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3.5 Positron Accelerators

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1 Introduction

In the last few years there has been increasing interest for high energy, high intensity and good quality positron beams; the requirements are very stringent particularly for the production of monochromatic γ-ray beams and for injection into e^+e^- storage rings.

The positrons are obtained in an electromagnetic shower produced by an electron beam striking a target; the positrons coming off the target have a large energy and angular spread, and usually only a fraction of them can be captured and accelerated further. The object of this chapter is to present, briefly, the methods sofar proposed and used for the production of high intensity positron beams; the linear accelerator has proved to be more convenient than a circular accelerator, as, when provided with suitable focusing, it has an acceptance in transverse phase space (expressed in cm $\times$ transverse momentum) constant with energy, and large enough; the acceptance of a circular accelerator increases with energy, so that the accelerator aperture, filled by positrons at injection, becomes uselessly large during the acceleration.

The positron conversion and acceleration efficiencies, expressed as the number of positrons accelerated in 1% energy bin per electron incident on the radiator, divided by the equivalent incident beam energy in GeV (see section 3, eq. (3.2)), range from 10^{-4} to $10^{-2} e^+/e^-$ GeV %, depending on the focusing systems used; total average intensities have recently reached the microampere range, which, only a few years ago, was considered to be a good result for electrons.

The first high energy positron beams accelerated in a linac were obtained at Stanford in 1960 by Yount and Pine [1962]; the conversion energy ranged from 350 MeV and the positron energy from 600 MeV to 300 MeV; the intensity achieved,

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with the 650 MeV conversion, was 3×10^7 positron per pulse in 2% energy bin, with an incident electron beam of 4×10^{10} e^- per pulse; the conversion efficiency was about 6×10^{-4} e^+/e^- GeV %. The same efficiency was achieved with conversion at 100 MeV and positron acceleration up to 800 MeV.

Similar values of conversion efficiency have been obtained at Orsay, $3\text{–}3.5 \times 10^{-4}$ e^+/e^- GeV % (Haissinski [1967]), with conversion at 580 MeV and acceleration to 240 MeV; very recently, the system has been improved and an increase by a factor of two in efficiency has been reported.

Low energy positron beams have been obtained at the General Atomic Laboratory (Sund *et al.* [1964]), in an L-band linac, using a focusing system with a matching lens and conversion at 8–15 MeV; similar low energy beams have also been obtained at Saclay and Livermore.

With the matched focusing system of the Frascati linac, a conversion efficiency of $2.8 \times 10^{-2} e^+/e^-$ GeV % has been achieved (Amman *et al.* [1966]); the primary electron beam has an energy of 80 MeV and the positrons are accelerated to 320 MeV.

The recent results obtained at SLAC with the 14 GeV positron beam (De Staebler, private communication, 1967) give an efficiency of $1.8 \times 10^{-3} e^+/e^-$ GeV %; it is expected that further work on the accelerator will improve it by a factor 2 or 3.

The primary electron beam power and its density are limited by thermal effects in the converter; new types of converters using liquid metal are under development (Wiedemann [1966]) and should allow a further increase in the available positron current.

In the following, the design criteria for a high intensity positron linac will be examined, and approximate equations for the conversion efficiency as a function of the focusing parameters will be derived; this approach is useful at an initial stage of the design and allows an easy evaluation and optimization of the costs. The theory of particle motion in the focusing system has been studied by many authors and will not be presented in detail here.

The following symbols will be used consistently:

p_x, p_y, p_t	transverse momentum in units of m_0c: $p_t^2 = p_x^2 + p_y^2$;
$P = (\gamma^2 - 1)^{\frac{1}{2}} \approx \gamma$	longitudinal momentum in units of m_0c (independent of transverse momentum in the paraxial approximation);
z	longitudinal coordinate, while (x, y) or (r, θ) are the transverse coordinates;
$\Theta = p_t/P \approx p_t\gamma^{-1}$	angle of the positron trajectory relative to the linac axis;
$\mathscr{E}$	beam emittance in one transverse plane (area in phase space) in units of cm $\times\, m_0c$;
$\mathscr{E}_G$	geometrical beam emittance, in units of cm $\times$ rad: $\mathscr{E}_G \approx \mathscr{E}/\gamma$;
Y	acceptance of a system with finite aperture, in units of cm $\times\, m_0c$;
R	radius of the limiting aperture in the linac;
ϱ	radius of the positron source;

λ, Λ	characteristic lengths of the focusing systems (betatron reduced wavelengths);
γ_c, E_c	design energy of the positrons at the converter, in units of m_0c^2 or in MeV;
V	4-dimensional volume in phase space accepted by a system of finite aperture, in units of $\text{cm}^2 \times (m_0c)^2$;
Ω	effective solid angle at the converter: $\Omega \approx V/(\gamma_c^2 \int \mathrm{d}x\, \mathrm{d}y)$;
ΔE	equivalent positron energy band accepted at the converter;
K	accelerating electric field in the linac in V/cm;
$\alpha = eK/m_0c^2$	accelerating electric field in the linac expressed in cm^{-1};
n_c	positron source density, in e^+/ster MeV or A/ster MeV;
X_0	radiation length in the converter material, in cm;
E_-	energy of the primary electron beam, in GeV.

The paraxial and the relativistic approximations will be used throughout the present chapter (P independent of p_t and $(\gamma^2-1)^{\frac{1}{2}} \approx \gamma$). The paraxial approximation is certainly justified in the focusing system along the linac, where the angles are small; it can be used also in the matching section, where the angles are very large, because the positron energy spectrum is assumed to be flat in the range of interest; as a consequence, the energy spread in the transition between the matching lens and the focusing system will become a little broader.

2 *General remarks on positron linear accelerators*

In the relativistic approximation no transverse forces are exerted on the particles by the rf waves in a linac; the effect of a focusing system can be represented by a 4×4 matrix relating positions and transverse momenta of the particles at a longitudinal coordinate z to positions and transverse momenta at another coordinate z_0. Usually the motion in the transverse plane (x, y) is decoupled, so that the 4×4 transfer matrix reduces to two 2×2 matrices, and one can consider the motion in one dimension. This is true not only for focusing systems made of quadrupole lenses, but also for solenoids, if a reference frame rotating around the axis of the solenoid is taken for the transverse coordinates (see section 4).

The general form of the transfer matrix of a periodic system between symmetry points is:

$$\boldsymbol{M} = \begin{vmatrix} \cos\omega & \lambda \sin\omega \\ -\lambda^{-1}\sin\omega & \cos\omega \end{vmatrix}. \tag{2.1}$$

For acceptance considerations, one has to choose the reference coordinate z_0 along the period in such a way to obtain the maximum value for λ; if the system has a finite aperture $2R$ (in the case of a linac R can be taken as a large fraction of the radius of the smallest iris) the acceptance Y of the system is (Helm [1962a]):

$$Y = \pi(R^2/\lambda). \tag{2.2}$$

If the transfer matrices in the two transverse planes in phase space are equal, one can easily obtain the 4-dimensional volume V transmitted by the system through a circular diaphragm of radius R; the locus of the largest transverse momenta accepted from a point of coordinates $x=x_0$, $y=0$ is an ellipse whose semi-axes are: $p_{yM}=R/\lambda$ and $P_{xM}=(R/\lambda)(1-x_0^2/R^2)^{\frac{1}{2}}$. Using polar coordinates, and integrating over r and θ, it results:

$$V = \int_0^{2\pi} \mathrm{d}\theta \int_0^R \pi p_{xM} p_{yM} r \,\mathrm{d}r = \tfrac{2}{3}\pi^2 R^4/\lambda^2 = \tfrac{2}{3}Y^2 . \tag{2.3}$$

If the matrices in the two planes are different, the 4-dimensional volume accepted by a system with a circular or elliptical aperture is

$$V = \int \mathrm{d}p_x \,\mathrm{d}p_y \,\mathrm{d}x \,\mathrm{d}y = \frac{1}{2}\frac{\pi a^2}{\lambda_x}\frac{\pi b^2}{\lambda_y} = \tfrac{1}{2}Y_x Y_y \tag{2.4}$$

where a and b are the semi-axes of the elliptical diaphragm.

According to the use foreseen for the positron beam, an upper limit for the useful beam emittance $\mathscr{E}$ can be set; therefore the maximum useful acceptance Y is fixed.

With an appropriate matching section between the converter and the linac focusing system, it is possible to accept all the positrons which are emitted from the converter within the 4-dimensional volume corresponding to the emittance $\mathscr{E}$; as the elements of the matching section and focusing system matrices depend on the particle energy, the acceptance will have a maximum for a certain energy E_c of the positrons from the converter (the design energy) and will fall off more or less rapidly at different energies. It is then convenient to define an equivalent energy band accepted ΔE, such that the 4-dimensional volume V integrated over the energy spectrum of the positrons coming from the converter is equal to ΔE times the volume V_c accepted at the design energy.

Two types of matching sections will be discussed later (section 5): the short lens matching, or narrow band system (NBS) which has an equivalent energy acceptance ΔE equal to $g\,(\varrho/R)\,E_c$, where ϱ is the radius of the positron source, and the value of the coefficient g is about 3; and the adiabatic matching, or wide band system (WBS), whose acceptance in principle does not depend on energy, and for which other considerations give an energy acceptance of the order of $(1\text{–}2)\,E_c$.

