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B. D'Ettorre Piazzoli: EFFECTS OF ELECTRON BREMSSTRAHLUNG IN THE e^+e^- ANNIHILATION REACTIONS AT ENERGIES NEAR THE RESONANT MASSES. -

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I. - INTRODUCTION. -

The aim of this work is to estimate the effects of the electron bremsstrahlung on the e^+e^- reactions near the resonant masses.

First we establish a general relation between the lowest-order "elastic" cross-section σ_0 for any e^+e^- process and the cross-section $d\sigma/d\vec{K}$ for the identical process with electron bremsstrahlung. Then, the consideration of the effects near the resonances requires only some mathematical effort.

The possibility to calculate higher order processes by relating them directly to simpler effects was first recognized by P. Kessler[(1)]. Following his results, we can write the cross-section for any e^+e^- reaction at the nominal total beams energy 2E, with electron bremsstrahlung

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(x) - Laboratorio di Cosmo-Geofisica del CNR - Torino.

of one photon with energy K, as the product of the probability of the virtual transition $e^+(e^-) \to e^+(e^-) + \gamma$ (x) times the elastic cross-section at the total C.M. energy $2E(1-K/E)^{1/2}$:

(1) $$\frac{d\sigma}{d\vec{K}} = \sigma_o(4E^2(1-K/E)) \times P(E,\vec{K})$$

where $P(E,\vec{K})d\vec{K}$ is the electron bremsstrahlung spectrum.

In fact after the transition the total energy in the C.M. of both electrons is exactly $2E(1-K/E)^{1/2}$, but one of them is off the mass shell with invariant mass given by $m_e^{x2} = m_e^2 - 2EK(1-\beta_L \cos\gamma)$; m_e = = electron mass, β_L = LAB electron velocity, γ = bremsstrahlung angle. However the frequency of the virtual state, $\nu = (m_e^x - m_e)/m_e/E \simeq \simeq K(1-\beta_L \cos\gamma)$ times the characteristic cross-time of the interaction region[2] $t \lesssim 1/E$, is very small compared to unity. Therefore the intermediate state is practically degenerate with the incident particle state and propagates for distances which are very large as compared to that of the interaction region.

U. Mosco[3], V.N. Baier and V.A. Khoze[5] have calculated by different techniques the cross section for the γ-emission by the initial electrons in the process $e^+e^- \to \mu^+\mu^-$. The inspection of the cross-section confirm the result (1) if we neglect in σ_o terms of the order $(m_e/E)^2 (1/1-K/E)(1/\beta_L)$ to respect to 1(+) irrespective of the bremsstrahlung angle. The electron bremsstrahlung spectrum in the relativistic limit m_e/E, $m_e/E-K \to 0$, is found to be

(2) $$P(E,\vec{K})\,d\vec{K} = \frac{\alpha}{(2\pi)^2}\frac{1}{K}\left[\left(\frac{P_+}{P_+\cdot K} - \frac{P_-}{P_-\cdot K}\right)^2 (1-K/E) + \frac{1}{2E^2}\left(\frac{P_+\cdot K}{P_-\cdot K} + \frac{P_-\cdot K}{P_+\cdot K}\right)\right] d\vec{K}$$

with

$$P_\pm \equiv (\vec{P}_\pm, E) \qquad\qquad K \equiv (\vec{K}, K)$$

which is, in the limit $K/E \ll 1$, just the soft spectrum.

(x) - We refer to the square of the sum of the amplitudes for the two virtual processes.

(+) - Some manipulation of the original formulas is necessary to get this result.

After integration over the angular variables of the photon[(x)] we get

(3) $$\frac{d\sigma}{dK} = \sigma_o(4E^2(1-K/E)) \cdot P(E,K)$$

where

(4) $$P(E,K)\,dK = \frac{2\alpha}{\pi}(2\log\frac{2E}{m_e} - 1)\,(1 - \frac{K}{E} + \frac{1}{2}(\frac{K}{E})^2)\,\frac{dK}{K}$$

which differs from the Kessler's spectrum[(1)] essentially for the small contribution of the interference between the e^+ and e^- bremsstrahlung.

We assume the formulas (1,2,3,4) as a well established result.

The differential cross-section $d\sigma/dK$ has to be integrated from a cut-off value ε[(+)], which separates the infrared region, up the maximum allowed photon momentum defined by the experimental resolutions. If σ_o is a slowly varying function of the energy in the range of integration we can make the useful "soft" approximation[(4, 8)].

$$\int_{\varepsilon}^{K_M} \frac{d\sigma}{dK}\,dK \simeq \sigma_o(4E^2)\,\frac{2\alpha}{\pi}\,(2\log\frac{2E}{m_e} - 1)\,\log\frac{K_M}{\varepsilon}$$

However for a resonant final state the soft approximation can fail if the form factor describing the resonance has a strong energy dependence. A more exact integration of (3) performed assuming the usual form factor parametrization[(9)] $|F(4E^2)|^2 \sim [(m^2-4E^2)^2 + m^2\Gamma^2]^{-1}$ leads to terms which add to the soft approximation results (Sect. II): these terms can be large and very energy dependent, enhancing the cross section when $2E >$ resonance mass (Sect. IV).

Moreover, the hadronic vacuum polarization[(10, 11)] factor has a strong energy dependence near the resonant masses. Therefore it is to be expected that its contribution at a given total energy 2E will be mo-

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(x) - That is not a limitation because the photon is emitted essentially in the directions of the electrons. Then the isotropic maximum allowed photon momentum is defined as the maximum along these directions, see ref.[(7)].

(+) - The actual choice of ε is rather arbitrary, but it is usually of the order of the electron mass[(3)].

dified by the electron bremsstrahlung. In fact we found that the vacuum polarization factor is suppressed around the Φ mass (Sect. IV).

The results are applied to the typical annihilation reactions

$$e^+e^- \to \mu^+\mu^-, K^+K^- \quad \text{around the } \Phi \text{ mass}$$
$$\to \pi^+\pi^- \quad \text{in the range } 2E = 650 \div 1050 \text{MeV}$$

We assume for the resonances parameters-mass m_R, width Γ_R, branching ratio $B_{R\to e^+e^-}$ the following values(12)

Resonance	m_R(MeV)	Γ_R(MeV)	$B_{R\to e^+e^-} \times 10^5$
ρ^0	770	111	6.2
ω	783.4	12.6	6.9
ϕ	1019.5	3.9	37.0

