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F. Cannata and F. Strocchi: TESTS OF T-INVARIANCE OF THE ELECTRO-MAGNETIC INTERACTIONS. -

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ABSTRACT. -

The experimental situation about C and T invariance in e.m. interactions is reviewed and the implications of the experiments performed so far are discussed. The general feature of the situation is that even if no definite CP(or T) violation effect has been observed in e.m. interactions, one cannot conclude that CP(or T) invariance holds. Tests of T invariance are suggested which look rather promising. Reasonable theoretical estimates show that T-violating effects are not depressed by kinematical or symmetry reasons and that they are expected to be of the order of 30-50%.

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I. - INTRODUCTION. -

Many experiments performed to detect the breakdown of C, CP and T symmetry show either conflicting results or inconclusive data. The degree of accuracy is sometimes very good but the theoretical models which have been forwarded to explain CP violation in the $K^o\bar{K}^o$ system often cannot be strictly verified in so far as they do not give sufficiently precise predictions. The "amount" of violation in the basic interaction cannot be easily related to the "amount" of violation expected in a given experiment without a detailed model and a fairly reliable method of calculation. Both of them are usually lacking. As a matter of fact a relatively large "amount" of violation in the basic interaction may produce small effects in the observation of particular asymmetries because of selection rules, low energy limits, vanishing of the lowest order contributions, summing over final states etc.(1).

These considerations are very important e.g. for an interpretation of the experiments on the e.d.m. of the neutron.

In general it seems very difficult to find reasonable theoretical estimates beyond which the experimental verification becomes really useful.

The aim of this talk is to discuss the hypothesis that C and/or T violation occurs in e.m. interactions. On one hand we will discuss the present experimental situation and the strength of its implications on this model. As we will see even if no definite CP violation effect has so far been observed in electromagnetic interactions, this can in no way be regarded as a proof that the model is wrong.

On the other hand we will suggest tests of T violation in e.m. interactions (photo and electroproduction of π) which look more decisive than the ones performed so far. In particular we will try to avoid as much as possible the difficulties of which we spoke above(2).

As a general feature of T-violation effects in photoproduction and electroproduction of pion (e.g. in the search of the correlation $\vec{\sigma}_{targ} \times (\vec{K}_{lepton} \times \vec{p}\pi))$. We expect a fairly large effect at threshold whereas in the N_{33} energy region the elimination of final state interaction effects reduces T-violating effects sensibly. This, we think, is the reason why the reciprocity relations discussed by Christ and Lee do not provide the best tests of T-violation: indeed they vanish at threshold. A similar argument may work for tests not involving final hadronic variables (e.g. in electroproduction of $\pi\,\vec{S}targ \times (K_{lept} \times K'_{lept})$. In this case indeed final state interaction effects vanish, but this elimination may sensibly depress T violation effects.

II. - GENERAL REMARKS. -

Before going into details it may be useful to recall the meaning of symmetry breaking.

Let V be the unitary (or antiunitary) operator which describes a symmetry transformation. This means that V leaves the Hamiltonian invariant and acts on the $\mathcal{T}$ matrix in the following way

$$V\mathcal{T}V^{-1} = \mathcal{T} \quad (V \text{ unitary})$$

or

$$V\mathcal{T}V^{-1} = \mathcal{T}^{+} \quad (V \text{ antiunitary})$$

Then, if $M_{i \to f}$ denotes the transition amplitude for the process $i \to f$, the invariance under the symmetry V implies

$$M_{i \to f} \equiv \langle f|\mathcal{T}|i\rangle = \langle V_f|\mathcal{T}|V_i\rangle \qquad (V \text{ unitary})$$

or

$$M_{i \to f} = \langle V_f|\mathcal{T}^{+}|V_i\rangle^{*} = \langle V_i|\mathcal{T}|V_f\rangle = M_{Vf \to Vi} \qquad (V \text{ antiunitary})$$

Hence, if q_i and q_f denote the quantities such as momenta spins etc. which characterize the initial and the final states, respectively, one has in the two cases

$$\left|M_{i \to f}\right|^{2} = P(q_i, q_f) = P(q_{Vi}, q_{Vf}) \qquad (V \text{ unitary})$$

and

$$\left|M_{i \to f}\right|^{2} = P(q_i, q_f) = P(q_{Tf}, q_{Ti}) = \left|M_{Vf \to Vi}\right|^{2} \qquad (V \text{ antiunitary})$$

In the first case (V unitary) one cannot measure a correlation between q_i and q_f which is odd under V. For example, a correlation like $\vec{\sigma}_i \times \vec{p}_f$, odd under space inversion, implies that parity is violated. Similarly in the $\eta \to \pi^{+}\pi^{-}\pi^{0}$ decay, a non vanishing asymmetry $(N^{+}-N^{-})/(N^{+}+N^{-})$, $N^{+}(N^{-})$ being the number of events in which $\pi^{+}(\pi^{-})$ is more energetic that the $\pi^{-}(\pi^{+})$, would imply that charge conjugation C is violated in this process.