The positron source density does not depend, in first approximation, on the source size, and is usually expressed as:

$$\frac{\mathrm{d}n}{\mathrm{d}E\,\mathrm{d}\Omega} = n_c \quad (\mathrm{e}^+/\text{ster MeV}). \tag{2.5}$$

The solid angle Ω at the converter, and the source size are related to the acceptance of the focusing system through the following equation (assuming perfect matching):

$$\pi\varrho^2\Omega\gamma_c^2 = V = h\mathscr{E}^2 \tag{2.6}$$

where h can take the values $\frac{2}{3}$ or $\frac{1}{2}$, see (2.3), (2.4).

With the assumption n_c=const., which is approximately valid for angles smaller than ≈ 0.2 rad and energy ranges of the order of a few tens of MeV, one obtains for the positron intensity:

$$\begin{aligned} &\text{NBS:}\quad I_+ = n_c \times \Delta E \times \Omega = n_c g E_c \frac{\varrho}{R} h \frac{\mathscr{E}^2}{\gamma_c^2} \frac{1}{\pi\varrho^2}, \\ &\text{WBS:}\quad I_+ = n_c \times \Delta E \times \Omega = n_c (1\text{–}2)\, E_c h \frac{\mathscr{E}^2}{\gamma_c^2} \frac{1}{\pi\varrho^2}. \end{aligned} \tag{2.7}$$

The limits of validity of (2.7) to angles smaller than ≈ 0.2 rad from the converter, can be expressed, in the case of perfect matching, as a limit on ϱ/R, that must be larger than $\approx 2\mathscr{E}/\gamma_c R$, and, eventually, on $\mathscr{E}$, that must not exceed $\approx \frac{1}{2}\gamma_c R$.

The source radius ϱ has also an absolute lower limit determined by the shower dimensions; with a point-like primary electron beam, one obtains 50% of the positrons of energy between 10 and 20 MeV in a radius of $\approx 0.16 X_0$, where X_0 is the radiation length of the converter material (see section 3). Heating problems in the converter may prevent making full use of this lower limit for the source dimensions, when using a high power primary beam, also if it is possible to focus it on the converter. This practically means that an upper limit for the ratio n_c/ϱ^2 is set, that depends on the nature of the converter. Liquid metal converters (Wiedemann [1966]) are now under development: they might allow for the ratio n_c/ϱ^2 higher values than those that can be obtained with solid converters.

As far as the best choice for energy of the positron at the converter is concerned, eqs. (2.7), together with the data on positron yields, show that the lowest values of γ_c give the highest intensity; a limit is determined by the dephasing in the focusing system, proportional to γ_c^{-1}, which is the cause of a broadening in phase of the positron bunch, and therefore of an energy spread during the acceleration; for very low values of γ_c ($\gtrsim$ 5–10) also the spread in velocity is an important cause of broadening of the bunch. A good compromise turns out to be $\gamma_c \approx 15$–20.

Eqs. (2.7) show that with the NBS one obtains a total positron intensity lower than what can be obtained with the WBS, for given values of the beam parameters, but within a smaller energy spread at the converter; the NBS (less expensive and simpler than the WBS) is then appropriate for an intermediate energy linac, when a narrow energy spectrum at the end is required, while the WBS is convenient for a very high energy positron linac, where the energy at the converter does not contribute essentially to the relative energy spread at the end.

If the geometrical beam emittance at the end of the linac $\mathscr{E}_G$ is given as a limiting requirement (this is the case, for instance, of the injectors in circular accelerators), then, as $\mathscr{E}_G\gamma=\mathscr{E}$, the positron intensity is proportional to the square of the final energy of the linac, γ.

Concluding these general remarks, one can say that existing positron linacs are already quite close to the angular acceptance limits at the converter (in the Frascati linac the largest angles accepted are ≈ 0.25 rad). There are two ways to obtain higher intensities: the increase of the positron beam emittance (which means more expensive focusing system along the accelerator, and worse beam quality at the end), and, more promising, the increase of electron beam power density on the converter, with a proportional increase in n_c/ϱ^2, which may eventually be possible with liquid metal converters.

3 *Positron yields*

In order to compute the positron intensity that can be achieved, one needs to know the emittance of the positron source as a function of energy γ_c, angle of emission Θ (or transverse momentum p_t) and distance r from the center of the source for low energy positrons (in the range from 5 to 20 MeV). Haissinski [1965, 1967] has collected the experimental results available, comparing them with the theoretical values computed by Crawford, Messel [1965] and arrived at analytical formulas that fit reasonably well the data, and are very useful for design purposes. The reader is referred to these papers for a more detailed discussion of the results.

The more relevant experimental results have been obtained at Orsay, by Aggson and Burnod [1962], who measured the total forward yield ($\Theta=0°$) as a function of positron energy E_c and converter thickness, with primary electron energy ranging from 55 MeV to 220 MeV, and at Stanford, by De Staebler [1965], who measured the yield and the angular distribution of positrons between 5 and 35 MeV, produced in lead and copper converters, about 3 radiation lengths thick, by 1 GeV primary electrons.

Measurements of positron forward yield with low energy primary beam (20–30 MeV) have been made also at Saclay, by Bernardini *et al.* [1962]; these results are less accurate than others, because of some uncertainties in the calibration of the spectrometer used.

No experimental results are available, so far, for the radial distribution.

3.1 Forward yield ($\Theta=0$, integrated over the radial dimension of the source)

The Orsay results for the low energy positron yield (from about 10 MeV to 0.2–0.3E_-, where E_- is the energy of the primary electron beam) can be summarized in the following formula:

$$n = 0.24E_{-}\left(1 - \frac{0.025}{E_{-}}\right) \quad (e^{+}/e^{-}\ \text{MeV ster}) \tag{3.1}$$

where E_{-} is expressed in GeV; the validity is limited to $E_{-} > 0.050$ GeV.

The yield is lower for higher positron energies, and this justifies the choice of low values of E_c to get the maximum positron density at the converter.

If one defines an equivalent primary energy E_{eq}:

$$E_{eq} = E_{-}(1 - 0.025/E_{-}) \tag{3.2}$$

the forward yield divided by the equivalent primary beam energy is approximately a constant:

$$n^{+} = n/E_{eq} = 0.24 \quad (e^{+}/e^{-}\ \text{GeV MeV ster}). \tag{3.3}$$

De Staebler [1965] gives a little lower value for n^{+}, namely 0.2 e^{+}/e^{-} GeV MeV ster, but the discrepancy with the Orsay results seems to be due to window scattering in the experiment; his results also show that the yield does not decrease appreciably for positron energy of the order of one half to one third of the critical energy of the converter.

The converter thickness must be increased with increasing energy, but the value is not critical.

3.2 Angular distribution

De Staebler [1965] obtains an angular dependence of the yield which can be approximated as:

$$n(\Theta) = n(\Theta = 0)\exp(-\Theta/\Theta_0) \tag{3.4}$$

with $\Theta_0 \approx 0.35$ rad.

Haissinski [1965, 1967] has compared this experimental result with the computations by Crawford and Messel [1965] considering the positrons in the range from 10 to 20 MeV, with primary beam energies of 200 MeV, 500 MeV and 1000 MeV (converter thicknesses: 1.5, 2 and 3 radiation lengths, resp.), and found a good agreement.

3.3 Radial distribution

Haissinski [1965, 1967] has derived from the Crawford and Messel [1965] computation the radial distribution for 10 to 20 MeV positrons produced by 200, 500 and 1000 MeV electrons; he finds that about 50% of the positrons are emitted in a circle of radius

$0.16X_0$, and about 75% in a circle of radius $0.32X_0$. These results are for a lead converter, but they do not change appreciably in the case of lower Z converter (emulsion, $X_0 = 11.08$ g/cm^2 = 2.89 cm).

For radii smaller than $0.16X_0$ the intensity is approximately proportional to $r^{\frac{1}{2}}$.

4 *Focusing systems*

In a relativistic linear accelerator the transverse forces tend to zero; some kind of magnetic focusing is therefore necessary if one wants to accelerate a large emittance beam. The system has a finite aperture, and the diaphragm radius can be taken as a fraction of the smallest waveguide iris radius; the radial components of the accelerating electric fields, very close to the iris radius, are such as to make the particle motion unstable. The experimental results so far obtained show that the useful aperture is about 90% of the physical aperture.

The focusing system, following the matching section, must have a large energy acceptance (this is more important in the first accelerating sections where the relative energy spread of the positrons is very large), and a transverse acceptance either constant or, for very long accelerators, constant in absolute value, but adiabatically changing shape, so that the radial beam dimensions damp somewhat at the expense of transverse momenta; this will avoid undesirable losses due to small misalignments or to cumulative effects of radial components of the electric field.

It is important to avoid abrupt non-matched transitions, if the beam emittance has to be kept constant and the full intensity has to be transmitted. In fig. 1 two cases of non-matched transitions from a system with acceptance Y to the systems with acceptance Y' and Y'' are represented: in the first case $Y' > Y$, with a diaphragm of radius R, and the beam within Y will increase its emittance to Y', because of the energy spread that differentiate the velocity of the particles in the phase-space trajectories; in the second case $Y'' < Y$ and part of the beam is lost.