II. - CORRECTIONS FOR A RESONANT FINAL STATE. -

We use the Putzolu's results(13) for the virtual and real soft-photons corrections. We carried out the integration in (3) assuming the Breit-Wigner parametrization $|F(4E^2)|^2 \sim [(m_R^2-4E^2)^2+m_R^2\Gamma_R^2]^{-1}$ to describe a resonance with mass m_R and width Γ_R. By neglecting the quadratic term in the bremsstrahlung spectrum (4), (linear approximation), we get:

$$\sigma_{exp}(4E^2) = \sigma_o(4E^2)\,|F(4E^2)|^2\,(1+\delta^{IN,P}+\delta^{FIN})$$

where:

$$\delta^{IN,P} = \frac{\alpha}{\pi}\left[2\left(2\log\frac{2E}{m_e}-1\right)\left(\log\frac{K_M}{E}-\frac{K_M}{E}-\delta_R\right)+\frac{13}{3}\log\frac{2E}{m_e}-\frac{28}{9}-\frac{2}{3}\pi^2\right]$$

with:

$$\delta_R = \frac{1}{2}\,\mathrm{LOG}(K_M,R)+\frac{m_R}{4E^2\Gamma_R}\,(m_R^2-4E^2+\Gamma_R^2)\,\mathrm{ARC}(K_M,R)-\frac{K_M}{E}$$

$$LOG(K_M, R) = \log \frac{\left[m_R^2 - 4E^2(1-K_M/E)\right]^2 + m_R^2\Gamma_R^2}{\left[m_R^2 - 4E^2\right]^2 + m_R^2\Gamma_R^2}$$

$$ARC(K_M, R) = \mathrm{arctg}\frac{m_R^2 - 4E^2(1-K_M/E)}{m_R\Gamma_R} - \mathrm{arctg}\frac{m_R^2 - 4E^2}{m_R\Gamma_R}$$

δ^{FIN} is the vertex and bremsstrahlung correction to the outgoing particles (final corrections)(4, 14).

$\sigma_o(4E^2)$ is the "perturbation theory"(9) cross section for particles without structure. $\delta^{IN,P}$ contains the e^+e^- vertex correction, the electron bremsstrahlung correction and finally the photon self-energy correction considering only electron pairs ("initial" and "propagator" corrections).

The resonance correction term $-(2\alpha/\pi)\left[2\log(2E/m_e)-1\right]\delta_R$ can be very large for $2E > m_R$ and even of the order of 1 for a narrow resonance like the Φ, therefore $\sigma_{exp} \sim 2\sigma_o|F|^2$ (Sect. IV). The explanation of this result is that the electron bremsstrahlung shifts the energy of the reaction toward the resonant mass where the cross-section exceeds greatly the cross-section at the "nominal" energy of the incoming electrons. Of course, if $|F(4E^2)|^2 = 1$ -no resonance- $\delta_R = 0$.

We note that a similar expression for δ_R was obtained by other authors in the soft approximation(15, 16).

III. - VACUUM POLARIZATION OF HADRONS AND EFFECTS OF ELECTRON BREMSSTRAHLUNG. -

The second order hadronic modifications to the photon propagator can be obtained in the vector meson dominance schemes by adding coherently the contributions from the virtual transitions $\gamma \to V \equiv \rho^o, \omega, \Phi$ (11).

The vacuum polarization factor $C_{VP}(4E^2)$, which multiplies the e^+e^- annihilation cross section irrespective of the particular final state(x), has been calculated in terms of the coupling constants g_V of the vector mesons to the hadrons.

(x) - Therefore in the resonance region we expect vacuum polarization effects of the resonance on itself(17).

By the well known relations[18]

$$\frac{4}{3}\pi\alpha^2 m_V = g_V^2 B_{V\to e^+e^-}\Gamma_V \quad (V\equiv \rho^0\omega,\phi)$$

we are able to put it in the more suitable form

$$C_{VP}(4E^2) \equiv 1+\delta_H(4E^2;\alpha)+\delta'_H(4E^2;\alpha^2) =$$

$$= 1+\frac{6}{\alpha}4E^2\sum_{i=\rho,\omega,\phi}\frac{\Gamma_i}{m_i}B_{i\to e^+e^-}\frac{4E^2-m_i^2}{(m_i^2-4E^2)^2+m_i^2\Gamma_i^2}+$$

$$+\frac{9}{\alpha^2}16E^4\left[\left(\sum_i\frac{\Gamma_i}{m_i}B_{i\to e^+e^-}\frac{m_i^2-4E^2}{(m_i^2-4E^2)^2+m_i^2\Gamma_i^2}\right)^2+\right.$$

$$\left.+\left(\sum_i B_{i\to e^+e^-}\frac{\Gamma_i^2}{(m_i^2-4E^2)^2+m_i^2\Gamma_i^2}\right)^2\right]$$

where we have specified the α contribution of the interference between the polarization and the basic graphs. In this form it can be generalized -leaving the vector meson dominance schemes out-to describe every neutral resonance $J^{PC}=1^{--}$ in the virtual state. In the region of the Φ the effect of the vacuum polarization is quite large, Fig. 1.

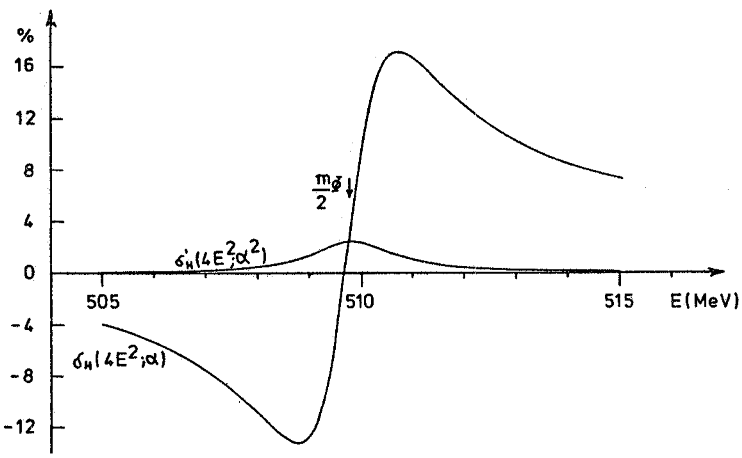

FIG. 1 - The α and α^2 contributions to the vacuum polarization factor around the ϕ mass.

