Laboratori Nazionali di Frascati

LNF-70/43

C. Castagnoli, E. Etim and P. Picchi : TESTS OF MODELS OF SCALE INVARIANCE WITH COSMIC-RAY NEUTRINO DATA.

Estratto da : Lett. Nuovo Cimento 4, 564 (1970)

C. CASTAGNOLI, *et al.*
26 Settembre 1970
Lettere al Nuovo Cimento
Serie I, Vol. 4, pag. 564-568

Tests of Models of Scale Invariance with Cosmic-Ray Neutrino Data.

C. CASTAGNOLI
Laboratorio di Cosmo-Geofisica del CNR - Torino

E. ETIM (*)
Istituto di Fisica Teorica dell'Università - Napoli

P. PICCHI
Laboratori Nazionali del CNEN - Frascati

(ricevuto il 5 Agosto 1970)

With the remarkable confirmation of scale invariance by the 6° and 10° data of the SLAC-MIT group (1), theoretical industry has laboured to provide a framework wherein the universal behaviour could be understood, and the outcome is a shift from the elementary nucleon constituent theory to a diffractive or multiperipheral mechanism (2). It is in the combined parton and diffractive scheme that DRELL, LEVY and YAN (3) (DLY) have interpreted the prediction of the field-theoretical model according to which the values of neutrino and antineutrino total and differential cross-sections with respect to the Bjorken variable $\omega = q^2/2M\nu$ are in the ratio 3 to 1. If confirmed such a prediction would severely distort the simple picture of the diffractive model since it would imply that the Pomeron is chiral in the sense that its weak coupling is helicity dependent (3). Since antineutrino experiments at high energy have not yet been performed, we have examined the available data (4) on cosmic-ray events for any

(*) Work supported by the INFN.

(1) E. D. BLOOM, D. H. COWARD, H. DESTAEBLER, J. DREES, G. MILLER L. W. MO, R. E. TAYLOR, M. BREIDENBACH, J. I. FRIEDMAN, G. C. HARTMANN and H. W. KENDALL: *Phys. Rev. Lett.*, **23**, 930 (1969); M. BREINDEBACH, J. I. FRIEDMAN, H. W. KENDALL, E. D. BLOOM, D. H. COWARD, H. DESTAEBLER, J. DREES, L. W. MO and R. E. TAYLOR: *Phys. Rev. Lett.*, **23**, 935 (1969).

(2) H. HARARI: *Phys. Rev. Lett.*, **22**, 1078 (1969); **24**, 286 (1970); H. D. I. ABARBANEL, M. L. GOLDBERGER and S. B. TREIMAN: *Phys. Rev. Lett.*, **22**, 500 (1969).

(3) S. D. DRELL, D. J. LEVY and T. M. YAN: SLAC-PUB no. 606 (June 1969) (*Phys. Rev.*, to be published). In our treatment of the property of scaling we shall consider in detail the model proposed in this reference because of its specific predictions.

(4) F. REINES: *Proc. Topical Conf. on Weak Interactions* (Geneva, 1969), p. 101; S. KRISHNASWAMY *et al.*: *Int. Conf. on Cosmic Rays* (Budapest, 1969); W. F. CROUCK *et al*: *Int. Conf. on Cosmic Rays* (Budapest, 1969).

possible difference between neutrino- and antineutrino-nucleon scattering and we report in this letter some evidence which, though not overwhelming, gives an indication in favour of the 3 to 1 ratio of DLY.

To better understand the hypotheses involved in the various models of scaling it is useful to express the weak structure functions $W_j^{\pm}(\nu, q^2)$ $(j=1, 2, 3)$ of neutrino $(+)$ and antineutrino $(-)$ scattering off nucleons in terms of the cross-sections $\sigma_L(\pm)$, $\sigma_R(\pm)$, $\sigma_S(\pm)$ for the absorption on nucleons of left-handed, right-handed and scalar intermediate vector bosons (IVB) $W^{\pm}$ respectively [5]:

$$(1)\qquad \begin{cases} \sigma_L(\pm)+\sigma_R(\pm)=\dfrac{2\pi}{\nu}\,W_1^{\pm}\,, \\[2mm] \sigma_L(\pm)+\sigma_R(\pm)+2\sigma_S(\pm)=\dfrac{2\pi}{\nu}\,\dfrac{\nu^2+q^2}{q^2}\,W_2^{\pm}\,, \\[2mm] \sigma_L(\pm)-\sigma_R(\pm)=\dfrac{2\pi}{\nu}\,\dfrac{(\nu^2+q^2)^{\frac{1}{2}}}{2M}\,W_3^{\pm}\,. \end{cases}$$