The case of symmetries described by antiunitary operators is more involved. Only when the $\mathcal{T}$ matrix is hermitian, i.e. when $\mathcal{T}$ may be approximated by an effective Hamiltonian, (first order processes) the discussion is similar to the previous one.

In fact one has

$$\left|M_{i\to f}\right|^2 = \langle f|\tau|i\rangle\langle f|\tau|i\rangle^{*} = \langle v_f|\tau^{+}|v_i\rangle^{*}\langle v_f|\tau^{+}|v_i\rangle \simeq$$

$$\simeq \langle v_f|\tau|v_i\rangle^{*}\langle v_f|\tau|v_i\rangle = \left|M_{Vi\to Vf}\right|^2$$

i.e. the same equation as in the case of unitary V. Therefore also in this case the detection of a V-odd correlation is a proof that the symmetry is violated.

For example, in $K^{+} \to \pi^{0}\mu + \nu$ decay any muon polarization along the normal to the decay plane, i.e. a non vanishing correlation $\vec{\sigma}_{\mu} \times \vec{p}_{\pi} \times \vec{p}_{\nu}$, would imply that time reversal invariance does not hold in weak interactions.

In general however one does not have an hermitian matrix mainly because of final state interactions and one cannot draw the above conclusions. We will discuss this more complicated situation later.

III.-C AND T INVARIANCE OF THE E.M. INTERACTIONS.-

As outilned in the Introduction our aim is to discuss the hypothesis that C is not symmetry of the e.m. interactions of hadrons. If we assume TCP invariance, which does not seem to be presently under discussion, C violation is equivalent to a violation of PT.

Now, all the presently available experimental data seem to indicate that strong and e.m. interactions are parity conserving.

As a matter of fact several experiments have been performed to look for P-violating effects in nuclear reactions in order to test V-A theory etc. These experiments may at the same time be regarded as tests for P violation in strong and e.m. interactions. Since all the results show P-violating effects of the order of 10^{-5} or less one may rather safely conclude that parity is conserved in strong and e.m. interactions. Tests of parity non conservation in nuclear physics involve a variety of processes such as irregular α-widths in nuclear α-decay, circular polarization of photons and asymmetries in the angular distribution of photons in nuclear electromagnetic transitions etc. Forbidden decays i.e. decays which can only occur if there is a parity mixing of nuclear states have transition probabilities of the order $10^{-13} \sim 10^{-14}$(3) compared with the intensity of similar allowed transitions. Alternative tests are obtained by measuring the interference between the normal e.m. decay and the decay resulting from parity mixing of nuclear states. In this case the effects are of the order of 10^{-6} compared to the regular processes(4).

In conclusion, as far as e.m. interactions are concerned, C non invariance is equivalent to T non invariance. As we shall see the latter symmetry seems less difficult to be tested provided one can eliminate final state interaction effects. As discussed in par. VI these spurious effects arise when one studies odd correlations under an antiunitary operator.

The main difficulty connected with tests of C invariance is that one has to compare charge conjugate processes $i \rightarrow f$, $C_i \rightarrow C_f$. Thus, except for the very special cases in which the initial state is an eigenstate of CP or C one has to compare two very different experiments. Normalization problems make this comparison rather difficult. Similar considerations may be in order when one uses detailed balance as a test of time reversal invariance. For example, when one tries to test the reciprocity relation in comparing the following processes

$$\gamma + \binom{P}{n} \rightleftarrows \binom{n}{P} + \pi^{\pm}$$

one finds serious difficulties because of the neutron involved. The very precise measurements of the g-2 of the muon and the Lamb shift, show that the e.m. interactions of the electron and of the muon are accurately described by the usual form $\mathcal{J}_\mu A_\mu$ where $\mathcal{J}_\mu = ie\psi^+ \gamma_4 \gamma_\mu \psi$. This form of interaction is explicitly invariant under, C, P and T. The hadronic electromagnetic current, instead is not known as in the case of leptons. In particular one does not know the symmetry properties of this current and it is not obvious that C and T invariance should hold for e.m. interactions.

As pointed out by Bernstein Feinberg and Lee(5), as a consequence of hermiticity and gauge invariance the C violating part of the e.m. current K_μ has vanishing matrix element between one particle states (spin 0 or 1/2) of equalmasses. Hence any effect of T-violation in nuclear physics must of the order at least of 10^{-2}. This may explain why no effect has so far been observed.