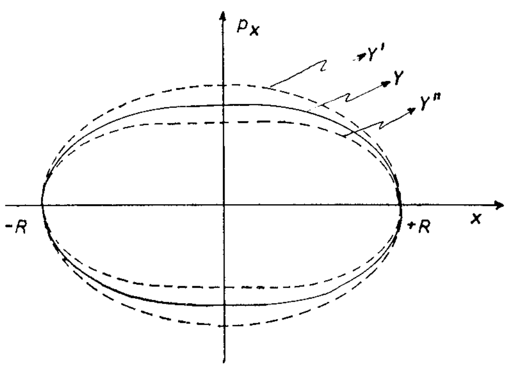

Fig. 1. Matching.

The particle motion in the focusing systems has been discussed in detail by many authors; reference is made here to the works of Helm [1962a, b; 1963a, b]; Ferlenghi and Mango [1963]; Haissinski [1965, 1967]; Lobb [1966]; Wiedemann [1966]; Lucci [1967]. It is here assumed that, when an accelerating field is present, it does not depend on the transverse coordinate and is constant along z.

The characteristics of different systems are examined in terms of their transfer matrices, which relate positions and transverse momenta in a generic point to the position and transverse momenta in a reference point.

4.1 Drift space

The transfer matrix through a section without transverse forces is given by:

$$\begin{vmatrix} 1 & l \\ 0 & 1 \end{vmatrix} \tag{4.1}$$

with:

$$l = \frac{1}{\alpha} \ln\left(1 + \frac{z - z_0}{\gamma_0} \alpha\right) = \frac{1}{\alpha} \ln \frac{\gamma}{\gamma_0}$$

$$\alpha = \frac{e}{m_0 c^2} K$$

and, for $K=0$, $l=(z-z_0)\gamma_0^{-1}$. γ_0 is the particle energy at the coordinate z_0, and $\gamma=\gamma_0+\alpha(z-z_0)$ is the particle energy at the coordinate z.

The acceptance of such a system is (see eqs. (2.1), (2.2)):

$$Y = \frac{\pi R^2}{l} = \frac{\alpha \pi R^2}{\ln \gamma/\gamma_0} = \frac{\gamma - \gamma_0}{\ln \gamma/\gamma_0} \frac{\pi R^2}{L} \tag{4.2}$$

where

$$L = z - z_0 .$$

4.2 Uniform solenoid

The 4×4 transfer matrix of a uniform solenoid (where the axial magnetic field B is uniform inside the solenoid and zero outside), between points where the field has dropped to zero, in the impulse approximation for the fringing, is defined by:

$$\begin{vmatrix} x \\ p_x \\ y \\ p_y \end{vmatrix} = \begin{vmatrix} |A| & |B| \\ -|B| & |A| \end{vmatrix} \begin{vmatrix} x_0 \\ p_{x0} \\ y_0 \\ p_{y0} \end{vmatrix} \tag{4.3}$$

where A and B are two 2×2 matrices:

$$|A| = \begin{vmatrix} \dfrac{1+C}{2} & \lambda\dfrac{S}{2} \\ -\dfrac{S}{2\lambda} & \dfrac{1+C}{2} \end{vmatrix}; \quad |B| = \begin{vmatrix} \dfrac{S}{2} & \lambda\dfrac{1-C}{2} \\ -\dfrac{1-C}{2\lambda} & \dfrac{S}{2} \end{vmatrix} \tag{4.4}$$

$$S = \sin\omega; \quad C = \cos\omega$$

and

$$\omega = \frac{e}{m_0 c} B \frac{1}{2\alpha} \ln\left(1 + \frac{z - z_0}{\gamma_0}\alpha\right) = \frac{eB}{m_0 c}\frac{1}{2\alpha}\ln\frac{\gamma}{\gamma_0}$$

$$\omega = \frac{eB}{m_0 c}\frac{z - z_0}{2\gamma_0} \quad \text{for} \quad K = \alpha = 0 \tag{4.5}$$

$$\lambda = \frac{2}{B}\frac{m_0 c}{e} \quad \left[\frac{m_0 c}{e} = 1703 \text{ G} \times \text{cm}\right] \tag{4.6}$$

It can be shown that, when the output coordinates and transverse momenta are referred to a frame of reference rotated by an angle ω, the non-diagonal blocks of the transfer matrix become zero, and the two diagonal 2×2 matrices become:

$$\begin{vmatrix} \cos\omega & \lambda\sin\omega \\ -\lambda^{-1}\sin\omega & \cos\omega \end{vmatrix}. \tag{4.7}$$

Given the cylindrical symmetry of the system, the rotation around the axis is non-essential, and the motion of the particles can be considered as decoupled in the two planes.

The acceptance of the solenoid is:

$$Y = \frac{\pi R^2}{\lambda} = \frac{\pi R^2}{2}\frac{eB}{m_0 c} \tag{4.8}$$

and does not depend on energy.

4.3 Thin focusing lens

The transfer matrix of a thin lens, focusing on both transverse planes, is of the form:

$$\begin{vmatrix} 1 & 0 \\ -Q & 1 \end{vmatrix}; \quad Q = Q_0/\gamma. \tag{4.9}$$

Helm [1963b] has shown that $Q = Q_0/\gamma$, where Q_0 is a quantity independent of energy; a thin lens can be obtained, for instance, with a quadrupole triplet; the transfer matrix of a thin quadrupole is:

$$\begin{vmatrix} 1 & 0 \\ \mp q & 1 \end{vmatrix}; \quad q = \frac{e}{m_0 c} \int_{\text{quad.}} \frac{\partial B}{\partial x} \mathrm{d}z . \tag{4.10}$$

The element 21 of the transfer matrix of a triplet is proportional to $q^2 t = q^2 T/\gamma$ (see eq. (4.1) assuming no acceleration along T), where T is the physical spacing between the lenses; $Q_0 \propto q^2 T$ is then independent of energy and proportional to the square of the quadrupole strength and to their physical spacing.

4.4 System of thin focusing lenses

A periodic system of thin focusing lenses of strength $2Q$ each, can be divided into equal periods composed of two focusing lenses of strength Q separated by a drift space l (fig. 2); the system is periodic if Q and l are constant, which means that, if there is acceleration, the physical spacing of the thin lenses and the quantity Q_0 must increase with γ according to (4.1) and (4.9).

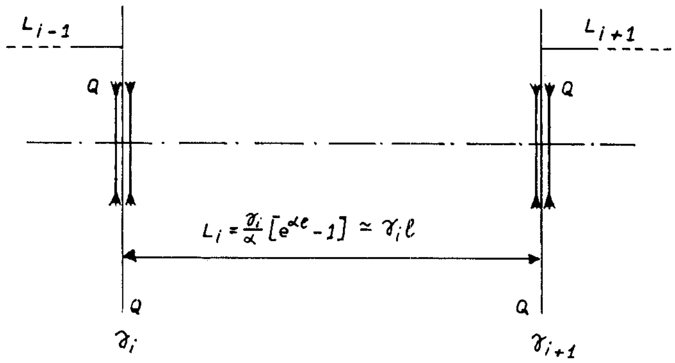

Fig. 2. Thin focusing lens system.

The transfer matrix for one period is

$$\begin{matrix} 1 - lQ & l \\ -Q(2 - lQ) & 1 - lQ \end{matrix} \tag{4.11}$$

and, using the notations of (2.1):

$$\cos\omega = 1 - lQ; \quad \sin\omega = (2lQ - l^2Q^2)^{\frac{1}{2}}$$

$$\lambda = (2Q/l - Q^2)^{-\frac{1}{2}}. \tag{4.12}$$

If the system has a large acceptance, the condition $\alpha l \ll 1$ is usually satisfied, so that the physical spacing L is given by:

$$L = \frac{\gamma_i}{\alpha}(\exp(\alpha l) - 1) \approx \gamma_i l \tag{4.13}$$

and

$$\lambda \approx \left(2\frac{Q_0}{L} - \frac{Q_0^2}{\gamma_i^2}\right)^{-\frac{1}{2}} \tag{4.14}$$

where Q_0 is the strength of the ith lens.

The acceptance of the system is given by:

$$Y = \frac{\pi R^2}{\lambda} = \pi R^2\left(2\frac{Q}{l}\left[1 - \frac{lQ}{2}\right]\right)^{\frac{1}{2}} \approx \pi R^2\left(2\frac{Q_0}{L}\left[1 - \frac{LQ_0}{2\gamma_i^2}\right]\right)^{\frac{1}{2}}. \tag{4.15}$$

The acceptance Y is constant along the system when Q and l are constant; but (4.15) shows that the system is chromatic, because if an energy band has to be transmitted the acceptance depends on energy and vanishes for $\gamma^2 \leqslant \frac{1}{2}LQ_0$; it can then be defined a "low energy cut-off" $\gamma_{ic} = (\frac{1}{2}LQ_0)^{\frac{1}{2}}$.

