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A. Małecki and P. Picchi: SHORT-RANGE DYNAMICAL CORRELATIONS FROM ELASTIC AND QUASIELASTIC SCATTERING OF HIGH-ENERGY ELECTRONS.

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ABSTRACT. -

We compare the influence of short-range correlations on elastic and quasielastic electron scattering on He^4.

Elastic electron scattering has recently aroused great interest[1] as a possible tool for studying short-range nucleon-nucleon correlations. It turns out that the elastic cross-section at large momentum transfers q is very sensitive to these correlations and a repulsive interaction produces a pronounced diffraction-like behaviour. This feature, which is supported by a close agreement with the experimental data, was perhaps somewhat unexpected since the elastic form factor $F_{ch}(q)$ being the Fourier transform of the single-particle density (ignoring nuclear recoil) depends on this correlations only indirectly.

Direct access to nucleon-nucleon spatial correlations may be had by the determination of a two-body operator expectation value. Two examples are the inelastic Coulomb sum rule[2] and that part of the transverse sum rule[3] which comes from the interaction of the electron with the spin current of the nucleus. Unfortunately, the total inelastic cross-section turns out to be a meager source of interesting information. Its behaviour at large momentum transfers is determined by very simple nuclear characteristics such as the number of protons[2] or nucleon magnetic moments[3]. This fact which has been well known for a long time, had focused the attention on the differential cross-section[4, 5]. The most promising idea, due

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to Czyz and Gottfried[4], regarded the study of the cross sections in the region of large electron energy loss, ω , where the dynamical correlations should lead to a considerable increase in the cross-section.

In this letter we discuss elastic and quasielastic scattering (the usual name for the deep nuclear inelastic scattering) employing in both cases the same model for internucleon correlations. By so doing we hope to be able to evaluate its relative merits in predicting the cross-sections for the two processes. A coherent interpretation necessitates the reexamination of quasileastic scattering since the existing calculations[4, 6] were based on methods assuming infinite nuclear matter which are clearly inadequate in the elastic case.

We describe the short-range correlations using a modified Jastrow[7] wave function: the correlations are introduced by means of the product of two-body unitary operators u(j, k)[8, 9]. The unitarity simplifies the extraction of the leading term of two-particle correlations and, by preserving normalization, assures their rapid headling as well as good convergence for the correlation method.

We apply the following form of the unitary correlation:

$$u(1,2)\,\alpha(1)\,\beta(2) = \sqrt{\frac{g^2(r_{12}) + (M_{\alpha\beta} - 1)g(r_{12})}{M_{\alpha\beta}}}\;\alpha(1)\,\beta(2) \qquad (1)$$

$$M_{\alpha\beta} = -\frac{\langle\alpha\beta|g(1-g)|\alpha\beta\rangle}{\langle\alpha\beta|1 - g\,|\alpha\beta\rangle}$$

α and β being single-particle states. The function $g(r_{ik})$ is a standard correlation factor defined to vanish within a repulsive core of radius c and to approach unity for $r_{jk} \gg c$. The correlations (1) are then healed at large mutual distances.

Since the application of the unitary formalism to elastic scattering has been widely discussed[10, 11] we will restrict ourserlves to a short description of the inelastic process. The name "quasielastic" reflects the basic mechanism at large energy and large momentum transfers. The electron scatters incoherently from individual nuclons inducing a single particle emission or a nucleon jump from the bound state to a continuum state in the average nuclear potential. The energy and momentum of the ejected nucleon depend on the momentum distribution W(p) in the initial state and the separation energy S. The outgoing nucleon may be described by a plane wave which is a good approximation[12] provided the probability of nuclear reabsorption is taken into account by means of the reduction factor D(ω). This quasielastic picture leads to the following prescription for computing the transition matrix elements[13]:

(2)
$$\sum_{|f\rangle} \delta(\omega - E_f + E_i) \left| \langle f | T | i \rangle \right|^2 =$$
$$= \frac{2\pi M}{q} \sum_{j=1}^{A} D_j(\omega) \int_{P_-}^{P_+} d\,p\,p \left| \langle (\vec{p}+\vec{q}, \alpha_h)_j | T | i \rangle \right|^2$$

with
$$(P\pm)_j = \frac{A-1}{A} q \left| 1 \pm \sqrt{1+\Omega_j} \right|$$
$$\Omega_j = \frac{2AM}{(A-1)q^2} \left(\omega - S_j - \frac{q^2}{2M} \right)$$

and where M is the nucleon mass.

