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A. MAŁECKI, *et al.*
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Investigations of Short-Range Correlations by Means of Electron Scattering from Nuclei.

A. MAŁECKI
Instytut Fizyki Jadrowej - Krakow

P. PICCHI
Laboratori Nazionali di Frascati del CNEN - Frascati

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1. – Introduction.

Electron scattering provides us with one of the most objective probes of nuclear structure. Its advantageous aspects are:

a) The electron-nucleus interaction is well understood. This is the electromagnetic interaction with nuclear charge and current densities.

b) The electromagnetic interaction is relatively weak and does not greatly disturb the structure of the target; therefore the reaction mechanism can be separated from the nuclear-structure effects.

c) For light nuclei it is possible to analyse the scattering in the convenient framework of first Born approximation.

d) For a fixed electron energy loss ω one can vary the three-momentum $\boldsymbol{q}$ with the only restriction that the four-momentum transfer should be space-like: $|\boldsymbol{q}| \geqslant \omega$. By varying $\boldsymbol{q}$ we can map out the Fourier transform of charge and current densities, whence their spatial distributions may be deduced (with real photons and for a given energy transfer ω, the only possible momentum transfer is $|\boldsymbol{q}| = \omega$ since the photon is massless).

The weakness of interaction results in small cross-sections. It makes the detection of the final electron in coincidence with final nuclear products very difficult and feasible only with sophisticated experimental techniques. Therefore, most experiments are noncoincidence ones with detection of the electron only.

In the past many electron scattering experiments have been carried out providing nuclear physicists with valuable information on nuclear structure. Indeed, our best information on nuclear sizes and charge distributions comes from elastic electron scattering. In general, the properties deduced from these experiments will be devoted to the more detailed study of the nucleus as a system of interacting particles. Of particular interest are: the single-particle binding energies and momentum distributions in different shells, the short-range correlations between nucleons, the charge distribution in the centre of the nucleus. In order to obtain this information we need experiments at high energies and the possibility of detecting final nuclear products together with the final electron. This requires very difficult experimental techniques: high energy, high intensity, high duty cycle, refined detection and preparation of the target.

The first important steps have been already done. Elastic electron scattering has been extended, at Stanford, to very large momentum transfers [1-3]. The single inelastic electron-nucleus scattering at large energy and momentum transfer has been studied at Kharkov [4]. The double-coincidence (e, e′, p) experiments performed at Frascati have investigated the shell structure of nuclei [5, 6]. The more one gets the more one wants; perhaps the extremely difficult triple-coincidence experiments—like (e, e′, pp)—will become available. First theoretical attempts in this direction have already been reported [7-9].

In this article we give a theoretical survey of electron scattering experiments dedicated to the study of short-range nuclear correlations. The relevant experiments are described in Sect. **2**. In Sect. **3** we discuss various methods of introducing the short-range correlations into the nuclear wave function. Calculations regarding the implications of correlations on elastic and inelastic electron scattering are reviewed in Sect. **4** and **5**. Finally, in Sect. **6** a summary of our analysis is given.

2. – Possibilities of experimental investigation.

We focus our attention on the dynamical properties of the nuclear ground state. The ground state can be investigated through:

1) Elastic scattering where we deal with the ground-state expectation values of charge and current operators. For spin-zero nuclei only the monopole moment of the charge density can contribute.

2) Sum rules. By means of the closure property one eliminates the final nuclear states from the analysis and is then left only with ground-state expectation values.

3) Inelastic scattering leading to final states whose properties can be easily related to the structure of the ground state. Emission of nucleons which occurs at sufficiently large energy transfers represents this case. It could be investigated in single scattering or in more complicated double and triple coincidence experiments.

We consider these three possibilities from the point of view of the short-range correlation study.

Let us recall that (in the Born approximation) the electron scattering cross-section may be divided in two terms which contain all the nuclear information and correspond, respectively, to the interaction with nuclear charge and current densities. The separation between these charge (Coulomb or longitudinal) and current (transverse) form factors can easily be performed. We do not reproduce here the complete expressions as they are well known and widely reported [10].

2·1. *Elastic scattering.* – If the nucleus has zero spin the transverse form factor in elastic scattering vanishes. The elastic cross-section is then given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big/\sigma_{\mathrm{M}} = F^2_{\mathrm{ch}} = \left|\int \mathrm{d}^3r \exp[i\boldsymbol{q}\boldsymbol{r}]\varrho_{\mathrm{p}}(\boldsymbol{r})\right|^2 \varphi^2_{E}\,, \tag{1}$$

where σ_{M} in the Mott cross-section for the point nucleus of charge number Z; φ_E being a correction due to finite proton size.

The most interesting quantity here is the one-proton density

$$\varrho_{\mathrm{p}}(r) = \frac{1}{Z}\langle\psi|\sum_{i=1}^{A} e_i\,\delta(\boldsymbol{r}_i - \boldsymbol{R} - \boldsymbol{r})|\psi\rangle\,, \tag{2}$$

where ψ is the ground-state wave function; $\boldsymbol{R} = (1/A)\sum_{j=1}^{A}\boldsymbol{r}_j$ being the nuclear c.m. vector.

Elastic electron scattering gives thus a direct (ground-state model-independent) access to the charge distribution of the nucleus under investigation. In other words, the elastic cross-section can always be explained in terms of a suitably chosen phenomenological charge density $\varrho_{ch}(r)$. The larger the range of momentum transfer accessible in experiment the more detailed is the knowledge about charge distribution that can be obtained. To know its properties in the centre of the nucleus one should go to very large q; we have the « sum rule »

$$\varrho_{ch}(0) = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}q\, q^2 F_{ch}(q)\,. \tag{3}$$

On the other hand, working at low q one determines the nuclear size only:

$$F_{ch}(q) \underset{q\to 0}{=} 1 - \tfrac{1}{6}\, q^2\langle R^2\rangle\,, \tag{4}$$

R being the r.m.s. radius of the nucleus.