Now we consider the effect of the electron bremsstrahlung. Experimentally one integrates $\delta_H(s,\alpha)$ over the bremsstrahlung spectrum from $s = 4E^2$ to $4E^2(1-K_M/E)$. Then, due to the particular shape of $\delta_H(4E^2,\alpha)$ which rapidly oscillates in sign, we expect the vacuum polarization contribution to be smoother. For simplicity we refer to a non-resonant final state; the quantity - to the order α^2:

$$\Delta^{IN}(4E^2, K_M/E, \alpha^2) = \delta_H(4E^2,\alpha)\frac{\alpha}{\pi}\left[2(2\log\frac{2E}{m_e}-1)\,x\right.$$

$$(5) \qquad x\left(\log\frac{K_M}{E}-\frac{K_M}{E}\right)+3\log\frac{2E}{m_e}-2-\frac{2}{3}\pi^2\Big] - \int_0^{K_M}\Big[\delta_H(4E^2,\alpha) -$$

$$-\,\delta_H(4E^2(1-K/E),\alpha)\Big]P(E,K)\,dK$$

represents the electron bremsstrahlung effect on $\delta_H(4E^2,\alpha)$. The infra red divergence has been regularized by adding the vertex correction(13): as a result, the integral is not divergent and can contribute substantially to the total correction if

$$\frac{d}{d(4E^2)}\,\delta_H(4E^2,\alpha) \qquad \text{is large.}$$

The purely electrodynamic α^2 corrections to the cross-section are usually neglected, but $\Delta^{IN}(4E^2, K_M/E, \alpha^2)$, can be relevant if the above condition is realized. This is the case for 2E around the Φ mass, as shown in Fig. 2: we have assumed the typical resolution $K_M/E = 10\%$. As expected, this correction decreases the vacuum polarization effects. (Of course, for the corresponding final term is simply $\Delta^{FIN}(4E^2, K_M/E,\alpha^2) \simeq \delta_H(4E^2,\alpha)\, x$ $x\,\delta^{FIN}$).

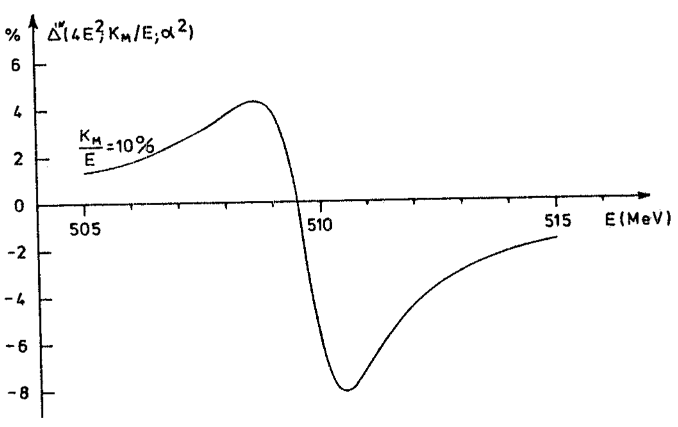

FIG. 2 - The correction $\Delta^{IN}(4E^2, K_M/E;\alpha^2)$ around the ϕ mass for $K_M/E=10\%$.

IV. - RESULTS. -

By simply adding the results of Sect. III to those of Sect. II we obtain in compact form (neglecting terms $< 0.5\%$):

$$\sigma_{exp}(4E^2) = \sigma_o(4E^2)\left|F(4E^2)\right|^2 (1+\delta_H(4E^2,\alpha))(1+\delta^{IN,P}+\delta^{FIN})$$

where:

$$\delta^{IN,P} = \frac{\alpha}{\pi}\left[2(2\log\frac{2E}{m_e}-1)(\log\frac{K_M}{E}-\frac{K_M}{E}-\delta_R-\sum_{i=\rho,\omega,\phi}\delta_{i,R}) + \right.$$

$$\left. + \frac{13}{3}\log\frac{2E}{m_e}-\frac{28}{9}-\frac{2}{3}\pi^2\right]^{(x)}$$

This correction holds for all e^+e^- annihilation reactions going through the one photon channel.

The subscripts R and i refer, respectively, to the resonance in the final state and to the virtual resonant states. The function $\delta_{i,R}$ is defined by the relation

$$\delta_{i,R} = \frac{1}{(1+\delta_H(4E^2,\alpha))\left|F(4E^2)\right|^2}\,\frac{6}{\alpha}\,\frac{\Gamma_i}{m_i}\,B_{i\to e^+e^-}\ x$$

$$x\int_0^{K_M}\left.\frac{Z(Z-m_i^2)}{(m_i^2-Z^2)^2+m_i^2\Gamma_i^2}\right]_{Z=4E(1-K/E)}^{Z=4E^2}\cdot\left|F(4E^2(1-K/E))\right|^2\cdot(1-\frac{K}{E}+\frac{1}{2}(\frac{K}{E})^2)\frac{dK}{K}$$

This is, apart from the normalization, just the single vector meson contribution to the integral in (5), generalized for a resonant final state.

We obtain in soft approximation, nonlogarithmic terms are of the order of $(\Gamma_i/m_i)^2$ to respect to the logarithmicterm for $2E\sim m_i$:

(x) - Near the ϕ mass also the α^2 correction $\delta_H'(4E^2;\alpha^2)$ is just different from zero, Fig. 1.

$$\delta_{i,o} = \frac{6}{\alpha\left[1+\delta_H(4E^2,\alpha)\right]} \frac{\Gamma_i}{m_i} \frac{B_{i\to e^+e^-}}{(m_i^2-4E^2)^2+m_i^2\Gamma_i^2} \left\{ \frac{1}{2} m_i^2(4E^2-m_i^2-\Gamma_i^2) \times \right.$$

(6')

$$\left. \times \, LOG(K_M,i) + 4E^2 m_i \Gamma_i \times ARC(K_M,i) \right\}$$ for a non-resonant final state, $R \neq 0$.

$$\delta_{R,R} = \frac{6}{\alpha\left[1+\delta_H(4E^2,\alpha)\right]} \frac{\Gamma_R}{m_R} B_{R\to e^+e^-} \left\{ \frac{2E^2}{m_R \Gamma_R} ARC(K_M,R) + \right.$$

(6'')

$$\left. + \frac{1}{2}\left[\frac{m_R^2(m_R^2+\Gamma_R^2) - 16E^4(1-K_M/E)}{\left[m_R^2-4E^2(1-K_M/E)\right]^2+m_R^2\Gamma_R^2} - \frac{m_R^2(m_R^2+\Gamma_R^2)-16E^4}{(m_R^2-4E^2)^2+m_R^2\Gamma_R^2} \right] \right\}$$

for a resonant final state, and $i \equiv R$.

$$\delta_{i,R} = \frac{6}{\alpha\left[1+\delta_H(4E^2,\alpha)\right]} \frac{\Gamma_i}{m_i} B_{i\to e^+e^-} \frac{(m_R^2-4E^2)^2+m_R^2\Gamma_R^2}{(m_i^2-4E^2)^2+m_i^2\Gamma_i^2} \left\{ \frac{1}{2} A(i,R) \times \right.$$

$$\times \left[LOG(K_M,i) - LOG(K_M,R) \right] + \frac{1}{4E^2 m_i \Gamma_i} \left[B(i,R) - 4E^2(m_i^2-4E^2)A(i,R) \right] \times$$

(6''')

$$\left. \times \, ARC(K_M,i) - \frac{1}{4E^2 m_R \Gamma_R} \left[C(i,R) - 4E^2(m_R^2-4E^2)A(i,R) \right] ARC(K_M,R) \right\}$$

for a resonat final state, and $i \neq R$.

