A. Bietti: RELEVANCE OF PHOTOPRODUCTION MULTIPOLE ANALYSIS IN ORDER TO TEST THE VALIDITY OF CURRENT ALGEBRA OR FIELD ALGEBRA COMMUTATORS. -

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A. Bietti[(x)]: RELEVANCE OF PHOTOPRODUCTION MULTIPOLE ANA LYSIS IN ORDER TO TEST THE VALIDITY OF CURRENT ALGEBRA OR FIELD ALGEBRA COMMUTATORS. -

In the last few years, several sum rules for photoproduction amplitudes have been derived using various current algebra[1] equal time commutators.

Most of these sum rules are obtained from charge density--charge density and charge-current density commutators in the so cal led "fixed q^2" approach. I will here sketch briefly the "fixed q^2" ap-proach and some of the useful results concerning photoproduction am-plitudes. Let us take for instance the commutator

$$(1) \qquad \left[\int e^{i\vec{q}\cdot\vec{x}} J_o^\alpha(x) d^3 x, \int e^{i\vec{q}\cdot\vec{y}} J_o^\beta(y) d^3 y \right]_{x_o = y_o} \underset{=}{\,} if_{\alpha\beta\gamma} \int e^{i(\vec{q}+\vec{q})\vec{x}} J_o^\gamma d^3 x$$

where $J_o^\alpha(x)$ is the vector charge density with SU_3 (or isotopic) index α. Sandwiching eq. (1) between proton states and inserting a set of inter-mediate states $|p_n\rangle$ one deals then with matrix elements of the type

$$\left\langle p \left| \int e^{i\vec{q}\vec{x}} J_o^\alpha(x) d^3 x \right| p_n \right\rangle$$

(x) - Istituto di Fisica dell'Università di Roma and Istituto Nazionale
di Fisica Nucleare, Sezione di Roma.

from which, by translational invariance, one obtains a $\delta(\vec{p}+\vec{q}-\vec{p}_n)$ factor, But

$$q_o = p_{on} - p_o = \sqrt{(\vec{p}+\vec{q})^2 + M_n^2} - \sqrt{\vec{p}^2 + M^2}$$

so that for fixed finite $\vec{p}$, q_o (and therefore q^2) increases with M_n^2, whereas for $\vec{p} \to \infty$, $q^2 = \vec{q}^2$ is fixed.

A well known sum rule for photoproduction amplitudes derived in the "fixed q^2 approach" from the commutator (1) is the Cabibbo-Radicati[2] sum rule

$$\left| F_2^V(0) \right|^2 + \frac{1}{2\pi^2\alpha} \int \frac{d\omega}{\omega} \left[2\sigma_{1/2}^V(\omega) - \sigma_{3/2}^V(\omega) \right] = \frac{\langle r^2 \rangle_V}{3}$$

$F_2^V(0)$ is the isovector anomalous magnetic moment, $\alpha = 1/137$, σ_I^V are the total cross sections (the sum rule is for immaginary parts of Compton amplitudes) for isovector photons on proton to states of isospin I, and $\langle r^2 \rangle_V$ is the isovector charge radius of the proton.

Other interesting sum rules for single π-photoproduction have been derived by Fubini et al.[3], using commutators between vector current densities and axial charges, under the PCAC assumption

$$F_2^{V,S}(0) + \frac{2M}{g_{N\pi}} \frac{1}{\pi} \int \text{Im} A^{V,S}(\nu) \frac{d\nu}{\nu} = 0$$

where $A^{V,S}(\nu)$ is an invariant amplitude (respectively for isovector and isoscalar photons) for single pion photoproduction.

More general sum rules for invariant amplitudes have been derived by Fubini[4], always in a dispersion theoretical approach and using a generalized Ward identity for current densities correlation functions (vector current and axial vector current densities in the case of photoproduction processes):

(2) $$\frac{1}{\pi} \int \text{Im } A(s, q_1^2, q_2^2, t)\, ds = F(t)$$

where q_1^2 and q_2^2 are the "masses" of the currents, $s = (p_1 + q_1)^2$ and $t = -(p_1 - p_2)^2$.

