can remind that such experiments provide some knowledge about the multipolarity, the reduced matrix element and the transition charge density.

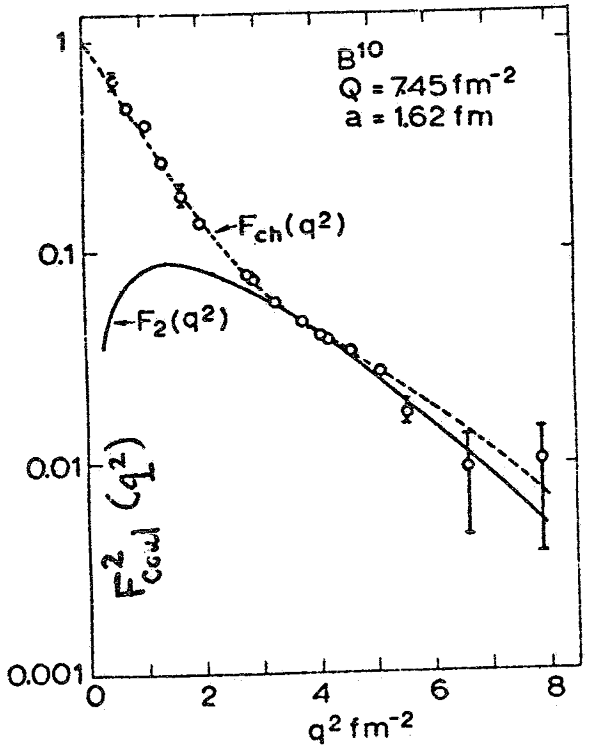

FIG. 7 - ^{10}B elastic form factor showing the importance of the quadrupole term $F_2(q^2)$.

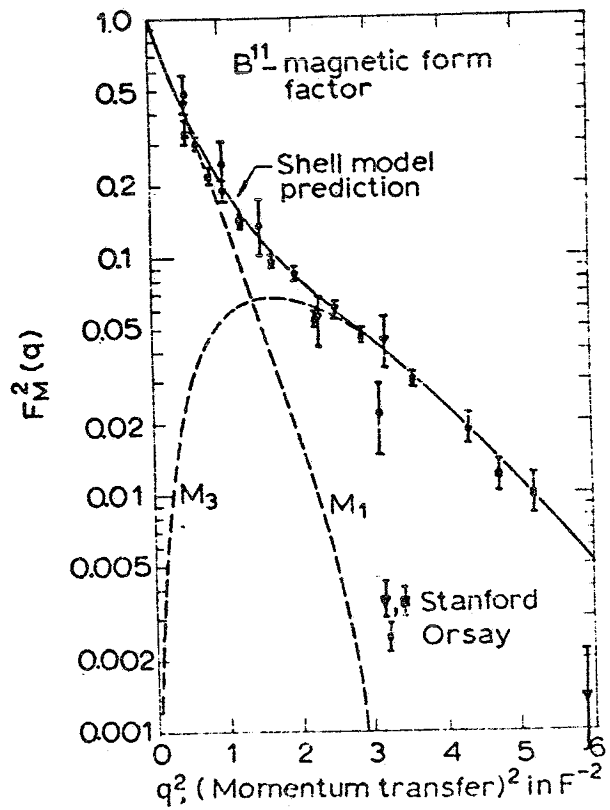

FIG. 8 - Magnetic elastic form factor of ^{11}B. The curves labelled M_1 and M_3 correspond respectively to the form factor of the dipole and octupole magnetic momentum.

Quasi-elastic scattering -

This type of experiment is very interesting as it gives us a mean to study the properties of bound nucleons. Obviously the experiments in which one detects in coincidence the scattered electron and one of the outgoing particles are more accurate. Nevertheless pure electron quasi-elastic scattering has also its value. Namely it allows us to look for effects such as short range correlations.

There are two ways to analyse those data:

- either by measuring sum-rules that can be compared to theoretical predictions. It was shown that the sum-rules are very insensitive to correlation effects but for 180^{o} measurements. In this case Czyz demonstrated that the sum-rule is proportional to the nucleon magnetic moments and any deviation between experience and theory could be interpreted as if the magnetic moments are different for free and bound nucleons.

- either by comparing the shape of the measured quasi-elastic peak to what is expected from the theory. This technique has been used both at Orsay and Stanford. It provides a mean to measure directly the value of the Fermi momentum. On the other hand we found in an experiment on Carbon that on the high energy side of this peak there excists at least one shoulder which is not predicted by any shell-model calculations.

One obvious interest of such measurements is to provide a check to any nuclear model which will be used later to analyse other experiments such as (e, e'p) for example.