For the same reasons, electron-nucleon elastic scattering cannot be used as a test of time reversal invariance and one has to analyze inelastic electron scattering or decays like $\Sigma^0 \rightarrow \Lambda^0 e^+e^-$. We will not discuss tests like polarization of deuteron in elastic scattering because these will be the subject of other talks.

IV. - C INVARIANCE OF E.M. CURRENT OF HADRONS. -

We summarize the most significant experimental data. As stressed above in order to avoid large experimental errors one has to discuss only those processes in which the initial state is an eigenstate of C or CP.

a)

$$\frac{\text{Rate}\,(\pi^0 \to 3\gamma)}{\text{Rate}\,(\pi^0 \to 2\gamma)/\text{exp}} \leq 5 \times 10^{-6}\,{}^{(6)}$$

Rough theoretical calculations give(7)

$$\frac{\text{Rate}(\pi^0 \to 3\gamma)}{\text{Rate}(\pi^0 \to 2\gamma)} \sim \alpha (K_\gamma R)^8 \frac{(\text{phase space})_{3\gamma}}{(\text{phase space})_{2\gamma}} \leq 10^{-8}$$

So the experimental limit is not accurate enough.

b)

$$\frac{\text{Rate}\,(\eta^0 \to \pi^0 e^+ e^-)}{\text{Rate}\,(\eta^0 \to \text{all})} \lesssim 10^{-4}$$

It is however not clear to what extent this excludes a possible T violation since this decay is inhibited possibly also by SU_3 selection rules (this point was stressed by Cabibbo). It would be completely forbidden at I order if K^μ were an isoscalar.

c)

$$\eta \to \pi^+ \pi^- \pi^0$$

the $\eta \to \pi^+ \pi^- \pi^0$ decay is not forbidden by C invariance but could contain a C violating contribution as well. The presence of a C violating amplitude could give rise to an interference between $\pi^+ \pi^-$ states of different C and angular momentum and yield a non vanishing asymmetry $A = (A_+ - N_-)/(N_+ + N_-)$.

It is worthwhile to stress that only the isovector part of K may contribute to this effect.

The last Columbia experiment gives(8)

$A = (1.5 \pm 0.5)\%$

$A = (1.66 \pm 0.63)\%$ (with more severe geometrical cuts).

The asymmetry, if it exists at all, is not expected to have a large value from a theoretical point of view due both to centrifugal barriers and to the fact that, by CPT invariance, the effect is proportional to sin being δ the relative $\pi\pi$ scattering phase shift of the interfering final states).

It has been suggested by Yuta and Okubo(9) that in the reaction

$$\pi^- p \to n \begin{array}{l} \eta^0 \\ \quad \pi^+\pi^-\pi^0 \end{array}$$

the presence of a small asymmetry is not necessarily an evidence of C violation. Indeed an asymmetry effect may be generated by an interference between resonant and non resonant $\pi^+\pi^-\pi^0$ states and its order of magnitude may be of the order of 1%. However Gormley et al.(10) claim that in their experiment the Yuta-Okubo effect cannot be larger than 0.23%.

A future possibility may be to eliminate the spurious asymmetry by looking at $e^+e^- \to \eta \to \pi^+\pi^-\pi^0$ in storage ring experiment. Indeed in this case the initial state is really an eigenstate of C.

d) Another possibility for studying C violation effect is given by $\eta^0 \to \gamma\pi^+\pi^-$ decays. The discussion is similar to that for $\eta \to \pi^+\pi^-\pi^0$. Now, however, the presence of an asymmetry gives information on the isoscalar part of K_μ. The last value given by Gormley et al. is $(1.2 \pm 1.6)\%$(11).

In conclusion, meson decays do not seem to give any decisive information on the hypothesis of C violation in the e.m. interactions.

Let us turn to barionic tests. Single baryon states cannot be eigenstates of C (or CP) and therefore one has to discuss either baryon antibaryon processes or to switch to tests of T invariance. The former type of tests so far performed did not yield sufficiently precise results, mainly because strong interactions are the dominant effect. We will therefore discuss T-violation tests in baryon electrodynamics.

V. - TIME REVERSAL INVARIANCE. -

In spite of the great level of precision, obtained in the measurement of neutron electric dipole moment

$$\frac{\mu_e}{e} < 5 \times 10^{-23}\,\mathrm{cm}^{(12)} \qquad (\mathrm{e.\,d.\,m.\,n.})$$

this limit is not yet able to rule out the hypothesis that e.m. interactions violate time reversal invariance. The point is that such a model can pretty well explain a value of $\sim 10^{-23}$(13) and therefore is not contradicted by the experiment. As a matter of fact the theoretical predictions are rather loosy even as far as the order of magnitude is concerned. So we will speak of other processes.