If the beam emittance at the input of the system is $\pi R p_M$, where R and p_M are the semi-axes of the ellipse representing the beam in phase space at the thin lens location, the acceptance will be matched to the emittance if the following condition holds:

$$\pi R p_M = \pi R^2\left(2\frac{Q}{l}\left[1 - \frac{lQ}{2}\right]\right)^{\frac{1}{2}} \tag{4.16}$$

or

$$l = \frac{2QR^2}{p_M^2 + R^2Q^2} = l_0$$

and $Y \gtrless \mathscr{E}$, when $l \lessgtr l_0$.

The acceptance ellipse at the lens is given by the equation:

$$\frac{x^2}{R^2} + \frac{\lambda^2}{R^2}p_x^2 = 1 \tag{4.17}$$

which, at the mid-point between lenses, is transformed into the following equation:

$$\frac{x^2}{R^2(1 - \frac{1}{2}Ql)} + \frac{p_x^2}{2R^2Q/l} = 1. \tag{4.18}$$

From (4.18) one obtains for the maximum value of x in the mid-point between lenses:

$$x_M = R(1 - \tfrac{1}{2}Ql)^{\frac{1}{2}} = \xi R. \tag{4.19}$$

If the value of $\xi = x_M/R$ in the waist is fixed, and the matching condition (4.16) between beam emittance and system acceptance holds, it results for Q and l_0:

$$Q = \frac{p_M}{\xi R}(1 - \xi^2)^{\frac{1}{2}}, \qquad l_0 = \xi \frac{R}{p_M}(1 - \xi^2)^{\frac{1}{2}}. \tag{4.20}$$

When the beam to be transmitted has a certain energy spread $\Delta\gamma$, constant in absolute value, the system can be made periodic for a single value of energy at the input, because the relative energy spread changes as the particles are accelerated. If the variation of the parameters can be considered adiabatic, which requires that the acceleration in one period and the energy spread $\Delta\gamma$ be small as compared to the particle energy, there will be a loss of particles whose energy is such that the acceptance is smaller than the beam emittance, and the density in phase space of the transmitted beam will remain constant.

If, on the contrary, the adiabatic condition is not fulfilled, there will be a dilution of the beam phase space density, when the acceptance is larger than the beam emittance.

4.5 Alternating gradient focusing systems

Helm [1962a, 1963b] has examined in detail the alternating gradient systems; the method is an extension of that used for the thin focusing lenses.

It must be noted, however, that the AG focusing has different acceptance in the two planes, and that in the plane where the first lens is defocusing, the maximum beam dimensions increase after the lens.

If one has then a beam whose emittance is an ellipse with semi-axes R and p_M, a matching section consisting of one half period of the system considered in section 4.4 can be conveniently used; at the waist the beam dimension is $\xi R < R$ and from there an AG focusing can start without losing beam intensity.

4.6 Considerations on the design of a positron focusing system

The typical beam quality in a high intensity positron linac, after the matching section, can be assumed to be: emittance $\mathscr{E} \approx \pi$ cm $\times\, m_0 c$, energy band from 5 to 15 MeV. The focusing system must then have a characteristic length $\lambda = \pi R^2/\mathscr{E} = 1$ cm, in the case of an S-band linac, where $R \approx 1$ cm.

Owing to the large relative energy spread it is convenient to start with a uniform solenoid; from (4.6) it results that to get $\lambda = 1$ cm an axial field of 3400 G is required.

The solenoid is more expensive than a quadrupole focusing, so it is convenient to make the solenoid as short as feasible. A condition that can be imposed is that the quadrupole focusing starts when the physical spacing between quadrupoles is of the order of the accelerating section length (typically 3 to 5 meters); from (4.20), assuming $\xi = 1/\sqrt{2}$, one obtains: $Q = 1$ cm^{-1}, $l = 0.5$ cm, and the physical spacing of the lenses (see (4.13): the approximation $\alpha l \ll 1$ is good, as α can range from 0.1 to 0.2 cm^{-1})

$L \approx 0.5\gamma_i$ cm. The lens spacing becomes 3 m when $\gamma_i = 600$; at this energy the relative energy spread is small enough and the chromatic effects of the thin lens or AG focusing system are negligible.

If the beam emittance is smaller, then l is proportionally increased, and the energy at which is possible the transition from solenoid focusing to quadrupole focusing is decreased, as $\gamma_i = L/l$.

The 20 GeV Stanford linear accelerator, for instance, has solenoid focusing up to about 40 MeV (the first accelerating section); the transverse momentum accepted is of the order of $0.3m_0c$, so that $l \approx 1.5$ cm and $L \approx 1.5\gamma_i$ (cm), which gives $L \approx 1.2$ m for $\gamma_i = 80$.

In the Frascati linac transverse momenta up to $0.7m_0c$ are accepted; in this case $l = 0.7$ cm and $L \approx 0.7\gamma_i$ (cm). As the accelerating sections are 5 m long, the transition would have been convenient at $\gamma_i = 500/0.7 = 700$, which is the final energy of the accelerator. If quadrupole focusing had been used, it should have been installed on the accelerating sections, and in this case the advantage in cost, as compared to the solenoid, would have been partially lost.

5 *Matching systems*

It has been shown, in section 4, that focusing systems with an acceptance in the range of π (cm $\times$ m_0c) are feasible; it has also been shown that the positron source density is still quite high up to transverse momenta of the order of (5–10) m_0c. If one wants an efficient positron capture it is necessary to obtain a small positron source, and to match its emittance (large transverse momenta, small dimensions) to the linac acceptance (small transverse momenta, large dimensions).

Two different methods have been proposed and used: a matching lens, with very short focal length, has been chosen for the Frascati and General Atomic linacs, and will be used in other linacs presently under construction; the tapered solenoid, which achieves an adiabatic matching, has been employed in the 20 GeV Stanford linac.

In the following only the linear lenses will be examined; recently Wiedemann [1966] has proposed to use a short nonlinear lens, similar to the magnetic horn used at CERN for the neutrino experiment. The computations indicate that its energy acceptance is higher, by about a factor of two, and the dephasing is smaller, by about a factor of two, than the corresponding values for a linear lens with similar transverse acceptance. This system presents some technical problems for the very high currents (15 kA) that must flow into the horn thin conductor; it is, however, very promising.

5.1 Matching lens

Many authors have studied the short lens matching: Garolla [1962] examined the possibility of using a circular lens and Sund *et al.* [1964] give experimental results obtained with such a lens; Varian [1963], Haissinski [1965, 1967], Lobb [1966] have discussed the two-solenoid system. Ferlenghi, Mango [1963, 1964] have made extensive computations, obtaining the positron current at the end of a linac, within a certain emittance and a definite energy spread, taking into account the dephasing and the non-uniform distribution of the accelerating field across the iris.

An approximate evaluation of the positron intensity achievable with a matching lens and a subsequent focusing system can be made as follows.

The properties of the focusing system can be defined by the focal length λ (see (4.6), (4.7) and (4.12)), and those of the matching lens by the similar quantity Λ; the argument ω_l of the trigonometric functions in the lens transfer matrix is very close to $\pi/2$ for the positrons coming from the converter at the design energy E_c, and the physical length of the lens can be derived using eq. (4.5), with $\omega_l=\pi/2$, obtaining $L_m=\pi\gamma_c m_0 c/eB$ or $L_m=\frac{1}{2}\pi\gamma_c\Lambda$.

One can transform back through the matching lens the acceptance of the focusing system as a function of energy, and integrate over the source dimensions and the energy to find the average value of the solid angle Ω times the equivalent energy band accepted ΔE. The result allows to obtain the total positron intensity, in the assumption that the radial, angular and energy distribution are uniform.

The integrations have to be made introducing approximations that lead to results in excess, as compared to the correct ones, by as much as 15–20%.

Integrating over the positron energy from $\frac{1}{2}E_c$ to ∞, one obtains:

$$\Omega\Delta E \sim \tfrac{2}{3}\pi \frac{R^2}{(\gamma_c\lambda)^2}\frac{R^2}{\varrho^2}\,3\frac{\varrho}{R}E_c \tag{5.1}$$

with the conditions: $\varrho/R \geqslant \Lambda/\lambda$ and $\varrho<R$; when $\varrho/R<\Lambda/\lambda$, eq. (5.6) has to be used. Remembering that the solid angle accepted by the focusing system, without matching, is (see (2.3) and (2.6)):

$$\Omega = \tfrac{2}{3}\pi\frac{R^2}{(\gamma_c\lambda)^2} \tag{5.2}$$

the gain in intensity obtained with the matching is proportional to R/ϱ.