It is worth mentioning that perhaps most of the interest of a sum rule like eq. (2) is beyond current algebra, in the so called superconvergence or strong interaction sum rules[5].

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In fact $\text{Im}\, A(s, q_1^2, q_2^2, t)$ has poles for $q_1^2 = m_1^2$, $q_2^2 = m_2^2$ that is at the physical masses of particles with the same quantum numbers of the currents. Taking then the residue of eq. (2) at the poles we obtain the strong interaction sum rules

$$(3) \qquad \frac{1}{\pi} \int \text{Im}\, A_{\text{strong}}(s, t)\, ds = 0$$

As it is well known the possibility of writing eq. (3) depend from the asymptotic properties in s of the amplitude A. In any case, by means of the finite energy sum rules[6] we can use relations like eq. (3) as a connection between the low energy part and the asymptotic behaviour of the amplitude.

This can be particularly important for pion photoproduction where the interpretation of the high energy data in terms of Regge parameters is rather difficult considering the presence of dip structures, conspiracy and factorization problems, and absorptive corrections.

Let us go back now to current algebra. The commutators that have been used in order to obtain sum rules in the "fixed-q^2" approach are always charge-charge or charge-current density commutators. One does not obtain, in the infinite momentum approach, sum rules from commutation relations involving the space components of the current densities because such sum rules would not be convergent, i.e. the $J_i^\alpha(x)$ are "bad" operators. The importance of sum rules based on the commutators between the $J_i^\alpha(x)$ is that quark model current algebra (C.A.) and field algebra[7] (F.A.) give different answers for such commutators:

$$(4) \qquad \left[J_i^\alpha(x), J_j^\beta(y) \right]_{x_0 = y_0} = \begin{cases} \text{if } \alpha\beta\gamma \; \delta_{ij}\, J_0^\gamma(x)\, \delta(\vec{x} - \vec{y}) & \text{C.A.} \\ \\ 0 & \text{F.A.} \end{cases}$$

In order to obtain some information on this kind of commutators we have thus to work with static i.e. q^2 variable sum rules. The first sum rule of this kind was the Dashen-Lee[8] sum rule for magnetic moments, derived from the commutation relation

$$(5) \qquad \left[M_3^+, M_3^- \right] = \frac{1}{6} \int d^3 r\, r^2\, J_0^3(r) - \frac{1}{12} \int d^3 r (3\, z^2 - r^2)\, J_0^3(r)$$

obtained from eq. (4) (in the C.A. hypothesis) for the operators

$$M_i^\alpha = \frac{1}{2} \int d^3 r \, \mathcal{E}_{ijk} \, r_j \, J_k^\alpha \qquad\qquad M_i^\pm = \frac{1}{\sqrt{2}} (M_i^1 \pm i \, M_i^2)$$

With SU(6) estimates for the nucleon-33 resonance transition magnetic moment μ^x they got the result

$$(6) \qquad\qquad (\frac{\mu_p}{2M})^2 = \frac{\langle r^2 \rangle_p}{6}$$

The two sides of eq. (6) agree within 15-20% so that this result could be considered a success for C.A. against F.A. Actually, the SU(6) esti mate for μ^x does not agree with experiment so that the validity of eq. (6) should be re-examined, as we shall see later. We will start in-stead considering the simpler commutator

$$(7) \qquad \left[\int d^3 r \, J_i^\alpha(r), \int d^3 r' \, J_j^\beta(r') \right] = \begin{cases} if_{\alpha\beta\gamma} \, \sigma_{ij} \int d^3 r \, J_o^\gamma(r) & \text{C.A.} \\[2ex] 0 & \text{F.A.} \end{cases}$$