In general time reversal invariance requires

$$\left|M_{i\to f}\right|^2 = M_{Tf\to Ti}{}^2 \qquad \text{where} \qquad M_{i\to f}(M_{Tf\to Ti})$$

is the amplitude for the process $i\to f(T_f\to T_i)$ and T_f and T_i are the final and initial states with momenta and spin reversed.

The comparison of the transition probabilities $|M_{i\to f}|^2$ and $|M_{tf\to T_i}|^2$ is often very difficult. We list here the main difficulties for looking at the reversed process $(T_f\to T_i)$:

i) Phase space factors
ii) the difficulty of reversing a decay processes
iii) the occurrence of additional interactions in the outgoing channel (final state interactions) so that it is almost impossible to obtain the final state of a given process as the initial state of the time inversed process.

Time reversal invariance implies

$$\left|M_{i\to f}\right|^2 = \left|M_{Ti\to Tf}\right|^2$$

only when the $\mathcal{T}$ matrix is hermitian. This last requirement is much easier to test(x).

However, this approximation is often an, justified for the occurrence of final state effects. This is the case when pions are photo-or electroproduced.

The easiest way to overcome this difficulty is to sum over the final and average over the initial hadronic variables, i.e. to measure only leptonic or photonic quantities. In this way one essentially gets the reciprocity relations proposed by Christ and Lee(14) as tests of T-invariance.

(x) - This is an application of the general remarks we mode before for an antiunitary operator.

However, performing sums and averages may easily destroy those delicate phase relations which are at the basis of T-violation effects. This seems to be the case in electro - or neutrino - induced production of pions near threshold. The detection of correlation referring to the various hadronic channels seems a much more sensitive test than the reciprocity relations suggested by Christ and Lee. As a matter of fact, we find that very large effect may be expected near threshold, whereas the asymmetries proposed by Christ and Lee(15) vanish in this case.

VI. - ELIMINATION OF FINAL STATE INTERACTION EFFECTS. -

A detailed analysis shows that final state interaction difficulties may be overcome even when one looks at a particular hadronic channel. We will discuss in particular the detection of a correlation like $\vec{\sigma}_N \times \vec{p}_N \times \vec{k}$ in photo - or electroproduction of pions.

Two different kinematical possibilities may be investifated

i) Pion production at N_{33}
ii) Pion production of threshold.

Other intermediate kinematical configurations are shown to give nothings new. In the ii) case effects of final states interactions become small in comparison with possible maximal T violation effects (indeed the ratio of the two effects behaves like to δ where δ is the relative πN phase shift) and at threshold phase shift are very small(16).

It is interesting to remark that near threshold we may have a very large effect while the reciprocity relation used by Christ and Lee gives no result. If the photon and the target were not polarized and we did not measure any polarization our correlation would also be vanishgly small. As a general feature of T-violation effects in photoproduction and electroproduction we get a fairly large effect at threshold, whereas at the N_{33} energy the elimination of final state interaction effects reduces T violation effects sensibly. This may suggest that the reciprocity relations discussed by Christ and Lee do not provide the best tests of T-violation as they vanish at threshold. A similar argument may work for tests not involving final hadronic variables (e.g. for the detection of the correlation $S \times k_1 \times k_2$ in pion electroproduction.

In this case in fact, final state interaction effects do not appear, but this elimination may sensibly depress T violation effects. So the recent experiments of CEA and Berkeley Slac collaboration are not a conclusive proof against T violation in e.m. interactions.

Electroproduction has a particular interest because it involves matrix elements in which the photon is off the mass shell and it is not unreasonable that T-violation effects may be enhanced when the photon is in these conditions. For example possible T-violation effects in $K \to 2\pi$ decays arising from e.m. interactions would involve virtual photons. Moreover we may have different values of the momentum transfer and the relative weights of longitudinal and transversal multipole amplitudes may be varied to get the best experimental conditions to test-invariance of e.m. interactions. For example at threshold we get the following results be choosing $\theta = (\pi/2) \varphi = 0$.

Final state interaction effects are of the order of 4% whereas a T-violating interaction with a strength of 50% contributes to the correlation $\sigma_{\text{target}}(k, p_f)$ with term of the order of 25%.

So this situation seems indeed very favourable. We want to devote finally sometime to the discussion of the experiment on deuteron photodisintegration which raised many discussion during the past year and still is not clear.