M is the nucleon mass, q^2 the invariant momentum transfer and ν the energy loss to the hadrons. From eq. (1) one gets the following inequalities between the structure functions:

$$(2)\qquad 0\leqslant\frac{(\nu^2+q^2)^{\frac{1}{2}}}{2M}\,|W_3^{\pm}|\leqslant W_1^{\pm}\leqslant\frac{\nu^2+q^2}{q^2}\,W_2^{\pm}\,.$$

In the field-theoretical model [3] these inequalities are saturated in the scaling limit, that is

$$(3)\qquad \begin{cases} W_1^{\pm}=\dfrac{\nu^2+q^2}{q^2}\,W_2^{\pm}\simeq(\nu/q^2)\,\beta_{\mathrm P}(q^2)\,\nu^{\alpha_{\mathrm P}-1}\,, \\[2mm] |W_3^{\pm}|=2M\,\dfrac{(\nu^2+q^2)^{\frac{1}{2}}}{q^2}\,W_2^{\pm}\simeq\left(\dfrac{2M\nu}{q^2}\right)\beta_{\mathrm P}(q^2)\,\nu^{\alpha_{\mathrm P}-1}\,, \end{cases}$$

where in the second half of eq. (3) the Regge parametrization of $\nu W_2^{\pm}$ in terms of the residue $\beta_{\mathrm P}(q^2)$ of the leading Pomeranchuk pole and its trajectory $\alpha_{\mathrm P}$ has been adopted [2]. Because of the equal coupling of the Pomeron to the $I_3=\pm 1$ weak hadronic current $\beta_{\mathrm P}^{(+)}(q^2)=\beta_{\mathrm P}^{(-)}(q^2)=\beta(q^2)$, from which it follows without further assumptions that

$$(4)\qquad \begin{cases} \sigma_S(+)=\sigma_S(-)=0\,, \\ \sigma_L(+)+\sigma_R(+)=\sigma_L(-)+\sigma_R(-)=|\sigma_L(\pm)-\sigma_R(\pm)|\,, \end{cases}$$

which together with the positivity condition (3)

$$(5)\qquad W_3^{+}=-W_3^{-}>0$$

yields for nontrivial asymptotic limit

$$(6)\qquad \begin{cases} \sigma_R(+)=\sigma_L(-)=0\,, \\ \sigma_L(+)=\sigma_R(-)\neq 0\,. \end{cases}$$

---

[5] J. D. BJORKEN and E. A. PASCHOS: SLAC-PUB no. 678 (December 1969).

It is important to note that eq. (6) is asymmetric in the strong interactions since the cross-sections $\sigma_R(+)$ and $\sigma_L(-)$ behave differently from $\sigma_L(+)$ and $\sigma_R(-)$ in the asymptotic limit. It is this asymmetry which is responsible for the 3 to 1 ratio. It gives rise to two other testable difference between ν-$\mathcal{N}$ and $\bar{\nu}$-$\mathcal{N}$ scattering:

i) The first is that the cross-section for $\bar{\nu}$-$\mathcal{N}$ scattering (—) vanishes in the backward direction ($\theta = \pi$); this is required by angular-momentum conservation [6] and is easily verified by substituting from (3) and (5) into the double differential cross-section

$$\frac{\mathrm{d}^2\sigma(\nu,\bar{\nu})}{\mathrm{d}\nu\,\mathrm{d}q^2} = \frac{G^2}{2\pi}\frac{E-\nu}{E}\left[2\sin^2(\theta/2)\,W_1^{\pm} + \cos^2(\theta/2)\,W_2^{\pm} + \frac{2E-\nu}{M}\sin^2(\theta/2)\,W_3^{\pm}\right], \tag{7}$$

where E is the incident-neutrino energy and θ the scattering angle.

ii) Secondly in the scaling limit eq. (7) gives for ν-$\mathcal{N}$ and $\bar{\nu}$-$\mathcal{N}$ scattering respectively

$$\begin{cases} \dfrac{\mathrm{d}^2\sigma(\nu\text{-}\mathcal{N})}{\mathrm{d}\nu\,\mathrm{d}q^2} = \dfrac{G^2}{2\pi}\,W_2^{+}, \\[2ex] \dfrac{\mathrm{d}^2\sigma(\bar{\nu}\text{-}\mathcal{N})}{\mathrm{d}\nu\,\mathrm{d}q^2} = \dfrac{G^2}{2\pi}\,W_2^{-}\left(1-\dfrac{2\nu}{E}+\dfrac{\nu^2}{E^2}\right), \end{cases} \tag{8}$$

from which it follows that the mean energy losses in ν-$\mathcal{N}$ and $\bar{\nu}$-$\mathcal{N}$ scattering are different and one finds