Radiative effect problems -

We have seen that the use of electron as nuclear probes has many advantages. Infortunately the analysis of the data is complicated by the fact that due to their small mass electrons radiate very easily.

The radiative effects that occur are the following:

- emission and reabsorption of soft photons
- internal bremsstrahlung or photon emission in the field of the scattering nucleus
- external bremsstrahlung or photon emission by the electron in the target on its way in or out
- ionization due to e-e collision on atomic electron inside the target.

This effect must be taken into account in two ways:

- if one is interested to measure the cross-section corresponding to the excitation of a given nuclear state, then due to the finite energy width of the detecting apparatus the raw data must be multiplied by the "radiative correction". This correction takes into account all the previou sly mentionned effects.

- if one wish to compute the extension of a given peak towards the low energy side of the spectrum, then the three last effects should be included in the so-called "radiative tail" calculation.

We refer the reader to the references given below if he is interested in this problem. We would like here to comment only on the ways that can be followed to analyse the experimental spectra. Two techniques can be used and the choice of the proper one depends on our theoretical knowledge of the wave function describing the nucleus under study.

If we deal with a complex nucleus for which we do not have at our disposal any accurate theoretical information, then we have to use the unfolding technique. The corresponding procedure is the following:

1 - we should measure all spectra at a given angle but for different incident energies
2 - from those spectra one extracts the elastic cross-section after applying the radiative corrections
3 - then for each spectrum one calculates the radiative tail of the elastic peak and subtracts it from the measured spectrum
4 - points 1) to 3) are then performed on the first inelastic level and so on for each level.

An example of such an unfolded spectrum is given in Figure 9 for Carbon.

For light nuclei such as hydrogen and helium isotopes, which have very few excited states if any, and for which the nuclear models provide relatively good wave functions then we can use a folding technique. In this case, a preticted spectrum is computed for a given nuclear model including all the radiative effects as well as the experimental resolution. This predicted spectrum is then directly compared to the measured as shown on Figure 10, for the case of 180^{o} electron scattering on deuterium.

It must be recalled that all the formulas used to take into account the radiative effects have been derived within the frame work of the first Born approximation. Recently some work has been done to take into account the second Born approximation terms. It will be interesting to see some more experimental work done in this field to verify the validity of the formulas in particular for large momentum transfer and for heavy nuclear targets.

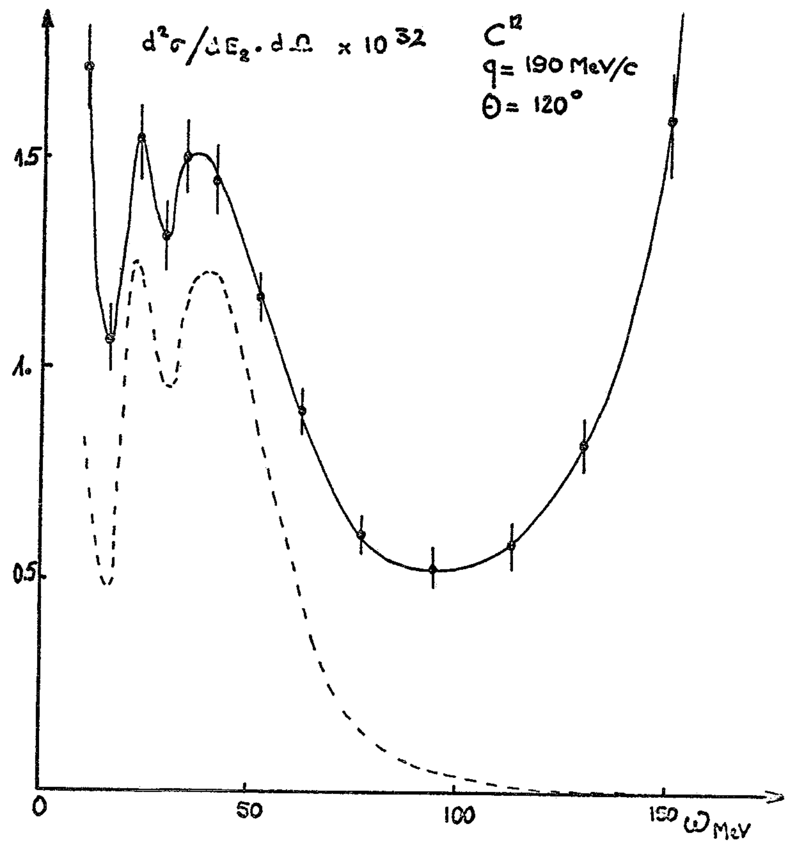

FIG. 9 - Comparison between a raw spectrum (full line) and a corrected data (dotted line) obtained using the "unfolding technique" for electron scattering on Carbon.