Of the tests that have been done among many that have no result$^{(17)}$ but also a small predictive power there is one which gives a 3 standard error effect but which is not clear i.e. the photodisintegration of the deuteron and the reverse process applying detailed balance.

It is known that the principle of "detailed balance" is a consequence of time reversal invariance and therefore if compares the processes $A + a \rightleftarrows B + b$ one should obtain

$$\frac{\left[\frac{d\sigma}{d\Omega}(E, \theta)\right]_{fi}}{\left[\frac{d\sigma}{d\Omega}(E, \theta)\right]_{if}} = \frac{(2S_A+1)(2S_a+1)}{(2S_B+1)(2S_b+1)} \left(\frac{P_a}{P_b}\right)^2$$

it T-invariance holds (here $d\sigma/d\Omega$ are considered at same energy and angle). The above relation establishes a correlation between the spin averaged matrix elements and has therefore some limitations. Moreover it is worth to remark that the above relation may be satisfied even T invariance does not hold$^{(18)}$. Leaving these difficulties aside we consider the process $\gamma + D \rightleftarrows n + p$ (N(1238) resonance region) the reaction $np \to \gamma D$ was measured by B.L. Schrock et al.$^{(19)}$ and the data vere compared with the results of the inverse reaction. In general the agreement is almost satisfactory. Only at the energy of about 500 MeV there is a disagreement of -3 standard deviation in the value of the parameter A_2/A_0 so defined

$$\frac{d\sigma}{d\Omega} \simeq A_0 + A_2 P_2(\cos\theta)$$

The theoretical and experimental uncertainties make this disagreement rather doubtful. The situation as in $\eta \to \pi^+ \pi^- \pi^0$ has to be cleared out by future experiments.

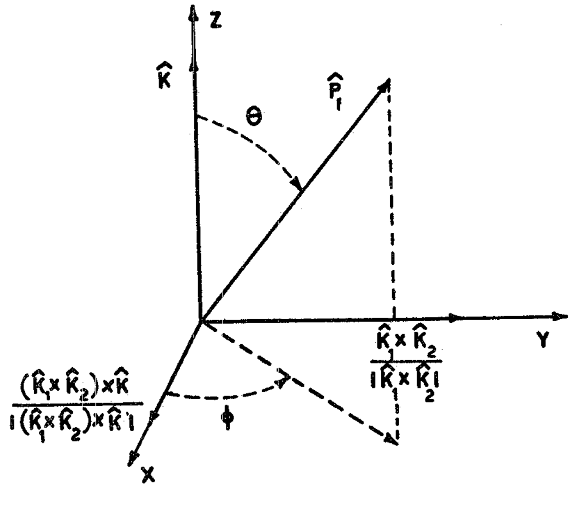
VII. - CONCLUSION. -

In conclusion it seems that the most decisive test of time reversal invariance in electromagnetic interactions is to look for T-odd correlations in electroproduction, without summing over all the final hadronic states. We recall that, with respect to other tests like η-decay etc., the above tests on electroproduction have the following advantages

a) A reasonable theoretical estimate shows that if T-violation occurs the expected effect is rather large. Therefore if no effect is seen it would be very difficult to defend the model.

b) One may test T-invariance over a fairly large range of momentum transfer (photon not on the mass-shell).

c) One can easily test T-invariance in channels with definite ΔI. For example, one can isolate isovector a isoscalar contributions. Thus one can test the isotopic structure of the T-violating electromagnetic current. It is perhaps interesting to note that all the tests performed so far ($\eta \to \pi^0 \pi^+ \pi^-$, photodisintegration of the deuteron in the N_{33} region) seems to suggest that possible T-violation effects are induced by an isovector current. Again this may easily checked in pion electroproduction.



Reference system for electroproduction of single pion

$$l^{\pm}(K_1) + N \to l^{\pm}(K_2) + N_+ \pi(P_f)$$

FOOTNOTES AND REFERENCES. -

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(13) - See for example A.T. Filippov et al., CERN 69-7 (1969).

(14) - N. Christ and T.D. Lee, Phys. Rev. 148, 1520 (1966).

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(16) - If we denote by $A_{if} = |M_{if}|^2 - |MTR_i \to TR_f|^2$ our asymmetry where R is a suitable rotation of π about the y axis, in general in order to isolate the correlation free of spurions effects other measurements have to be performed, for example $A_i \to f \pm A_i \to P_f$, where P_f is the state obtained from f by space inversion: in this way one may eliminate even or odd parity interferences. In a similar way one may eliminate interferences between different isotopic spin channels by suitably combining the experimental data. For a detailed analysis see Ref. (2).

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