$$\begin{cases} \langle \nu_+ \rangle = \dfrac{\int \nu\big(\mathrm{d}^2\sigma(\nu\text{-}\mathcal{N})/\mathrm{d}\nu\,\mathrm{d}q^2\big)\,\mathrm{d}\nu}{\int\big(\mathrm{d}^2\sigma(\nu\text{-}\mathcal{N})/\mathrm{d}\nu\,\mathrm{d}q^2\big)\,\mathrm{d}\nu} = \dfrac{E}{2}, \\[2ex] \langle \nu_- \rangle = \dfrac{\int \nu\big(\mathrm{d}^2\sigma(\bar{\nu}\text{-}\mathcal{N})/\mathrm{d}\nu\,\mathrm{d}q^2\big)\,\mathrm{d}\nu}{\int\big(\mathrm{d}^2\sigma(\bar{\nu}\text{-}\mathcal{N})/\mathrm{d}\nu\,\mathrm{d}q^2\big)\,\mathrm{d}\nu} = \dfrac{E}{4}. \end{cases} \tag{9}$$

As a test of the DLY model eq. (9) predicts that neutrino- and antineutrino-derived cosmic-ray muons underground should fall into two distinct groups of mean energies in the ratio 2 to 3 and production cross-sections in the ratio 3 to 1. There being no measurement with a magnetic spectrometer, we have compared the results predicted by the various models for the experimental total fluxes of neutrinos and antineutrinos, using the formula

$$\Phi_\mu = N\int \Phi_\nu\sigma(\nu)\,\Delta x\,\mathrm{d}E + N\int \Phi_{\bar{\nu}}\sigma(\bar{\nu})\,\Delta x\,\mathrm{d}E = \Phi_\mu^{(+)} + \Phi_\mu^{(-)}\ (\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}), \tag{10}$$

where N is Avogadro's number, Φ_ν, $\Phi_{\bar{\nu}}$ [7] are the neutrino and antineutrino fluxes, $\sigma(\nu)$, $\sigma(\bar{\nu})$ the corresponding total cross-sections on nucleons averaged over proton and

(6) D. J. GROSS and C. H. LLEWELLYN SMITH: CERN preprint TH 1043 (June 1969).

(7) For the differential neutrino and antineutrino spectra we have used those computed by COWSIK *et al.*: *Proc. Ind. Acad. Sci.*, **63** A, 217 (1966). These spectra differ by about 20% from similar ones given by J. L. OSBORNE, S. S. SAID and A. W WOLFENDALE: *Proc. Phys. Soc.*, **86**, 93 (1965). The values of $\Phi_{\nu(\bar{\nu})}(E)$ have been averaged for θ between 60° and 90°.

neutron states and Δx, the mean distance from the starting point of an event to that of observation, depends on the mean energy loss $\langle v \rangle$ and is given by (8)

$$(11) \qquad \Delta x = 545 + \int\limits_{10^3 (\mathrm{MeV})}^{E - \langle v \rangle} \frac{\mathrm{d}E}{1.88 + 0.077 \ln (E^2/m_\mu(E + m_\mu^2/2m_e)) + 4 \cdot 10^{-6} E} \; (\mathrm{g/cm^2})$$

with m_μ the muon mass and m_e that of the electron. Table I gives the values of the muon flux Φ_μ so calculated under various hypotheses on the virtual IVB-nucleon cross-sections $\sigma_L(\pm)$, $\sigma_R(\pm)$, $\sigma_S(\pm)$ on which the cross-sections $\sigma(\nu)$, $\sigma(\bar{\nu})$ and Δx depend (9).

TABLE I. – *Entries for the muon flux* Φ_μ *calculated from eq.* (10) *of the text under various assumptions on* $\sigma_{L,R,S}(+)$. The models considered satisfy $\sigma_L(+) = \sigma_R(-)$, $\sigma_L(-) = \sigma_R(+)$ and $\sigma_S(+) = \sigma_S(-)$.

Model and/or hypothesis	$\Phi_\mu^{(+)}$ (*)	$\Phi_\mu^{(-)}$ (*)	$\Phi_\mu = \Phi_\mu^{(+)} + \Phi_\mu^{(-)}$ (*)
Diffraction: $\sigma_S(+) = 0$, $\sigma_R(+) = \sigma_L(+) \neq 0$	2.9 ± 0.8	2.0 ± 0.6	4.9 ± 1.0
DLY: $\sigma_S(+) = \sigma_R(+) = 0$, $\sigma_L(+) \neq 0$	2.68 ± 0.6	0.86 ± 0.2	3.54 ± 0.7
$\sigma_S(+) = \sigma_L(+) = 0$, $\sigma_R(+) \neq 0$	4.0 ± 1.0	5.5 ± 1.3	9.5 ± 1.6
$\sigma_R(+) = \sigma_L(+) = 0$, $\sigma_S \neq 0$	3.8 ± 0.9	2.55 ± 0.6	6.35 ± 1.1
$\sigma(\nu) = \sigma(\bar{\nu}) = (0.6 \pm 0.15)(G^2 ME/\pi)$, $E \leqslant 10$ GeV $\sigma(\nu) = \sigma(\bar{\nu}) = \text{const}$, $E > 10$ GeV	0.83 ± 0.2	0.7 ± 0.2	1.53 ± 0.3

(*) Only errors from the experimental cross-sections are taken into account.