The functions A(i,R), B(i,R) and C(i,R) are defined as follows:

$$A(i,R) = \frac{(m_R^2 - m_i^2)\, G_o^i - (F_o^R - F_o^i)\, G_1^i / 8E^2}{D(i,R)}$$

$$B(i,R) = \frac{(m_R^2 - m_i^2)\left[G_o^i F_1^i - G_1^i F_o^i\right] - 2\,E^2 (F_o^R - F_o^i)\, G_o^i}{D(i,R)}$$

$$C(i,R) = \frac{(m_R^2 - m_i^2)\left[G_o^i F_1^R - G_1^i F_o^R\right] - 2E^2 (F_o^R - F_o^i)\, G_o^i}{D(i,R)}$$

with

$$D(i,R) = (m_R^2 - m_i^2)\left[F_o^R F_1^i - F_1^R F_o^i\right] - 2E^2 (F_o^R - F_o^i)^2$$

where

$$F_o^j = (m_j^2 - 4E^2)^2 + m_j^2 \Gamma_j^2 \, ; \qquad F_1^j = 8E^2 (m_j^2 - 4E^2); \qquad (j = i, R)$$

$$G_o^i = 4E^2\, m_i^2 (8E^2 \Gamma_i^2 - F_o^i)\, ; \qquad G_1^i = 16\,E^4\, m_i^2 (4E^2 - m_i^2 - \Gamma_i^2)$$

The functions $LOG(K_M, i)$ and $ARC(K_M, i)$ are the natural extensions of $LOG(K_M, R)$ and $ARC(K_M, R)$ defined in Sect. II.

We applied this results to some typical annihilation reactions:

1) - $e^+e^- \to \mu^+\mu^-$. -

The study of this reaction around the Φ mass was proposed(19) as an indirect but straight-forward measurement of the branching ratio $B_{\phi \to e^+e^-}$ exploiting the effects of vacuum polarization. The correction $-(2\alpha/\pi)\left[2 \log(2E/m_e) - 1\right] \sum_{i = \rho, \omega, \phi} \delta_{i,o}$ defined by formula (6') is shown in Fig. 3 for various cases of energy resolution K_M/E. The Φ contribution is dominant, the ρ and ω parts being $< 0.1\%$: all the corrections and the shape of the cross-section are determined by the intermediate virtual Φ. In Fig. 4 is plotted the ratio $\sigma_{exp}(4E^2)/\sigma_o(4E^2)$.

For an "asymmetric arrangement(8)" the maximum energy K_M of a photon emitted by e^+ or e^- depends on the production angle ϑ and

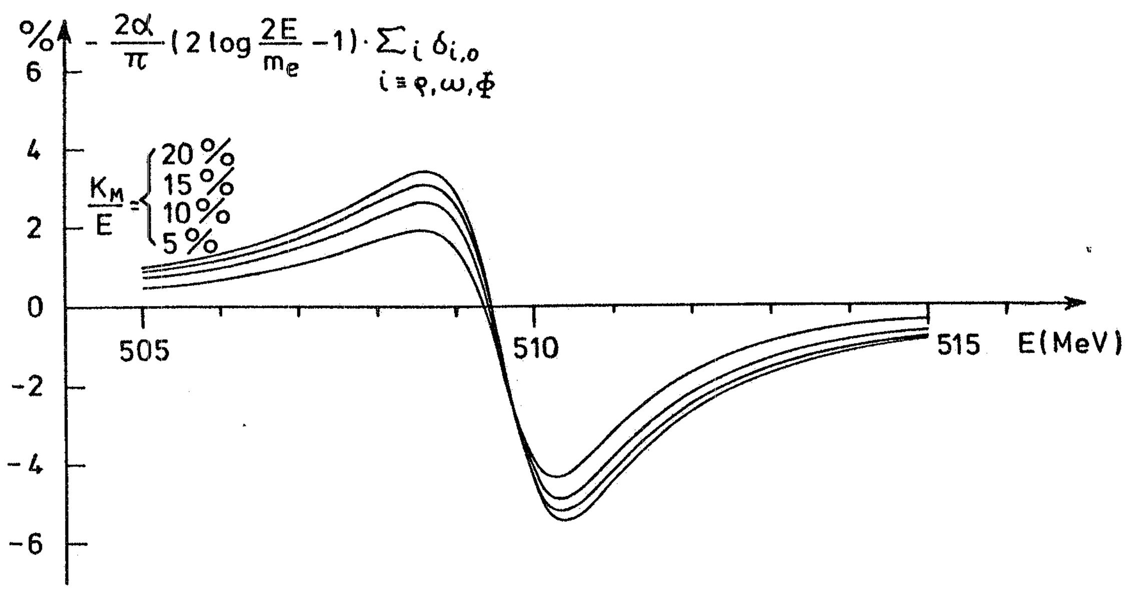

FIG. 3 - $\mu^+\mu^-$ production around the ϕ mass: the correction $-(2\alpha/\pi)[2\log(2E/m_e)-1]\ (\sum_{i=\rho,\omega,\phi}\delta_{i,o})$ is shown for $K_M/E=$ = 5%, 10%, 15%, 20%.