The integrals of the currents as we shall see soon, induce strong pari ty and angular momentum restrictions in the sum rules. Moreover, pos sible Schwinger terms in the r.h.s. of eq. (7) are ruled out by the inte grations. We will take $= +$ and $= -$, $i = j = 3$ in eq. (7) sandwiching the commutator between proton states at rest and we obtain the sum rules

$$(8) \qquad \sum_{\mathcal{N}} \left| \langle p | J_z^3(0) | \mathcal{N} I = \frac{1}{2} \rangle \right|^2 - \frac{1}{2} \sum_{\mathcal{N}} \left| \langle p | J_z^3(0) | \mathcal{N} I = \frac{3}{2} \rangle \right|^2 = \begin{cases} \frac{1}{4} & \text{C.A.} \\[2ex] 0 & \text{F.A.} \end{cases}$$

It should be stressed again that in these sum rules the four momentum k^2 of the current J_z varies with energy. In fact the integra tions in eq. (7) give $\vec{k} = -\vec{p} + \vec{p}_n = 0$ while $k_o = W - M$, since the nucleon is at rest and so is the intermediate state of mass W. We see thus that the four momentum squared of the "photon" is timelike. We can see now the different contributions to such "variable k^2" sum rules.

Type I - The usual "direct channel" contribution, also present (as a mat ter of fact the only one present) in the infinite momentum frame sum rules. In our case, since all the intermediate states are at rest, these

will be only $J = 1/2^-$ and $J = 3/2^-$ states.

Type II - "Mass" singularities due to the fact, as we observed, that k^2 varies in the time-like region one gets therefore contributions from boson resonances (in our case the φ-meson for instance) with the quantum numbers of the current.

Type III - The so called "pair" or "Z-graph" contribution: for instance the incoming current creates a pair: the particle is the final nucleon where the antiparticle annihilates with the incoming nucleon in the outgoing current.

In our case type III contributions are negligible compared to the two others because of the low value of the electromagnetic form factor of the nucleon, as can be seen by the $\bar{p}p \to e^+e^-$ experiments[9].

It has to be noted that this discussion on the different types of contribution to "variable q^2" sum rules has been recently presented in a very elegant paper by Fubini and Furlan[10].

Since the φ-meson singularity is a rather broad resonance, instead of extracting it going on the φ-pole residue, like in the Fubini--Furlan method, we prefer integrate over it in the sum rule, as we did for some rest frame sum rules a few years ago[11], using therefore photoproduction amplitudes and the vector dominance for the electromagnetic form factor. We have thus a sum rule for the imaginary part of a virtual Compton scattering so that the main problem is the knowledge of definite parity and angular momentum amplitudes for the process:(virtual) $\gamma + p \to$ anything.

As a first approximation we will saturate the sum rule eq. (8) with the multipoles of the dominant process, i.e. single pion photoproduction, for which we have reasonable multipole analysis. In terms of the conventional CGLN[12] multipoles we have for eq. (8)

$$(9) \quad \frac{137}{\pi M} \int Wq\, dW \left| F(k_o^2) \right|^2 \left[\left| E_{o+}^{v\frac{1}{2}} \right|^2 + 2 \left| E_{2-}^{v\frac{1}{2}}(o) \right|^2 \right] - \frac{1}{2}(\text{same with}) = \begin{cases} 0.25 & \text{C.A.} \\ \\ 0 & \text{F.A.} \end{cases}$$
$$E^{v\frac{3}{2}} \qquad E^{v\frac{1}{2}, \frac{3}{2}}$$

q is the C.M. pion momentum, $E_{2-}(o)=E_{2-}+M_{2-}$ and the notation $E^{v\frac{1}{2}, \frac{3}{2}}$ refers respectively to the isovector $I = 1/2$ and $I = 3/2$ electric dipoles. $F(k_o^2)$ is the isovector form factor. We will use at this point Walker's analysis[13] of single pion photoproduction. An evaluation of l.h.s. of