The final nuclear state has a "hole" in the bound state α_h and a free "particle" with momentum $\vec{p} + \vec{q}$.

In the evaluation of the inelastic cross-section we have included the following points:

1) we have considered the transitions due to interaction with the nuclear charge, the spin current and the convection current calculated through order M^{-2} (McVoy-Van Hove interaction[2]);

2) we have correctly described the nuclear c. m. motion by performing the Gartenhaus-Schwartz transformation[14];

3) by employing a projection operator $P = 1 - |i\rangle\langle i|$, we have accounted for the nonorthogonality of final and initial state in the model with plane waves;

4) as in the case of elastic scattering, the introduction of short range correlations consists in the unitary transformation of the transition operators. We assumed here the same correlations in the final and initial nuclear states except for the interaction with an ejected nucleon which was neglected.

In our computations, the single-particle potential was represented by a harmonic-oscillator well. The correlation function was assumed to be

(3)
$$g(s) = \begin{cases} 0 & s \leq c \\ 1 - \exp\left[- B^2(s^2 - c^2) \right] & s \geq c \end{cases}$$

The numerical results for the He^4 nucleus are given in Figs. 1 and 2. In Fig. 1 we compare the influence of the short range correlations on elastic and quasielastic cross-sections. In Fig. 2 the applied correlations are visualized by presenting the separation density $\rho_s(s)$ which is

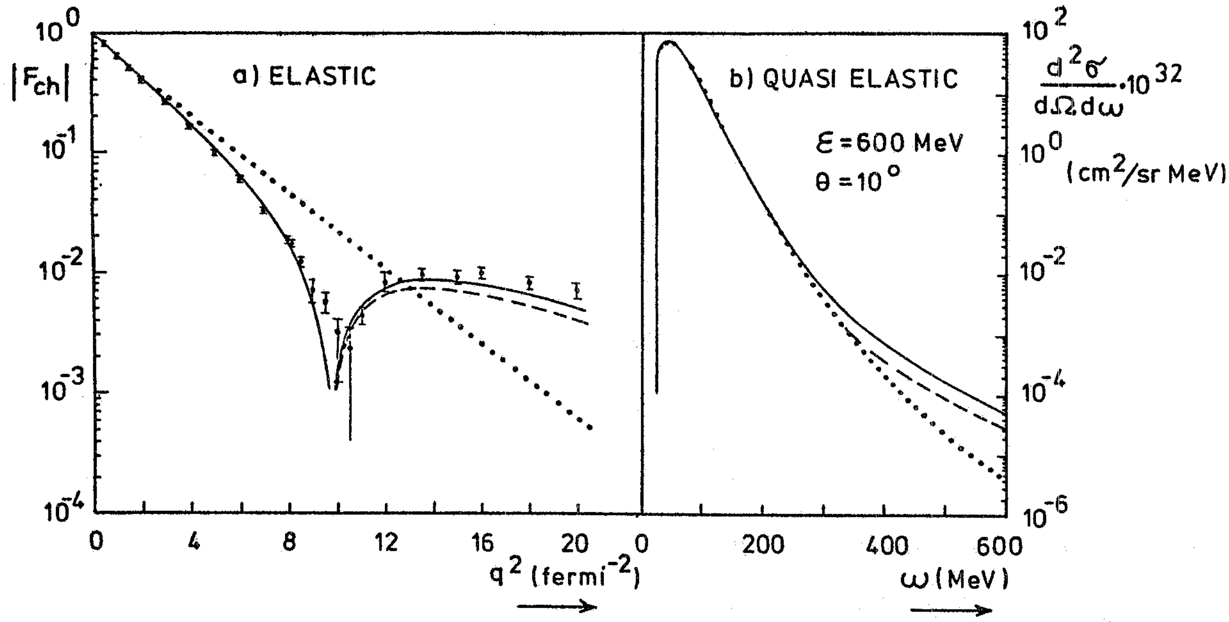

FIG. 1 - Elastic (a) and quasielastic (b) electron scattering cross-sections for He^4. The dotted curves represent predictions without correlations where the oscillator parameter α - 156 MeV is used. The solid curves correspond to the hard-core interaction with c=0.56 fermi, B=0.77 fermi, while the dashed curves are given by the soft-core correlation with c=0, B=0.64 fermi^{-1}. Elastic experimental data are taken from ref. 15.