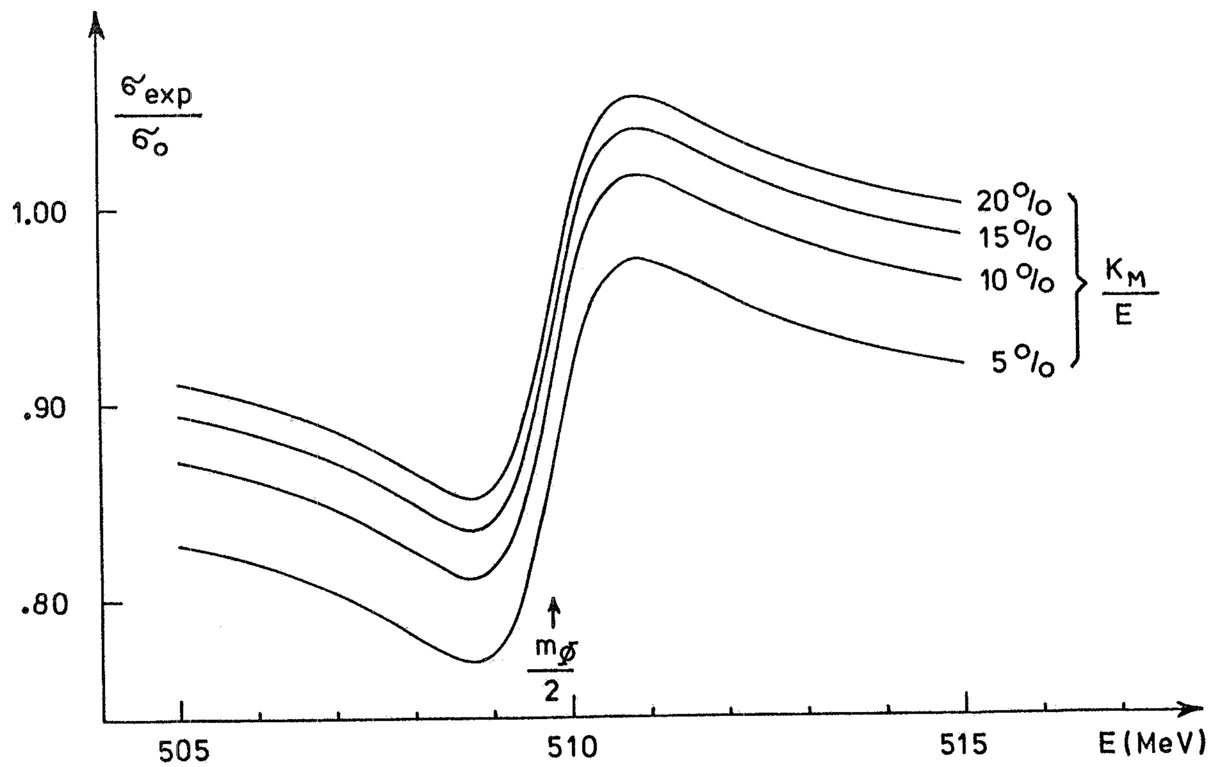

FIG. 4 - The ratio σ_{exp}/σ_o for $e^+e^-\to\mu^+\mu^-$ around the ϕ mass. The curves employ the same energy resolutions as Fig. 3.

the maximum non-collinearity angle $\Delta\vartheta_M$

$$K_M(\vartheta,\Delta\vartheta_M) = 2E\,\frac{\sin\Delta\vartheta_M}{\sin\Delta\vartheta_M+\left(1+\frac{m_\mu^2}{2E_\mu^2}\right)\sin\vartheta\,(1+\cos\Delta\vartheta_M)}$$

In Fig. 5 the ratio σ_{exp}/σ_o for $K_M/E=10\%$ is compared with that obtained by not taking into account the α^2 correction; the apparent relative change of $B_{\phi\to e^+e^-}$ is relevant at $2E\simeq m_\phi \pm \Gamma_\phi/2$ where $\delta_H(4E^2,\alpha)\approx\pm(3/\alpha)\,x$ $x\,B_{\phi\to e^+e^-}$ (-43% and -27 % respectively).

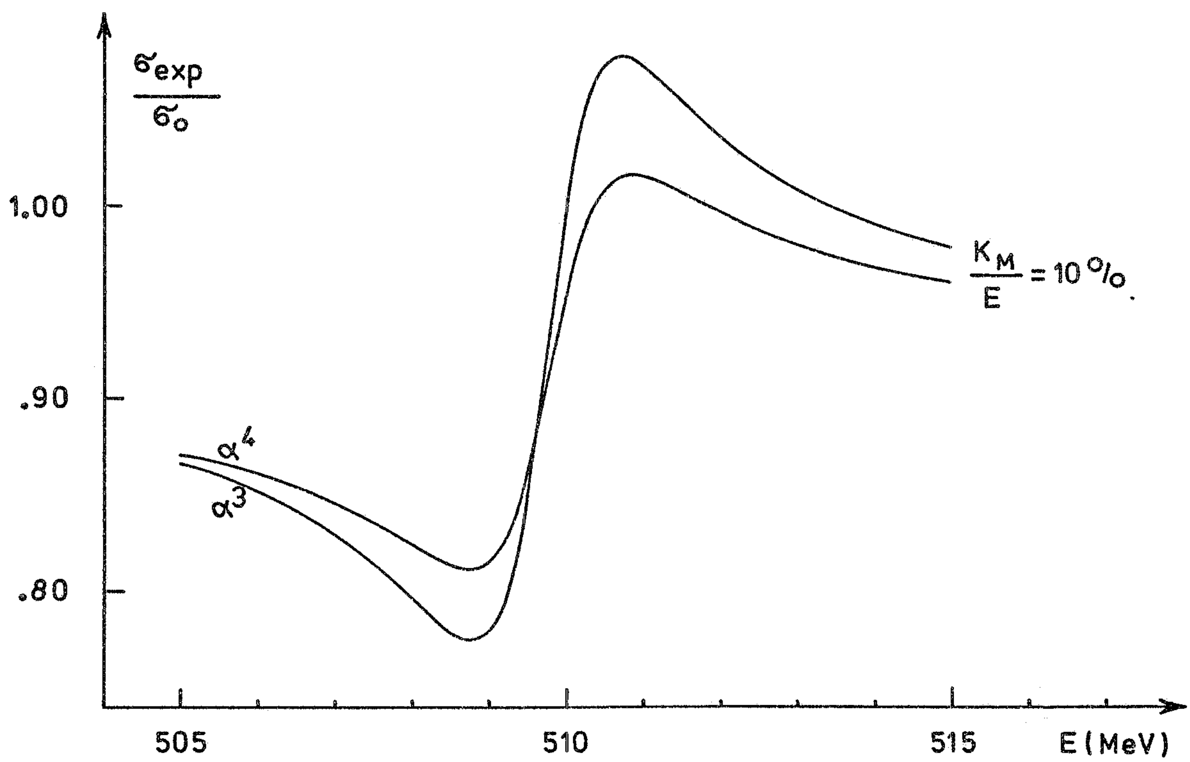

FIG. 5 - Comparison of the ratio σ_{exp}/σ_o (Labelled "α^4") with the same ratio without the α^2 correction (Labelled "α^3") for $e^+e^-\to\mu^+\mu^-$ around the ϕ mass. The energy resolution is $K_M/E=10\%$. In the absence of this contribution the shape of the cross-section plot is noticeably distorted.

2) - $e^+e^- \to K^+K^-$. -

The study of $e^+e^- \to \Phi$ by means of the K^+K^- decay mode has been performed with the Novosibirsk and Orsay colliding rings[20]. In Fig. 6a is reported the K_M - dependent correction

$$\delta(K_M) = \frac{2\alpha}{\pi}\left(2\log\frac{2E}{m_e} - 1\right)\left(\log\frac{K_M}{E} - \frac{K_M}{E} - \delta_\phi - \sum_i \delta_i\right)$$

for K_M=5, 10, 15 MeV around the Φ mass. When $2E > m_\phi$ the emission of radiation from the incoming electrons shifts the C.M. energy toward the Φ mass; as a result, we have a strong positive correction due mainly to δ_ϕ. The correction

$$-\frac{2\alpha}{\pi}\left(2\log\frac{2E}{m_e} - 1\right)\sum_i \delta_{i,\phi}$$

- formulae (6"), (6"') - is given in Fig. 6b; the ρ and ω parts are not important (0.1%). These results agree only qualitatively with those given by V. N. Baier and V. S. Fadin[15] who have calculated δ_ϕ in logarithmic approximation without considering the terms - $\sum_i \delta_{i,\phi}$. In Fig. 7 we compare the "experimental" form factor $\sigma_{exp}(4E^2)/\sigma_0(4E^2)$ to $|F(4E^2)|^2$ - the normalization is quite arbitrary: the radiative correction lowers the resonance peak (which measures the $B_{\phi\to e^+e^-} B_{\phi\to K^+K^-}$ product) by ≈ 30% irrespective of the particular K_M value.