eq. (9) in the narrow width approximation has already been done [14];
here we will give the results performing the integrals in eq. (9) up to
$W \simeq 2\,\mathrm{GeV}$ by means of a computer (the University of Rome 1108 UNIVAC
Computer). Looking to the essential features of Walker's analysis we see
that in the $I = 1/2$ part we have the $S_{11}(1560)$ and $D_{13}(1520)$ resonances
plus the background and the Born terms, in the $I = 3/2$ part we do not have
resonances. We obtain the following result:

$$
\underbrace{\overbrace{\underset{0.27}{\text{S wave}} \quad \underset{0.372}{\text{D wave}}}}_{I = \dfrac{1}{2}} \qquad \underbrace{\overbrace{\underset{-0.21}{\text{S wave}} \quad \underset{-0.085}{\text{D wave}}}}_{I = \dfrac{3}{2}} \quad = 0.347
$$

to be compared with 0.25 from C.A. and 0 from F.A. The relatively
high value for the S wave contribution in the $I = 3/2$ part is most certain
ly due to great importance of the electric Born term. In the $I = 1/2$ part
if we consider only the pure S_{11} and D_{13} resonances we get respectively
0.2 and 0.45.

The result of this sum rule does not agree very well with C.A.,
but seems nevertheless to be in contradiction with F.A. We do not think
it would be very easy to establish or not a more precise agreement with
the 0.25 value given by C.A. In fact, "inelastic" contributions (that is
multipion photoproduction and so on) are lacking and the vector dominance
hypothesis should not work very well for values of $k_o = W-M$ appreciably
higher than m_ρ. On the other hand around $k_o = m_\rho$, not very far from the
region in W where we have the S_{11} and D_{13} resonances, the ρ-dominance
is a rather good approximation. One could in principle use other expres
sions for the form factor, for instance expressions that are based on
the Veneziano model [15], but which are unfortunately to a big extent ar
bitrary in the timelike region because of problems of unitarization.

In any case we think that adding inelastic contributions and con
sidering more realistic expression for the form factor should not chan
ge drastically the situation in favour of F.A. In fact there is in princi-
ple no reason for which the $I = 3/2$ isovector S waves and D waves should
be much greater than the corresponding $I = 1/2$ ones, and we know on the
other hand that there is an appreciable inelastic contribution to the S_{11}
and D_{13} resonances. Anyway it is clear that a better knowledge of pho
toproduction (not only single π, but multiple π, η, etc.) multipoles would
be very important for our purposes.

We have also varied the multipoles, using for instance the Proia
and Sebastiani [16] parameters for the S_{11} and D_{13} resonances (their ana
lysis is similar to the one given by Chau, Dombey and Moorhouse [17]),
but the results do not change very much, because the difference with

Walker's[13] fit amounts only to weighting more the importance of the S_{11} compared to the D_{13} resonance, so that the net result in eq. (9) is practically unchanged. It is maybe worth mentioning a sum rule very similar to eq. (9), that was derived already a few years ago[11], and which is the same for C.A. or F.A., from an electric dipole commutation relation

$$(10) \qquad \left[E_3^+, E_3^-\right] = \frac{1}{3} \int r^2 J_0^3 d^3 r \qquad \text{with} \qquad E_i^\alpha = \int d^3 r \, r_i \, J_0^\alpha$$

If we take the expectation value of eq. (6) between proton states at rest we have a sum rule which is identical, in the l.h.s., to eq. (9) a part of a k_0^2 factor in the denominator that arises from the fact that

$$J_0(\vec{k}) = \frac{\vec{k} \cdot J(k)}{k_0}$$

Thus we have from eq. (10)

$$(11) \qquad \frac{137}{\pi M} \int \frac{Wq \, dW}{k_0^2} \left|F(k_0^2)\right|^2 \left[\left|E_{0+}^{v\frac{1}{2}}\right|^2 + 2\left|E_{2-}^{v\frac{1}{2}}(o)\right|^2 \right] -$$

$$- \frac{1}{2} \text{(same with } E^{v\frac{3}{2}}) = \frac{\langle r^2 \rangle v}{12}$$