It is often convenient to introduce a cut-off in the energy band accepted, defining the limits of the band as $E_c(1-k)$ and $E_c(1+k)$; in this case we obtain:

$$\begin{aligned}\Omega\,\Delta E \sim \tfrac{2}{3}\pi\frac{R^2}{(\gamma_c\lambda)^2}\frac{R^2}{\varrho^2}E_c\Big\{&2k(1-\tfrac{4}{3}k^2)(1-A^3)+3\frac{\varrho}{R}\\ &-3\frac{\varrho^2}{R^2}\left[\left(\frac{2}{\pi}\right)^2\left(1+\frac{1}{k}\right)A+\frac{2}{\pi}\frac{R}{\varrho}\arcsin\left(\frac{2}{\pi}\frac{\varrho}{R}\left(1+\frac{1}{k}\right)\right)\right.\\ &\left.+\frac{8}{3\pi^2}\frac{A}{1+1/k}\right]+\frac{8}{\pi^2}\frac{\varrho^3}{R^3}\Big\}\end{aligned} \tag{5.3}$$

where

$$\Lambda = \left[1 - \left(\frac{2}{\pi}\frac{\varrho}{R}\left\{1 + \frac{1}{k}\right\}\right)^2\right]^{\frac{1}{2}}.$$

(5.3) is valid under the following conditions:

$$\frac{\varrho}{R} \geqslant \frac{\Lambda}{\lambda}; \quad \varrho < R; \quad k \ll 1$$
$$\frac{2}{\pi}\varrho\left(1 + \frac{1}{k}\right) < R. \tag{5.4}$$

If $2\varrho\pi^{-1}(1+k^{-1}) \geqslant R$, $\Lambda = 0$ and only the first term in parentheses in eq. (5.3) must be retained so that:

$$\Omega\,\Delta E \sim \tfrac{2}{3}\pi \frac{R^2}{(\gamma_c\lambda)^2}\frac{R^2}{\varrho^2} E_c 2k(1 - \tfrac{4}{3}k^2) \tag{5.5}$$

for

$$\frac{2}{\pi}\varrho\left(1 + \frac{1}{k}\right) \geqslant R; \quad \frac{\varrho}{R} \geqslant \frac{\Lambda}{\lambda}; \quad k \ll 1.$$

The case of point sources has been integrated by Ferlenghi and Mango [1963], who obtain:

$$\Omega\,\Delta E = 2\frac{R^2 E_c}{(\gamma_c\lambda)^2}\frac{\lambda}{\Lambda}\left[\operatorname{arctg}\frac{\Lambda}{\lambda}\operatorname{tg}\frac{\frac{1}{2}\pi}{1-k} - \operatorname{arctg}\frac{\Lambda}{\lambda}\operatorname{tg}\frac{\frac{1}{2}\pi}{1+k}\right]. \tag{5.6}$$

If a positron energy band from $\frac{1}{2}E_c$ to ∞ is considered, (5.6) becomes:

$$\Omega\,\Delta E = 2\pi\frac{R^2}{\gamma_c^2\lambda\Lambda}E_c. \tag{5.7}$$

Eqs. (5.6), (5.7) have to be used, instead of (5.1), (5.3), (5.5) whenever $\varrho/R < \Lambda/\lambda$.

From these approximate equations the following conclusions can be drawn:

a) if the source radius ϱ is fixed by other considerations (for instance heating in the converter), the matching lens should have a characteristic length Λ such that $\Lambda/\lambda = \varrho/R$; it is useless to have a smaller value for Λ (see (5.1), that does not depend on Λ as long as $\varrho/R \geqslant \Lambda/\lambda$);

b) in a low energy positron accelerator, when a small final energy spread is required, it is convenient to operate with $\Lambda/\lambda = \varrho/R \ll 1$: about half of the intensity is within an energy band of the order of $4\varrho E_c/\pi R$ (compare (5.5) and (5.1));

c) for a constant value of λ and R (which means also constant value of final emittance), the positron intensity increases linearly with R/ϱ, if one can make Λ small enough to keep $\lambda/\Lambda = R/\varrho$; this is true only as far as the source density can be considered uniform with angles. When the largest emission angles, equal to $R/\gamma_c\Lambda$ for converters outside

of the lens field, and to $(R+\varrho)/\gamma_c\Lambda$ for converters immersed in the field, become comparable with $\Theta\approx0.35$ rad (see section 3.2) or bigger, the gain in intensity tapers off, with decreasing values of Λ, and eventually vanishes when $R/\gamma_c\Lambda$ is about 0.7. Taking into account also the dephasing in the matching lens, which is proportional to $R^2/\Lambda\gamma_c$, see eq. (6.5), it turns out that there is not much to be gained in making $R/\gamma_c\Lambda$ larger than $\frac{1}{3}$ to $\frac{1}{2}$.
d) for high energy positron linacs, where the energy spread at the converter does not contribute appreciably to the final relative energy spread, and a beam with larger emittance can be used, a further increase in intensity can be obtained decreasing the value of λ, while leaving Λ at the limit set in point c) above; the maximum is obtained for $\lambda=\Lambda$, in which case the focusing system (if it is a solenoid) is achromatic, and the source radius can be made equal to the iris radius. Such an arrangement would be expensive (as it would require a very long solenoid at high field), but it could allow an increase in acceptance of about one order of magnitude, as compared to the best results so far obtained, and a much higher thermal limit for the primary beam power, owing to the larger source dimensions.

5.2 Adiabatic matching

Helm [1962b] has proposed to use, as a matching section, an adiabatically tapered solenoid, where the characteristic length changes slowly from Λ at the input end, to λ at the output, with $\Lambda<\lambda$. This system is achromatic; the 4-dimensional acceptance is given by

$$V = \tfrac{2}{3}\pi^2 R^2 \frac{R^2}{(\gamma_c\lambda)^2} \tag{5.8}$$

and the solid angle accepted at the converter:

$$\Omega = \frac{V}{\pi\varrho^2} = \tfrac{2}{3}\pi\left(\frac{R}{\varrho}\right)^2 \frac{R^2}{(\gamma_c\lambda)^2} \tag{5.9}$$

with the condition:

$$(\varrho/R)^2 = \Lambda/\lambda . \tag{5.10}$$

The condition of the adiabatic taper can be expressed as follows:

$$-\gamma_c \frac{\overline{m_0}c}{eB^2}\frac{\mathrm{d}B}{\mathrm{d}z} = \varepsilon \ll 1 . \tag{5.11}$$

Integration of (5.11) over z, for ε and γ constant, gives the axial field law; if B_0 is the maximum field at the input of the matching section, B_f the field at the end, L_m the length of the matching section, it results that:

$$\gamma_c \frac{m_0 c}{e} \left[\frac{1}{B} - \frac{1}{B_0} \right] = \varepsilon z \,, \tag{5.12}$$

$$\gamma_c \frac{m_0 c}{e} \left[\frac{1}{B_f} - \frac{1}{B_0} \right] = \varepsilon L_m \,. \tag{5.13}$$

But $2m_0c/eB_f = \lambda$ and $2m_0c/eB_0 = \Lambda$, so that

$$L_m = \frac{\gamma_c}{2\varepsilon} (\lambda - \Lambda) \quad \text{with} \quad \varepsilon \ll 1 \,. \tag{5.14}$$

One can see that, for a given value of ε (which can be of the order of 0.1–0.2) the physical length of the matching section is directly proportional to the positron energy γ_c, and to the difference $(\lambda - \Lambda)$.

The energy acceptance is limited by the breakdown of the validity condition (5.11) or (5.14) on the high energy side, and by the excessive dephasing and velocity spread on the low energy side; for purpose of comparison let us assume that a reasonable range for the energy band accepted is $(1\text{–}2)\,E_c$. We get then:

$$\Omega \, \Delta E \sim \tfrac{2}{3}\pi \frac{R^2}{(\gamma_c \lambda)^2} \left(\frac{R}{\varrho} \right)^2 (1\text{–}2) \, E_c \tag{5.15}$$

which can be written, using (5.10):

$$\Omega \, \Delta E \sim \tfrac{2}{3}\pi \frac{R^2}{(\gamma_c \lambda)^2} \frac{\lambda}{\Lambda} (1\text{–}2) \, E_c \,. \tag{5.16}$$

The transverse momenta are reduced, through the adiabatic matching section, by a factor $(\Lambda/\lambda)^{\frac{1}{2}}$, so that the maximum angle Θ_M accepted at the converter, for a converter immersed in the field, is given by (see section 6 for the maximum angle inside a solenoid):

$$\Theta_M = \frac{2R}{\gamma_c \lambda} \left(\frac{\lambda}{\Lambda} \right)^{\frac{1}{2}} . \tag{5.17}$$

Equations (5.10), (5.16) and (5.17), compared with (5.1), (5.7), show that for the same values of λ and Λ, the positron intensity is very similar in the two cases of matching lens and adiabatic matching but in this second system the source radius is larger by a factor $(\lambda/\Lambda)^{\frac{1}{2}}$, and the accepted angles are smaller by a factor $\approx 2(\Lambda/\lambda)^{\frac{1}{2}}$; both these two differences are at the advantage of the adiabatic matching, as a higher thermal limit is achieved, because of the larger source size, and a higher positron intensity is obtained, because of the smaller angles accepted at the converter.

The disadvantages of the adiabatic matching are the higher cost (the typical length of the matching section is 10–20 times that of a short lens) and, for low energy positron linacs, the broader energy spectrum accepted at the converter.

6 *Dephasing of the positrons in the matching and focusing systems*

The path length of the positrons during acceleration depends on their transverse motion; the differences in path length cause a broadening of the positron bunch, and, as a consequence, a large energy spectrum at the linac end. We shall consider the dephasing in the two cases of interest: the matching section without acceleration and the uniform solenoid with acceleration.