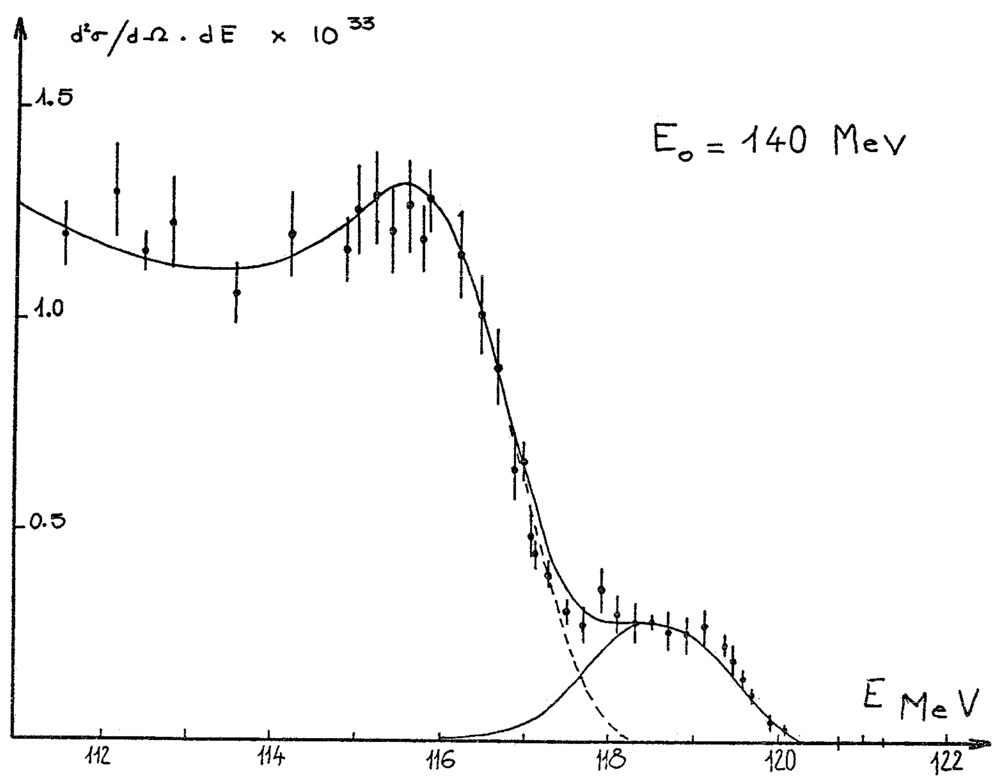

FIG. 10 - Comparison between a spectrum of 140 MeV electron scattered on Deuterium at 180° and at the expected spectrum obtained by the "folding technique".

Conclusion -

We believe that electron scattering has become a very power tool and accurate way of studying the properties of nuclear matter.

We must admit that the experimentalist should start to be more careful in the analysis of their data. In particular the use of the same nuclear wave functions should be tried to compare the results obtained from various nuclear reactions (e, e') (α,α'), (p, p')....

As to the use of the Frascati synchrotron for electron scattering experiments one must realize that the extracted beam intensity is at least ten times smaller than at the old Orsay linac, while the momentum resolution is of the order of 3 MeV. If someone wished to undertake anykind of measurements in a reasonable time he should performed forward angle scattering on heavy nuclei to study quasi-elastic scattering and radiative correction effects. Another interesting problem will be to use a polarized electron beam and a polarized target different from hydrogen. This last type of experiments will surely be the most interesting. Nevertheless we believe that the best use of the extracted electron beam at Frascati is for the (e, e'p) coincidence experiments for which the large duty cycle is a first order necessity while the low intensity beam does not imply limitations. On the other hand if some electron scattering program was started at Frascati it should make use of the very good properties of the electron linac used part of the time as an injector to Adone.

REFERENCES -

- General reviews about electron scattering:
  - T. de Forest and J.D. Walecka, Advan. Phys., 15, 1 (1966).
  - J.D. Walecka, in High-Energy Physics and Nuclear Structure (S. Devons Ed.) (Plenum Press, 1970), p. 1.
  - D.B. Isabelle, in High-Energy Physics and Nuclear Structure (S. Devons Ed.) (Plenum Press, 1970) p. 18.
  - Proc. MIT 1967 Summer Study (W. Bertozzi and S. Kowalsky Ed.) Report TID 24667 (1969)
  - Proc. NATO Advanced Institute on Electron Scattering and Nuclear Structure, Cagliari 1970 (B. Bosco Ed.) to be published by Gordon and Breach.
- Review articles about radiative corrections:
  - L.C. Maximon, Rev. Mod. Phys., 41, 193 (1969).
  - L.W. Mo and Y.S. Tsai, Rev. Mod. Phys., 41, 172 (1969).