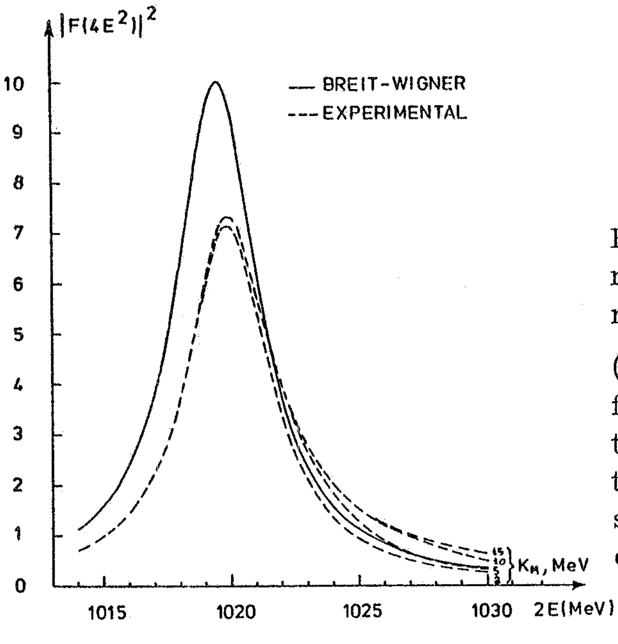

FIG. 7 - K^+K^- production around the ϕ mass: comparison between the experimental form-factor $\sigma_{exp}(4E^2)/\sigma_0(4E^2)$ (dotted Line) and $|F(4E^2)|^2$ (solid Line) for some values of the maximum photon energy. The normalization is arbitrary. The radiative correction decreases the peak cross-section by ≈30% and enhances the ϕ-tail.

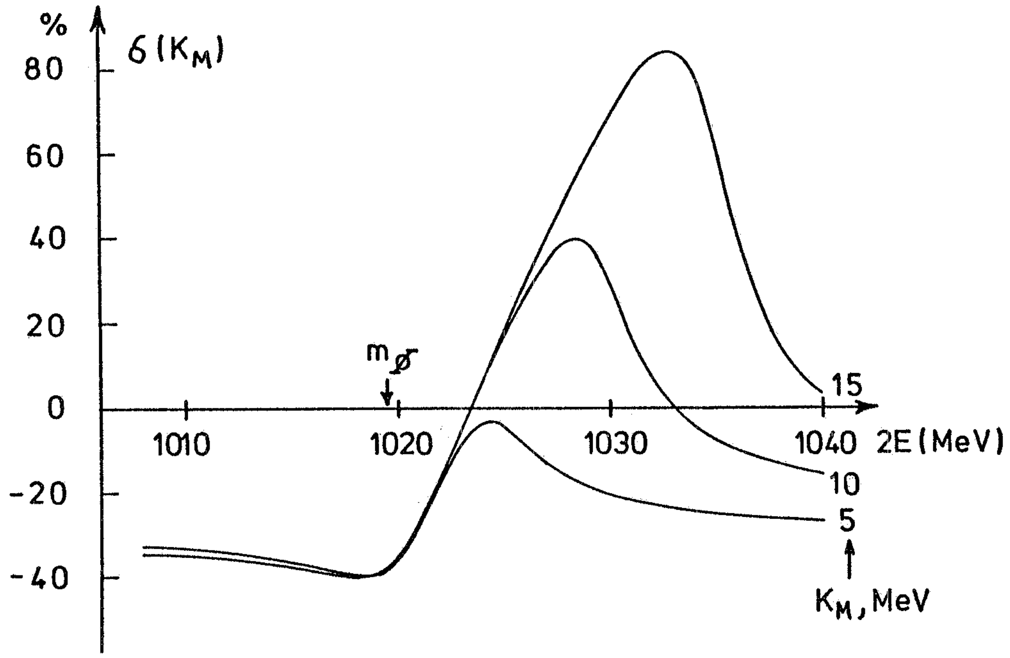

FIG. 6 - a) K^+K^- production around the ϕ mass: the K_M-dependent correction

$$\delta(K_M) = \frac{2\alpha}{\pi}\left(2\log\frac{2E}{m_e}-1\right)\left(\log\frac{K_M}{E}-\frac{K_M}{E}-\delta_\phi-\sum_i \delta_{i,\phi}\right)$$

for some values of the maximum photon energy. For $2E > m_\phi$ the correction increases rapidly to positive values as explained in the text.

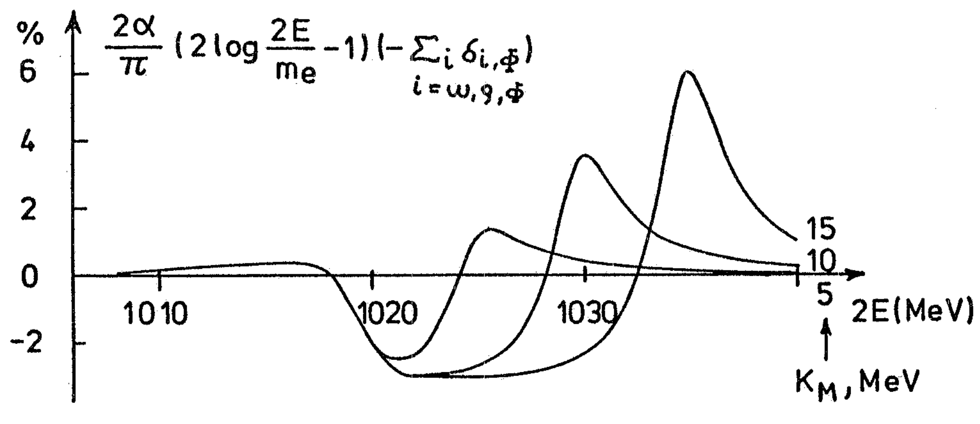

FIG. 6 - b) The correction

$$\frac{2\alpha}{\pi}\left(2\log\frac{2E}{m_e}-1\right)\left(-\sum_i \delta_{i,\phi}\right)$$

around the ϕ mass for some values of the maximum photon energy.