6.1 Dephasing in the matching section

Let us assume that in the matching section there is no acceleration; the difference in path length δL between a particle which is emitted at an angle Θ relative to the axis and a particle emitted at zero angle is:

$$\delta L = \int_0^L \mathrm{d}z \sqrt{1 + x'^2 + y'^2} - L = \int_0^L \mathrm{d}z \sqrt{1 + \Theta^2} - L \tag{6.1}$$

assuming $\Theta^2 \ll 1$:

$$\delta L \approx \int_0^L \tfrac{1}{2}\Theta^2 \,\mathrm{d}z . \tag{6.2}$$

In a uniform field solenoid Θ is a constant; if the solenoid is used as a matching lens, its length L_m will be such that $\omega = \pi/2$ (see eq. (4.5)) for the central energy γ_c; combining eq. (4.5) and (4.6) for $\omega = \pi/2$ we obtain:

$$L_\mathrm{m} = \tfrac{1}{2}\pi \Lambda \gamma_\mathrm{c} \tag{6.3}$$

and the maximum angle in the solenoid is $\Theta_\mathrm{M} = (R+\varrho)/\gamma_\mathrm{c}\Lambda$; (6.2) can then be written:

$$\delta L \approx \frac{\pi}{4} \frac{R^2}{\gamma_\mathrm{c}\Lambda}\left(1 + \frac{\varrho^2}{R^2}\right) \tag{6.4}$$

or, introducing the dephasing $\delta\varphi$ expressed in radians, if l_RF is the accelerating field wavelength, and assuming $\varrho/R = \Lambda/\lambda$:

$$\delta\varphi = \frac{2\pi}{l_\mathrm{RF}} \delta L \approx \frac{\pi^2}{2 l_\mathrm{RF}} \frac{R^2}{\gamma_\mathrm{c}\Lambda}\left(1 + \frac{\Lambda^2}{\lambda^2}\right). \tag{6.5}$$

For the tapered solenoid, one has to remember that Θ^2 scales like B (see Helm [1962b]), so that Θ can be expressed in terms of Θ_f, the angle at the end of the matching section, and $B(z)$ (eq. (5.12)); (6.2) becomes, using also eq. (5.14):

$$\delta L \approx \frac{1}{2} \int_0^{L_m} \frac{\lambda}{\Lambda} \frac{\Theta_f^2}{1 + \dfrac{2\varepsilon z}{\gamma_c \Lambda}} \mathrm{d}z = \frac{\gamma_c \lambda}{4\varepsilon} \Theta_f^2 \ln \frac{\lambda}{\Lambda}. \tag{6.6}$$

Θ_f, the maximum angle at the end of the tapered solenoid, inside the solenoid, is $\Theta_f = 2R/\lambda\gamma_c$ (the factor of two as compared to the maximum angle outside the solenoid is due to the effect of the fringing fields):

$$\delta L \approx \frac{R^2}{\gamma_c \Lambda} \frac{\Lambda}{\varepsilon \lambda} \ln \frac{\lambda}{\Lambda} \tag{6.7}$$

$$\delta\varphi = \frac{2\pi}{l_{RF}} \delta L \approx \frac{2\pi}{l_{RF}} \frac{R^2}{\gamma_c \Lambda} \frac{\Lambda}{\varepsilon \lambda} \ln \frac{\lambda}{\Lambda} \tag{6.8}$$

Comparing (6.5) and (6.8), which refer to the matching lens and to the adiabatic matching, assuming γ_c, λ and Λ equal in the two cases, which means the same beam emittance in the linac and similar values for $\Omega\Delta E$ (see eqs. (5.7), (5.16)), but with different ϱ and transverse momenta, it results that:

$$\frac{(\delta\varphi)_{m.l.}}{(\delta\varphi)_{ad.}} = \frac{\pi}{4} \frac{1 + (\Lambda/\lambda)^2}{(\Lambda/\lambda) \ln \lambda/\Lambda} \varepsilon. \tag{6.9}$$

This equation fails at the limit $\lambda/\Lambda = 1$ (no matching), because the dephasing for the matching lens remains finite, as it is assumed that a solenoid of physical length $\frac{1}{2}\pi\Lambda\gamma_c$ is always present, whereas in the case of adiabatic matching $L_m \to 0$ as $\lambda/\Lambda \to 1$. Eq. (6.9) shows that, when $\varepsilon \leqslant 0.2$, the dephasing in a matching lens is smaller than in the adiabatic matching for $0.05 < \Lambda/\lambda < 0.73$, that is in the range of Λ/λ which is commonly used; eq. (6.9) has a quite flat minimum for $\Lambda/\lambda \approx 0.2$, where the right hand side is equal to 2.42ε.

6.2 Dephasing in a uniform field solenoid with acceleration

The maximum angle inside the solenoid, at the input, is given by $\Theta_i = 2R/\gamma_c\lambda$; if there is a uniform electric field K, parallel to the linac axis, the angle Θ as a function of the longitudinal coordinate z will be:

$$\Theta = \frac{\Theta_i}{1 + \dfrac{ek}{\gamma_c m_0 c^2} z} = \frac{\Theta_i}{1 + \dfrac{\alpha}{\gamma_c} z} = \frac{2R}{\gamma_c \lambda} \frac{1}{1 + \dfrac{\alpha}{\gamma_c} z} \tag{6.10}$$

and eq. (6.2) becomes:

$$\delta L \approx \frac{1}{2}\int_0^L \left(\frac{2R}{\gamma_c \lambda}\right)^2 \left(1+\frac{\alpha z}{\gamma_c}\right)^{-2} dz = \frac{2R^2}{\alpha\gamma_c\lambda^2}\left[1-\left(1+\frac{\alpha L}{\gamma_c}\right)^{-1}\right]; \tag{6.11}$$

as $\alpha L = \gamma_f - \gamma_c$, and $\delta\varphi = (2\pi/l_{RF})\delta L$, we obtain:

$$\delta\varphi \approx \frac{4\pi}{l_{RF}} \frac{R^2}{\alpha\gamma_c\lambda^2} \frac{\gamma_f - \gamma_c}{\gamma_f} \tag{6.12}$$

which, when $\gamma_f \gg \gamma_c$, tends to the limit

$$\delta\varphi_{\lim} \approx \frac{4\pi}{l_{RF}} \frac{R^2}{\alpha\gamma_c\lambda^2}. \tag{6.13}$$

The major contribution to the dephasing comes from the first accelerating sections after the converter; it is convenient to run them at the highest possible value of accelerating field (proportional to α) in order to reduce the dephasing.

The dephasing in a thin lens focusing system with the same acceptance of a solenoid is smaller by about a factor 8 (in the limit $\gamma_f \gg \gamma_c$) if $(\alpha\lambda)^2 \ll 1$ (otherwise the factor approaches 4), because the maximum angle is a factor of two smaller ($R/\gamma_c\lambda$ instead of $2R/\gamma_c\lambda$), and because the trajectory is a sinewave instead of a helix. This advantage however cannot be made use of, as on the first sections after the converter one must have a solenoid focusing to obtain achromaticity, large acceptance and simpler construction problems (see section 4.3).

The total dephasing is the sum of two terms: one due to the matching section (eq. (5.5) or (6.8)), and the second due to the focusing system on the accelerating waveguides (eq. (6.13)): for the case of matching lens one obtains:

$$\delta\varphi_{tot} \approx \frac{2\pi}{l_{RF}} \frac{R^2}{\gamma_c\lambda} \left\{\frac{\pi}{4}\frac{\lambda}{\Lambda}\left(1+\frac{\Lambda^2}{\lambda^2}\right) + \frac{2}{\alpha\lambda}\right\} \tag{6.14}$$

or, recalling that $\mathscr{E} = \pi R^2/\lambda$:

$$\delta\varphi_{tot} \approx \frac{2\mathscr{E}}{\gamma_c l_{RF}} \left\{\frac{\pi}{4}\frac{\lambda}{\Lambda}\left(1+\frac{\Lambda^2}{\lambda^2}\right) + \frac{2}{\alpha\lambda}\right\}. \tag{6.15}$$

The positron phase spread, which has to be combined with the primary electron beam phase spread, eventually sets a limit to the positron intensity that can be accelerated within a certain energy bin.

Assuming that the relative energy spread η at the end of the accelerator is due only to the dephasing, and that $\lambda/\Lambda \ll 2/\alpha\lambda$, from (6.15) one can obtain a limit on the beam emittance $\mathscr{E}$ as a function of η:

$$\mathscr{E}_{\lim} \approx (8\eta)^{\frac{1}{4}} (\pi\gamma_c l_{RF}\alpha)^{\frac{1}{2}} R. \tag{6.16}$$

If the parameters are chosen in such a way that (6.16) is the limiting factor on $\mathscr{E}$, then the positron current in a certain energy bin at the end of the accelerator does not depend on γ_c, and is proportional to $R^2 l_{RF}$, which means that lower frequency linacs are favoured in this respect.

7 *The converter*

It has been shown that a necessary condition to obtain a high intensity positron beam is the small emittance at the source; as the angular distribution cannot be changed, one has to achieve very small beam spot size, at high primary electron beam power. The energy lost in the converter is a sizable fraction of the primary beam energy, and this gives rise to a thermal problem, which has two aspects: average heating, and peak heating (during the pulse).