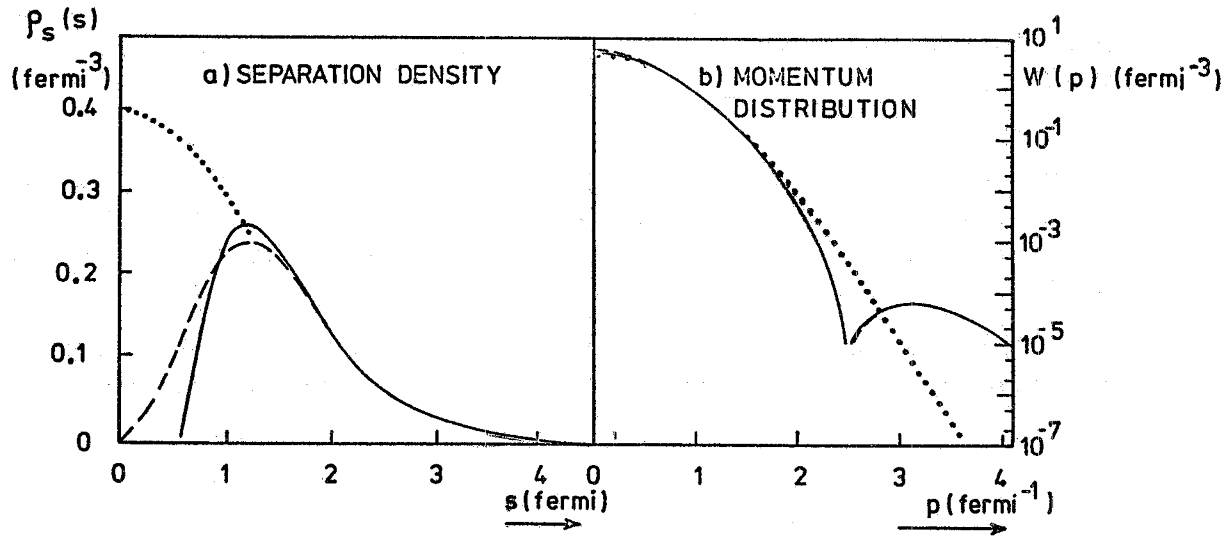

FIG. 2 - The separation density (a) and momentum distribution (b) in He^4. The three types of curves correspond to the correlations described in the caption of Fig. 1.

the probability of finding a nucleon pair with mutual distance s and the momentum distribution W(p). All the distributions are normalized to unity.

A number of comments are relevant here:

i) In the case of elastic scattering the correlated fit to the experimental charge form factor is very good, where as the uncorrelated prediction is inconsistent with the data. Our "healed" version of the unitary correlation (Eq. 1) is helpful in achieving a good fit, and moreover it decides about the very small difference between two-particle and multi-particle correlations.

ii) In quasielastic scattering we observe a distinct correlation effect but only at very large energy transfers. We were unsuccessful in shifting the interesting domain below the threshold for meson electroproduction by choosing suitable electron kinematics. The attempt appears to be incompatible with the need of having a momentum transfer which is large enough to probe short-range effects.

iii) The type of the repulsive interaction (hard-or soft-core) is felt rather weakly. The most important features are, the node of the two--particle wave function at $r_{12} = 0$ and the quickness of recovery to the unperturbed value. The healing distance ($\sim$ 1. 1 fermi - see Fig. 2a) better characterizes the interaction than the radius of repulsive core.

In summary, elastic scattering appears to be prefered as a tool for studying the short-range correlations. It is free of final-state interactions and electroproduction complications. The extraction of the correlation effects in quasielastic scattering however is more difficult as it must be preceded by a careful separation between nuclear and meson physics. We hope that the situation may be somewhat improved if e e'p coincidence experiments (or quasi elastic experiments where one measures nucleons instead of electrons) can be performed. We are currently studying these possibilities.

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