Using again the Walker's analysis[13] we have in units 10^{-28}cm^2

	$I = \dfrac{1}{2}$		$I = \dfrac{3}{2}$		
	S wave	D wave	S wave	D wave	= 6.58
	4.92	4.65	-2.29	-0.7	

to be compared with $6.4 \times 10^{-28} \text{cm}^2$ which is the value of $\langle r^2 \rangle v/12$. If we use only the pure S_{11} and D_{13} resonance parameters we get (of course for the $I = 1/2$ part) 1.7 for the S wave and 4.4 for the D wave contribution. Again we see that the S-wave Born term gives an important contribute not only to the $I = 3/2$ part but also to the $I = 1/2$ part.

The good agreement obtained is probably accidental, owing to the previous discussion about the inelastic contributions and the uncertainties in the form factor. We should however note that the k_0^2 factor

in the denominator makes the l.h.s. of eq. (11) converge much faster than the corresponding l.h.s. of eq. (9). We see also that we have in some sense a cancellation between the S_{11} contribution and the $I = 3/2$ contribution, justifying therefore in some way the results of ref. (11) based on the D_{13} resonance dominance in the sum rule eq. (11).

We want to add now a few words on the magnetic moment sum rule eq. (6) derived from the commutation relation eq. (5). Such a commutation relation is more uncertain (and more interesting in some respects) because of the possible presence of Schwinger terms. Unfortunately the phenomenological situation is rather bad: eq. (6) is badly violated, because the experimental value for μ^x, that is $1.23 \div 1.26 \left[(2\sqrt{2})\mu_p/3\right]$ according Dalitz and Sutherland[18], R. Walker[13], Grilli et al.[19] is appreciably greater than the SU(6) value $\mu^x = (2\sqrt{2})\mu_p/3$. In fact we can write from eq. (5) the sum rule

$$(12) \qquad 2\sum_{\mathcal{N}} \left| \left\langle p \left| M_3^3 \right| \mathcal{N} I = \frac{1}{2} \right\rangle \right|^2 - \sum_{\mathcal{N}} \left| \left\langle p \left| M_3^3 \right| \mathcal{N} I = \frac{3}{2} \right\rangle \right|^2 = \begin{cases} \dfrac{\langle r^2 \rangle v}{12} & \text{C.A.} \\[2em] 0 & \text{F.A.} \end{cases}$$

If we take only the nucleon contribution for the $I = 1/2$ part and the 33 resonance for the $I = 3/2$ part we get eq. (6) if we use for μ^x the SU(6) value and essentially zero if we use the experimental value.

The situation would then favour F.A. against C.A. We should however note that even small variations in the value of μ^x could alter this result[20], and we should also take in account the π-N $I = 1/2$ intermediate states as well as the π-N $I = 3/2$ continuum states for energies higher than 33-resonances. We have thus the following situation, in units of 10^{-28} cm^2

Nucleon	$I = \dfrac{1}{2} \pi N$	$I = 3/2 \; \pi N$	
13	5.5	- 17	= 1.5

In the $I = 3/2$ part the pure 33-resonance gives -15×10^{-28} cm^2. It should be noted that Walker's fit contains a very small amount of P_{11} resonance even if the background is very large, so that the figure for the $I = (1/2) \pi N$ contribution could be increased[21]. There are also the multipion contribution to be taken in account, which are probably more important for the $I = 1/2$ case being the 33-resonance totally elastic. We see thus once more the need we have for accurate analysis for single and multiple pion photoproduction, that we hope will still play an important role in the experimental programs of the existing intermediate energy electron synchrotrons in the near future.

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