Crawford and Messel [1962] give the ionization losses due to an electron at different energies on lead converter, as a function of thickness; in their computations the electrons in the shower with energies lower than 10 MeV are not traced, and their ionization losses are not considered. The energy deposited in the converter can be evaluated summing the ionization losses, the total energy of the electrons with energy <10 MeV lost in the half radiation length (r.l.) under consideration and a contribution from the photons with energy <10 MeV, which can be assumed to have an absorption mean free path of 2 r.l. in lead.

The energy deposited divided by the primary beam energy, or the fraction of primary beam power deposited in the last half radiation length (where the power deposited per unit length is greater) and in the whole converter are shown in fig. 3; the converter thickness is chosen to be 1 r.l. for 50 and 100 MeV primary beam energy; 1.5 r.l. for 200 MeV; 2 r.l. for 500 MeV and 3 r.l. for 1000 MeV.

Within the approximation of this evaluation, one can assume that the fraction of power deposited in the converter is constant for primary beam energy higher than 100 MeV, and equal to about 16%; the fraction deposited in the last half radiation length of the converter is about 6–7%. The situation is worse when the primary beam energy is lower than 100 MeV.

This result leads to the conclusion that the heating problems in the converter depend on the primary beam power, when the energy is higher than 100 MeV; as the positron density at the converter depends also essentially on the primary beam power, in the high energy limit, eq. (3.3), this turns out to be the parameter that determines both the positron intensity and the thermal limitation.

The limitation is usually due to pulse heating, more than to the average power; moving converters can handle without much difficulty very large average beam power (several hundreds of kW). The pulse heating limits the power density, and, as a consequence, the positron source emittance; the allowable power density on the converter $W/\pi\varrho^2$ can be expressed as a function of the temperature rise ΔT during.

the pulse, for the last half radiation length of the converter, as follows:

$$w = \frac{W}{\pi \varrho^2} = \frac{X_0 \delta}{2} \frac{C_{sp}}{\nu \tau} \Delta T \tag{7.1}$$

where:

X_0 is the radiation length of the converter material, in cm;
δ is the converter specific density, in g/cm^3;
C_{sp} is the specific heat of the converter material, in J/°C g;
ν is the fraction of beam power deposited in the last half radiation length of the converter;
τ is the pulse duration, in μsec.

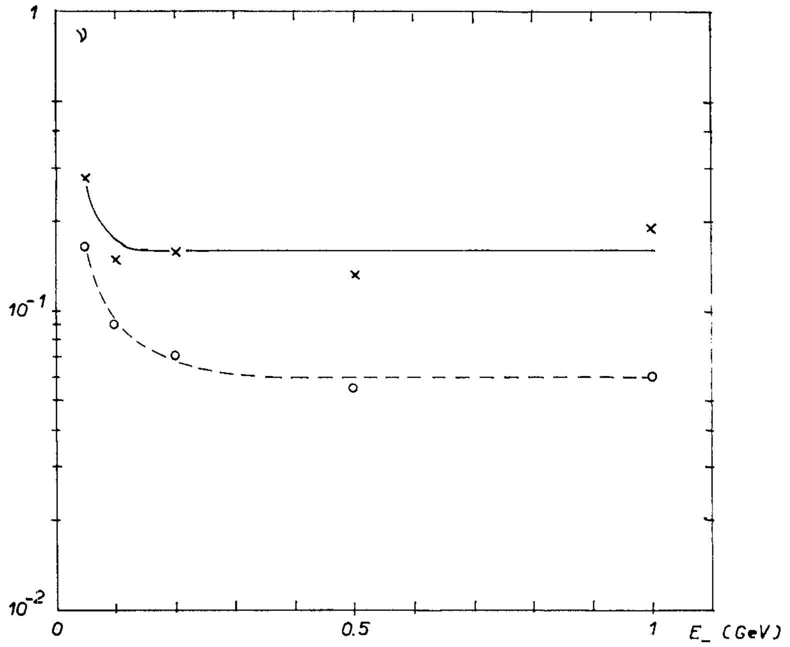

Fig. 3. Fraction of the primary electron beam power deposited in the converter. × —— Fraction deposited in the whole converter. ○ ---- Fraction deposited in the last half radiation length of the converter.

If it is assumed that $\Delta T = \frac{3}{4} T_m$, where T_m is the melting temperature in °C, and $\nu = 6\%$, for tungsten ($X_0 = 0.33$ cm) and tantalum ($X_0 = 0.38$ cm) τw is equal to about 180 MW μsec/mm^2 and for platinum ($X_0 = 0.28$ cm), $\tau w = 90$ MW μsec/mm^2. In lower Z converters, with the same assumption for ΔT and with $\nu = 10\%$, one gets higher power densities: in copper ($X_0 = 1.42$ cm) $\tau w = 200$ MW μsec/mm^2, and in titanium ($X_0 = 3.58$ cm) $\tau w = 500$ MW μsec/mm^2; $\nu \approx 10\%$ has to be considered a very rough approximation and it has been obtained from the value for lead, scaling with Z

the ionization losses and leaving unchanged the other contributions. It must be remembered that the positron radial distribution at the source is very large with low Z converters (see section 3.3): with a point-like primary beam, 75% of the positron intensity is within a circle of radius 4.5 mm for copper and 11.4 mm for titanium.

In the high energy limit for the primary beam ($E_- \approx E_{eq}$, see eq. (3.2)) the positron current density at the converter n_c can be written as:

$$n_c = 0.24 \times 10^{-3} W \quad \text{(A/MeV ster)} \tag{7.2}$$

with W in megawatts.

Using (7.1) one can rewrite eqs. (2.7) as:

$$\begin{aligned} \text{NBS:} \quad & I_+ = 0.24 \times 10^{-3} g E_c \frac{\varrho}{R} h \frac{\mathscr{E}^2}{\gamma_c^2} w \quad \text{(A)} \\ \text{WBS:} \quad & I_+ = 0.24 \times 10^{-3} (1\text{–}2)\, E_c h \frac{\mathscr{E}^2}{\gamma_c^2} w \quad \text{(A)} \end{aligned} \tag{7.3}$$

and obtain the limit on the positron current set by the pulse heating in the converter; the limit is inversely proportional to the pulse duration.

The liquid metal converter of the type proposed by Wiedemann [1966] might be developed in such a way as to withstand very high pulse temperature rise (in the present design the temperature rise is limited to $\approx 150\,°\text{C}$).

8 *Positron accelerator operation and experimental results*

The efficient operation of a positron linac as compared to that of an electron linac presents some additional problems that must be taken care of in the design.

The converter requires a very careful mechanical construction, to avoid frequent failures; the level of induced radioactivity in the converter and in the first section downward, when the primary beam average power is in the 10 to 100 kW range, is so high that the maintenance is difficult. The necessity to install it as close as possible to the accelerating section, to avoid a large dephasing of the positrons, and the presence of the high axial magnetic field (either for the short lens or for the adiabatic matching) complicates the design. It may be convenient to use, immediately after the converter, a special short accelerating section, at very high electric field, powered by an independent klystron, as a rigid unit with the converter, to be changed, with remote operation, in case of a failure.

The rf field in the positron sections must be shifted in phase by about 180° as compared to that in the primary electron beam sections (it is not exactly 180° because of the dephasing in the matching and focusing system); Haimson (private communication, 1965) has computed the bunching effect in the first section after the converter for 5–10 MeV positrons, and found that it is not negligible; the experimental results with the Frascati linac show that the highest positron intensity in 1% energy bin is achieved when the phase of the first section after the converter is delayed by an amount of the order of 20° as compared to that of the following sections. It is therefore convenient to have an independent phase control for all the positron sections and an additional control for the first positron section.

It has been experimentally observed that the electron current accelerated together with the positrons is about a factor of 2 to 3 higher; the electrons are first decelerated and then, after a phase slippage of 180°, are captured and accelerated by the rf wave; the presence of the electrons makes impossible to detect the positron beam without a magnetic or rf selection. At the 20 GeV Stanford linac an rf chopper, a few sections after the converter, sweeps the electrons out; in the Frascati linac the electrons are swept out by the magnets at the end, and total direct positron current measurements cannot be performed.

Through the same process, the positrons can be accelerated when the rf phase is set for electron acceleration; the positron intensity and energy in this case are somewhat lower: in a test made at Frascati a positron current was obtained whose intensity was about 75% of that obtained with the correct phasing, and the energy was lower by about 10 MeV.

The alignment of the focusing system with respect to the linac axis presents very stringent requirements; Helm [1963a] has considered the case for a quadrupole focusing system: a typical alignment tolerance of 0.25 mm reduces the useful linac aperture by 1 mm, if steering corrections every two quadrupole pairs are available. In the Frascati linac, where each accelerating section has a separate solenoid, soft iron plates, concentric with the linac axis within 0.1 mm, are mounted at each end of the sections; two steering fields per section compensate the transverse magnetic field component, due to the solenoid being tilted with reference to the linac axis. This system allows a complete correction of the misalignments, if each solenoid can be considered ideal and rigid. The steering fields are set by means of the electron beam at low energy (about 100 MeV) coasting through the linac on the axis.

The design of the mechanical supports must therefore allow for a more precise and stable positioning of the sections and of the focusing elements in a positron linac than in an electron linac; it is also desirable to have a sufficient number of beam position monitors along the linac that permit, when using the electron beam, an accurate compensation of the residual alignment errors.