3) - $e^+e^- \to \pi^+\pi^-$. -

A similar treatment holds for this reaction. It has been studied at Orsay and Novosibirsk in connection with ρ production[20]. The K_M-dependent correction is reported in Fig. 8a and 8b for the two extreme cases of the $\pi^+\pi^-$ collinearity requirement, K_M/E=5% and 20%, from 2E=650 to 1050 MeV. The positive correction for $2E > m_\rho$ is smooth, due to the large width of the resonance. The correction around the Φ mass, on the ρ tail, diminishes the vacuum polarization as in reaction (1). The $(\delta_{\rho,\rho} + \delta_{\omega,\rho})$ contribution is nowhere relevant (the maximum is $\sim$-1% at 2E$\simeq$785 MeV). In Fig. 9a and 9b we compare the experimental form factor to $|F(4E^2)|^2$; the radiative corrections reduce the resonance peak by $\approx$15%. At $2E \sim m_\phi$ the vacuum polarization effects are superimposed on the ρ tail, in principle allowing one to extract information on the branching ratio $B_{\phi \to e^+e^-}$.

The Orsay results are also fitted with a modified Breit-Wigner formula, given by G. J. Gounaris and J. J. Sakurai[21], taking into account the finite width of the ρ mass; moreover, there is some indication for a non-zero amplitude for the G violating decay $\omega \to 2\pi$[20]. In both cases the corrections have to be calculated by the appropriate parametrization of the resonance.

CONCLUSIONS. -

In conclusion we remark that a careful estimate of e^+e^- annihilation cross-section characteristics around the resonant masses requires a judicious consideration of the electron bremsstrahlung effects, which we have calculated in a general way in terms of the resonance parameters. The foregoing numerical applications have made evident this point.

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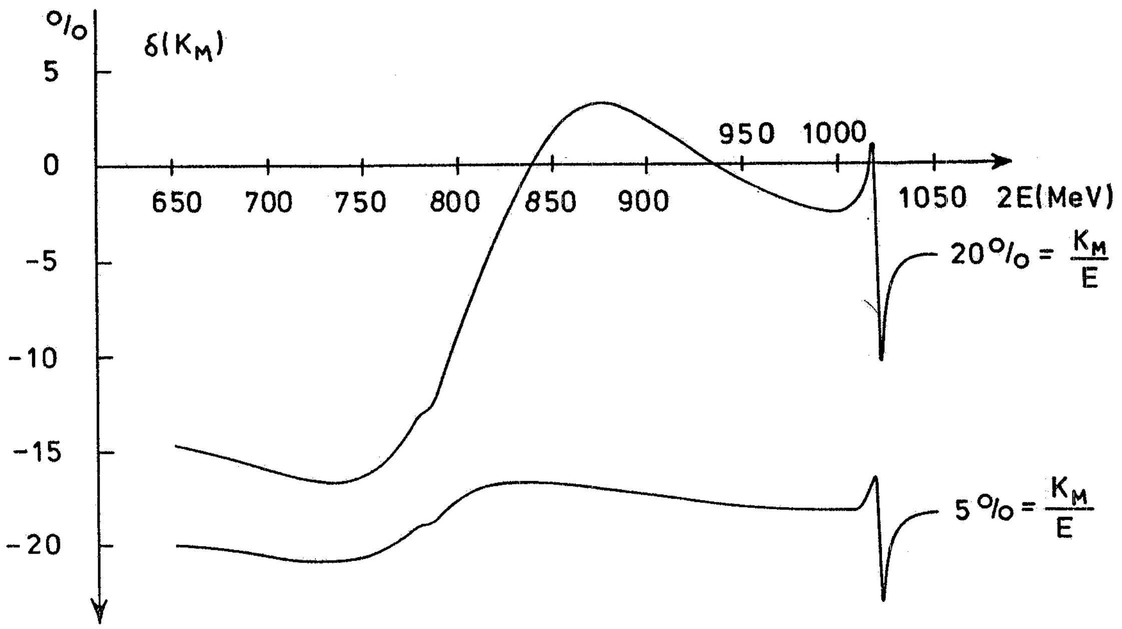

FIG. 8 - a) $\pi^+\pi^-$ production around the ρ mass: the K_M- -dependent correction

$$\delta(K_M) \equiv \frac{2\alpha}{\pi}\left(2\log\frac{2E}{m_e}-1\right)\left(\log\frac{K_M}{E} - \frac{K_M}{E} - \delta_\rho - \sum_i \delta_{i,\rho}\right)$$

for $K_M/E = 5\%$ and 20%. For $2E > m_\rho$ the correction increases rapidly. For $2E \approx m_\phi$ the term $\delta_{\phi,\rho}$ produces the oscillating shape of the correction.