Note that scale invariance implies a linear dependence of the total cross-sections $\sigma(\nu)$, $\sigma(\bar{\nu})$ on the energy E and that use has been made of the linearly increasing cross-sections well above $E = 10$ GeV up to which energy a linear rise has been verified experimentally (10). To test this extrapolation we have used $\sigma(\nu) = \sigma(\bar{\nu}) = (0.6 \pm 0.15)(G^2 ME/\pi)$ up to $E = 10$ GeV and $\sigma(\nu) = \sigma(\bar{\nu}) = \text{const}$ for $E > 10$ GeV to compute the muon flux; the corresponding entry in Table I compared with the experimental value (4) displayed in Table II shows that a linear rise of the cross-sections with energy is possible above 10 GeV in agreement with the prediction of scale invariance.

---

(8) M. G. K. MENON and P. V. RAMANA MURTHY: *Prog. in Elementary Particle and Cosmic-Ray Physics*, Vol. 9 (1965), p. 163.

(9) All models considered here satisfy $\sigma_L(+) = \sigma_R(-)$, $\sigma_L(-) = \sigma_R(+)$, $\sigma_S(+) = \sigma_S(-)$ from which it is clearly sufficient to state the hypotheses on $\sigma_{L,R,S}(+)$ only.

(10) I. BUGADOV, D. C. CUNDY, C. FRANZINETTI, W. B. FRETTER, H. W. K. HOPKINS, C. MANFREDOTTI, G. MYATT, F. A. NEZRICK, M. NIKOLIC, T. B. NOVEY, R. B. PALMER, J. B. M. PATTISON, D. H. PERKINS, C. A. RAMM, B. ROE, R. STUMP, W. VENUS, H. W. WACHSMUTH and H. YOSHIKI: *Phys. Lett.*, **30 B**, 364 (1969).

TABLE II. – *Experimental total muon flux from neutrinos and antineutrinos underground in units of* $10^{-13}\,\text{cm}^{-2}\,\text{s}^{-1}\,\text{sr}^{-1}$.

Reference	F. REINES CERN (1969)	W. F. CROUCH *et al.* Budapest (1969)	S. KRISHNASWAMY *et al.* Budapest (1969)
Experimental results	3.5 ± 0.6	3.7 ± 0.6	3.4 ± 0.8

As seen from Table I the DLY and the purely diffractive models give results which are not in conflict with experiment; more critical tests of these models are therefore necessary in order to be able to distinguish between them.

As a concluding remark we note that, although the field-theoretical prediction agrees with experiment, it is difficult on theoretical grounds to accept the view that the weak structure functions $W_3^{\pm}(\nu, q^2)$, which in the DLY model must be large in order to get the 3-to-1 ratio, are dominated by the Pomeranchuk pole (cf. eq. (3)). Since $W_3^{\pm}$ arise from the interference of the vector and axial vector matrix elements, the Pomeranchuk pole cannot contribute to them because of G-parity conservation ([6]). Actually $W_3^{\pm}$ have the quantum numbers of ω and φ Regge exchanges in the crossed channel and from duality or directly from Adler's third sum rule ([11])

$$-g_A(q^2)\cdot\left(F_{1v}(q^2) + \mu_v F_{2v}(q^2)\right) = \frac{1}{2}\int\limits_{M+m_\pi}^{\infty} \frac{K\,\mathrm{d}K}{M}\,(W_3^- - W_3^+)\,, \tag{12}$$

which must be valid if the dual diffractive model of scaling is to make sense ([12]), $W_3^{\pm}$ should not scale in the Bjorken limit and should therefore be negligibly small compared to $W_{1,2}^{\pm}$. It is however possible to accomodate the 3-to-1 ratio and still retain a nonchiral Pomeron if the two aspects of scaling and Pomeron dominance of the structure functions are decoupled as proposed in ref. ([12]).

([11]) S. ADLER: *Phys. Rev.*, **143**, 1144 (1966).

([12]) E. ETIM: *Adler's neutrino sum rules in the scaling limit and the possibility of a chiral Pomeron*, to be published.