When a narrow energy spectrum is required for the positrons, the primary electron beam must have a phase spread as small as possible; this is, as a matter of fact, the only stringent requirement on the primary beam: its emittance and energy spread are quite easily made small enough to obtain it focused on the converter within a spot 1 to 2 mm in diameter.

The experimental results on the positron current have been summarized in section 1; the positron linacs have not yet been operating for a time long enough to collect all the information required to compare the theory with the results.

As far as the Frascati linac is concerned, the best energy spectrum obtained is presented in fig. 4, and the corresponding conversion efficiency is $2.8 \times 10^{-2} e^+/e^-$ GeV %; at present the injection system is not accurately tuned, the efficiency is dropped to $2.4 \times 10^{-2} e^+/e^-$ GeV % and the energy width at the half intensity points is about 2% (as compared to the previous 1.5%). The lack of a total positron intensity monitor makes difficult a measurement of the beam profile, because of the energy-position correlation, due to the spiralling in the solenoids.

The relative beam intensity in 1% energy bin, at the energy which gives the maximum intensity without diaphragm, through a square diaphragm of side a at the end of the linac is given in fig. 5; the measurement was done at 340 MeV, with a

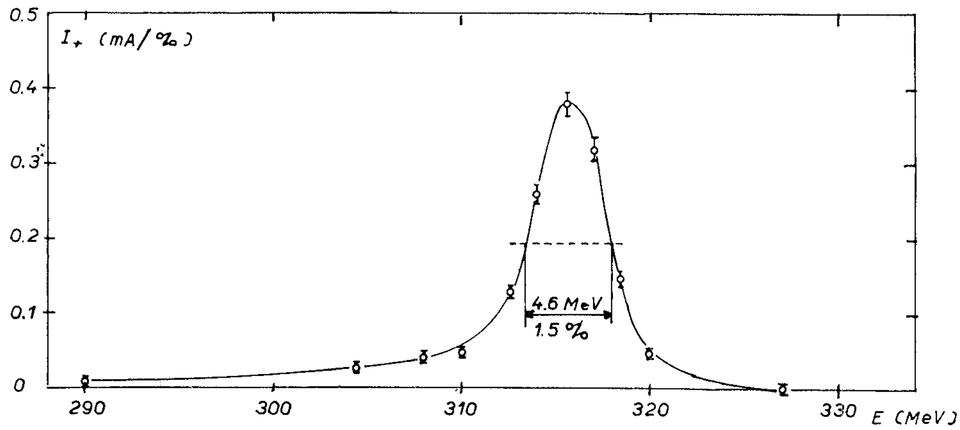

Fig. 4. Positron current distribution (mA per 1 % energy bin). Analyzing slits: 1 %. Primary electron beam: 250 mA, 80 MeV. Total positron current between 290 and 327 MeV: 0.85 mA.

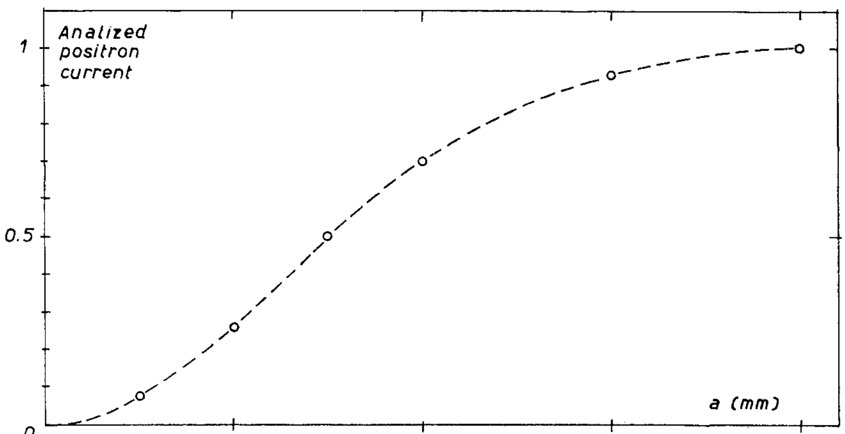

Fig. 5. Analyzed positron current transmission through a square diaphragm of side a.

positron current of 0.32 mA peak in 1% energy bin without diaphragm. It is interesting to note that in this energy bin 70% of the current is within 1 cm^2.

The transmission of the total positron current through a variable diaphragm would allow to obtain information on the useful linac aperture; a lower limit is given by the transmission through a diaphragm in one dimension, of positrons within 1% energy bin, assuming uniform positron density in the 4-dimensional phase space; the transmission factor α, if a is the diaphragm total aperture (in one dimension), is:

$$\alpha = \frac{3a}{4R}\left(1 - \frac{1}{12}\left[\frac{a}{R}\right]^2\right). \tag{8.1}$$

A measurement of α on the Frascati linac has given for R a value between 0.8 cm and 0.9 cm; it is therefore assumed that the effective value of R, for the total positron current, is ≈ 0.9 cm. The effective linac aperture takes into account the presence of the short drift spaces between the solenoids (in the Frascati linac there are 20 cm between each accelerating section), whose effect has been calculated to reduce the aperture by about 7%; the result can then be interpreted as if the useful aperture in the linac were $R \approx 1$ cm, reduced to 0.9 cm by the presence of drift spaces; the smallest iris in the accelerating sections has a radius of 1.045 cm.

The characteristics of the matching and focusing systems of the Frascati linac are the following:

L_m length of the matching lens, about 6.5 cm;
B_1 magnetic field in the matching lens, variable up to a maximum value of 16 000 G;
B_2 magnetic field in the focusing solenoids, variable up to a maximum value of 2 250 G;
R effective linac radial aperture, 0.9 cm.

The physical length of the matching section cannot be varied; from eqs. (4.5) and (4.6) one obtains, remembering that $\omega = \pi/2$:

$$\begin{cases} \Lambda = \dfrac{2m_0c}{eB_1} = \dfrac{2}{\pi}\dfrac{L_m}{\gamma_c} \\ \gamma_c = \dfrac{eB_1}{m_0c}\dfrac{L_m}{\pi} \end{cases} \tag{8.2}$$

so that when B_1 is varied, both Λ and γ_c change according to (8.2); eq. (5.1) can then be written, as a function of the magnetic fields in the solenoids B_1 and B_2:

$$\Omega\,\Delta E = \frac{\pi^2 R^2}{2L_m} m_0c^2 \frac{eB_2}{m_0c}\left[\frac{B_2}{B_1}\frac{R}{\rho}\right]. \tag{8.3}$$

The term in brackets in (8.3) cannot be larger than 1: when $B_1/B_2 = \lambda/\Lambda$ is smaller than R/ϱ, this term must be taken equal to 1, as in this case the point source approximation applies (see eq. (5.1) that becomes equal to (5.7) when $R/\varrho = \lambda/\Lambda$). For given ϱ, R and B_2 the positron intensity, according to (8.3), should then remain constant for all values of B_1 smaller than B_2R/ϱ: this apparently paradoxical result can be understood remembering eq. (8.2), which shows that γ_c is proportional to B_1, and the hypothesis of uniform energy distribution and energy independent angular distribution of the positron source. When B_1 is decreased the central energy γ_c of the positrons accepted at the converter decreases, and the total intensity in a given solid angle tends to decrease; moreover the dephasing increases, determining a larger energy spectrum of the positrons.

Taking into account these effects, for a given source size ϱ, a maximum in the analyzed positron current versus B_1 is expected; the value of B_1 at the maximum allows to evaluate the ratio R/ϱ, which, for the Frascati linac, turns out to be approximately 6–6.5, corresponding to $\varrho=0.14$–0.15 cm.

From (8.3), when $B_1=14\,500$ G (corresponding to $\gamma_c=18$) and $B_2=2250$ G, we have $\Omega\,\Delta E\approx 0.41$ ster MeV, using the numerical values for R and L_m given above. With a primary electron beam of 250 mA peak at 80 MeV the central positron density n_c gives 3.3 mA/ster MeV; a correction factor that takes into account the nonuniform angular distribution of the positrons at the converter must be applied, and has been evaluated to be about 0.8. The expected total positron current is then 1.1 mA peak.

This value has to be compared with the 0.85 mA obtained, resulting from the integral of the positron spectrum in fig. 4; the agreement is good, considering the approximations made (the nonuniform radial distribution at the source is probably responsible for a large fraction of the difference) and the uncertainty on the useful aperture.

The beam emittance is determined by R and B_2, and it turns out to be $\mathscr{E}=0.54\pi$ (cm m_0c); the corresponding geometrical emittance at 340 MeV ($\gamma=660$) is $\mathscr{E}_G=0.8\pi$ (cm mrad).

The positron energy spectrum is due to the combination of many causes: the primary beam phase spread, the energy spread accepted at the converter, the dephasing in the focusing system and the radial distribution of the accelerating electric field across the iris; in the case considered here the result of 45% of the current in 1% energy bin and about 70% of the current in 2% energy bin, is close to what one would expect.

These experimental results, while not yet adequate to allow the accurate determination of the various parameters, are sufficient to prove that the approximate approach presented in this chapter can be used in the design of high intensity positron linacs.

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