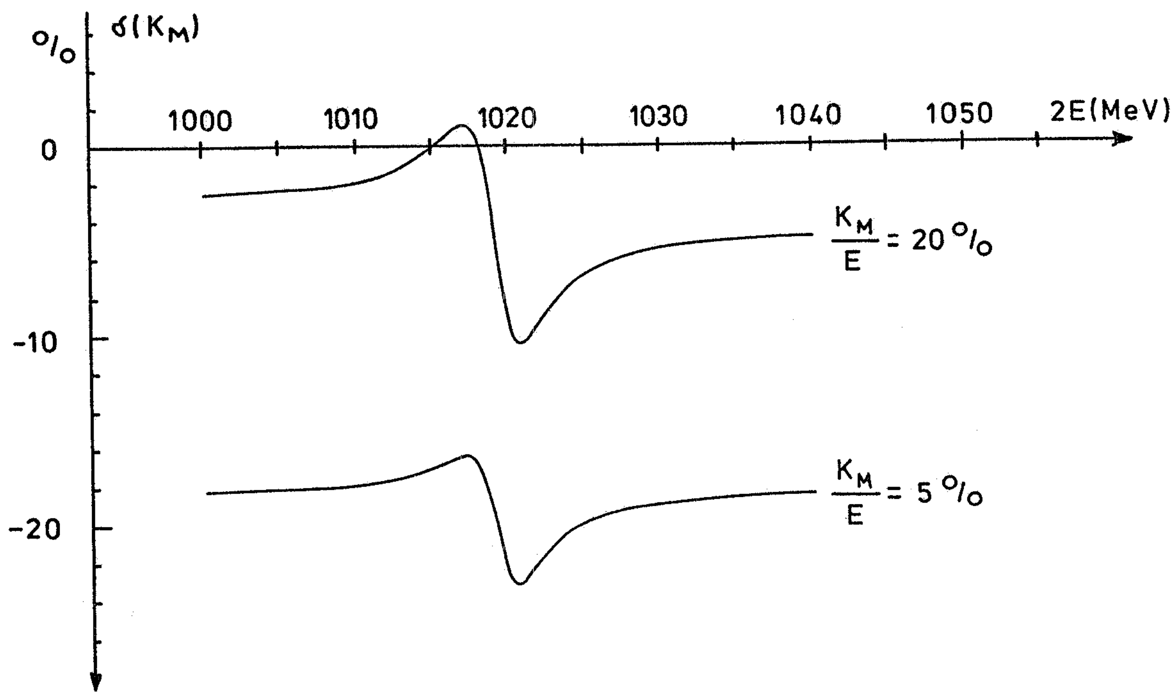

FIG. 8 - b) As in Fig. 8a), the correction $\delta(K_M)$ for $2E \sim m_\phi$

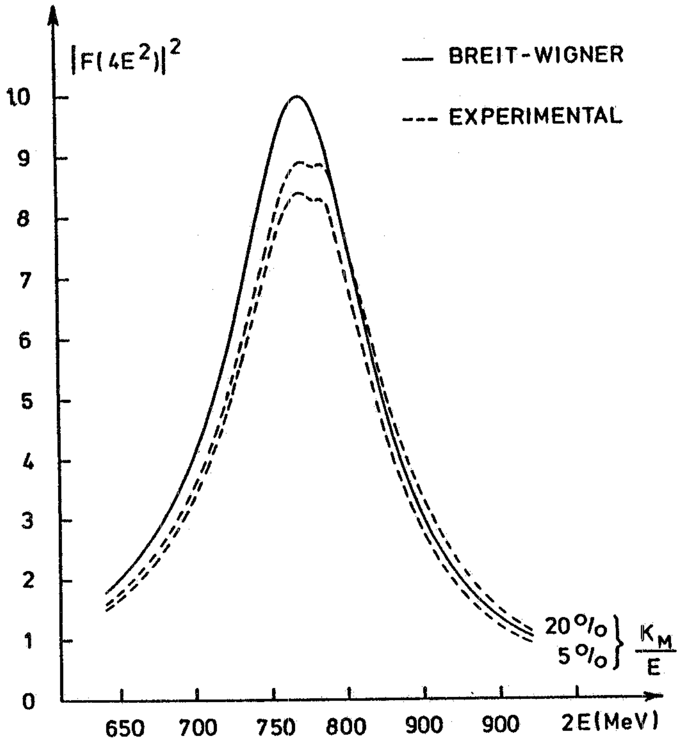

FIG. 9 - a) $\pi^+ \pi^-$ production around the ρ mass: comparison between the experimental form-factor $\sigma_{exp}(4E^2)/\sigma_o(4E^2)$ (dotted Line) and $|F(4E^2)|^2$ (solid Line) for K_M/E=5% and 20%. The normalization is arbitrary. The radiative correction decreases the peak cross-section by ≈ 15%.

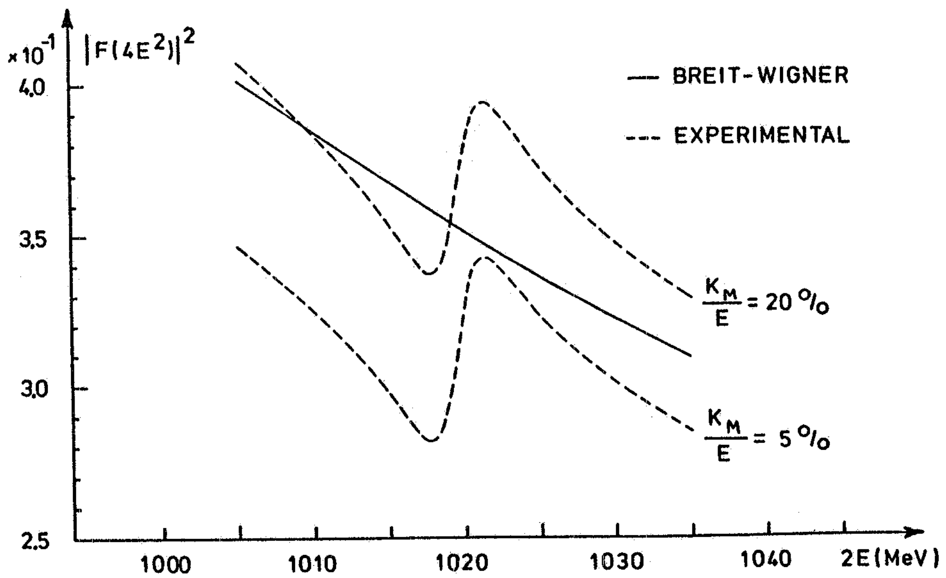

FIG. 9 - b) $\pi^+ \pi^-$ production: comparison between $\sigma_{exp}(4E^2)/\sigma_o(4E^2)$ and $|F(4E^2)|^2$ at $2E \sim m_\phi$. The vacuum polarization effects superposed on the ρ -tail are shown.

REFERENCES. -

(1) - P. Kessler, Nuovo Cimento 17, 809 (1960).
(2) - E. Etim, G. Pancheri and B. Touschek, Nuovo Cimento 51B, 276 (1967).
(3) - U. Mosco, Nuovo Cimento 33, 115 (1963).
(4) - G. Longhi, Nuovo Cimento 35, 1122 (1965).
(5) - V. N. Baier and V. A. Khoze, Soviet Phys. -JEPT 21, 629 (1965).
(6) - V. N. Baier and S. A. Kheifets, Nuclear Phys. 47, 313 (1963).
(7) - Y. S. Tsai, Radiative corrections to colliding beam experiments, Proc. Intern. Symp. on Electron and Photon Interactions at High Energy, Hamburg (1965), p. 387.
(8) - G. Altarelli, S. De Gennaro, E. Celeghini, G. Longhi and R. Gatto, Nuovo Cimento 47A, 113 (1967).
(9) - R. Gatto, Theoretical aspects of colliding beam experiments (Springer Tracts in Modern Physics, Berlin, 1965), Vol. 39, p. 106.
(10) - R. Gatto, Nuovo Cimento 28, 658 (1963).
(11) - E. Etim and P. Picchi, Phys. Letters 28 B, 272 (1968).
(12) - E. Lohrmann, Electromagnetic Interactions, Lund International Conference on Elementary Particles, 1969 - Report DESY 69/21 (1969).
(13) - G. Putzolu, Nuovo Cimento 20, 542 (1961).
(14) - E. Cremmer and M. Gourdin, Nuclear Phys. B 9, 451 (1969).
(15) - V. N. Baier and V. S. Fadin, Phys. Letters 27 B, 223 (1968).
(16) - G. Pancheri, Nuovo Cimento 60 A, 321 (1969).
(17) - R. Gatto, Lettere Nuovo Cimento 4, 117 (1970).
(18) - N. Kroll, T. D. Lee and B. Zumino, Phys. Rev. 157, 1376 (1967).
(19) - U. Amaldi, A. De Gasperis and P. Stein, A Proposal for the Investigation of the ϕ Resonance using Adone - Report ISS-65/48 (1965).
(20) - See Comprehensive review in Vector meson production and omega-rho interference, Proceedings of the Daresbury Study Weekend June 1970 - DNPL/R7 (1970).
(21) - G. J. Gounaris and J. J. Sakurai, Phys. Rev. Letters 21, 4 (1969).