To know the nuclear structure in more detail means to understand which features of the charge density (or cross-section) depend on the correlation properties of the nuclear wave function ψ, such as the shape of average nuclear potential (long-range correlations), symmetry properties of ψ (statistical correlations) and its behaviour at small inter-nucleon distances r_{jk} (short-range correlations). As we saw the elastic electron scattering measures only $\varrho_p(r)$ and the establishment of any relation « experimental cross-section $\leftrightarrow$ nuclear wave function » can be done only in a model-dependent way. In particular, to see the effect of the short-range correlations one should first choose the average nuclear potential and then introduce these correlations by modifying the wave function (constructed as a Slater determinant of single-particle states in that potential) at small inter-nucleon separations. The procedure may seen to be highly arbitrary. In fact, the overall properties of the potential can be determined from the elastic cross-section at low and medium momentum transfers, while the short-range effects are expected to show up only when working with large q. To establish true correlations one can also attempt to give a coherent explanation of various experiments which would emphasize different features of the ground-state wave function.

On the basis of these ideas numerous works related to the influence of the short-range correlations on the elastic electron scattering have been done recently. These were stimulated by the recent experiments (and in turn stimulated other experiments) at large momentum transfers which showed interesting features of the elastic form factor F_{ch} and charge density ϱ_{ch}, namely the central depression in the latter and additional minima in the former. The two things are connected to each other according to eq. (3). The central depression suggests that short-range repulsion may be responsible for the effect. The new minima in the cross-section have been found for ^{4}He, ^{6}Li, ^{16}O, ^{40}Ca and (with doubt) ^{12}C. We report various papers on the subject in Sect. 4.

2'2. *Sum rules.* – Let us proceed now to the investigation of the total cross-section obtained as a sum over a large range of energy loss ω.

The use of sum rules has many attractive features. Experiments are easy to carry out since the final state of the target nucleus need not be identified. Sum rules do not require any knowledge of the much too complicated final-state wave functions; the information obtained is only related to the nuclear ground state.

Consider, as an example, the Coulomb sum rule (including both elastic and inelastic scattering) at fixed momentum transfer:

$$Z\int \mathrm{d}\omega \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega'\,\mathrm{d}\varepsilon'}\right)_c (\sigma_M \varphi_{\text{in}}^2)^{-1} = C(q) = 1 + (Z-1)\int \mathrm{d}^3 r_{12} \exp[i\boldsymbol{q}\boldsymbol{r}_{12}]\, \varrho_{pp}(r_{12})\,; \tag{5}$$

φ_{in} is a relativistic correction to the nuclear charge operator. The nuclear structure is represented here by the two-proton density

$$\varrho_{pp}(r_{12}) = \frac{1}{Z(Z-1)} \langle\psi| \sum_{j\neq k}^{A} e_j e_k\, \delta(\boldsymbol{r}_{jk} - \boldsymbol{r}_{12})|\psi\rangle\,. \tag{6}$$

Thus the total Coulomb cross-section measures directly the spatial distribution of nucleon pairs. Hence the investigation of the sum rule is expected to give information on the two-body interaction between nucleons immersed in the nuclear matter. The effect of the repulsive core in the nucleon-nucleon interaction on the second term in eq. (5) is even more important than in the case of elastic scattering. The total effect (integrated over a large range of momentum transfer) of short-range correlations is given in both cases as follows:

$$\begin{cases} \dfrac{\int \mathrm{d}q\, q^2[F_{ch}(q) - \widetilde{F}_{ch}(q)]}{\int \mathrm{d}q\, q^2 F_{ch}(q)} = 1 - \dfrac{\widetilde{\varrho}_{ch}(0)}{\varrho_{ch}(0)} < 1\,, \\[2ex] \dfrac{\int \mathrm{d}q\, q^2[C(q) - \widetilde{C}(q)]}{\int \mathrm{d}q\, q^2[C(q) - 1]} = 1\,, \end{cases} \tag{7}$$

where the tilde denotes the « correlated » quantities, and we have put $\widetilde{\varrho}_{pp}(0) = 0$.

Unfortunately at large momentum transfers, when the properties of the second term in eq. (5) are most interesting, its value and q dependence are masked by the trivial first term which becomes then overwhelmingly large. Therefore the measurement of the proton pair distribution $\varrho_{pp}(r)$ from the sum rule is very difficult and requires a high-precision detection to be available.

The reason why the pair correlation effect is just a small ripple over the main contribution to the sum rule is quite clear [7]. The interesting short-range correlations are only due to very closely lying nucleons. Therefore, if one considers a total cross-section which is affected by all nucleon configurations the correlation effect gets canceled.

The main effect, as the first term in (5) shows, corresponds to single-nucleon excitations which are emphasized because of the kinematical combination of a large momentum transfer and a relatively small energy transfer (not too small in order to prevent closure but also not too large to exclude the possibility of meson production).

2'3. *Nucleon emission.* – The single-nucleon emission (referred to as the quasi-free or quasi-elastic scattering) is the only possible process that could occur if the nucleons were noninteracting. This follows from the validity of the one-photon exchange approximation, which means that there is only one electron-nucleon scattering, and cascade processes are eliminated. If nucleon interactions are included, one has the possibility of emitting two or more nucleons. Due to short-range correlations between the nucleons, not only the nucleon which interacts with the projectile but also some of the neighbouring nucleons may be ejected. The observation of a nucleon pair extracted from the nucleus would represent an ideal method for the study of correlations. Such an experiment (*e.g.* the (e, e′, pn) reaction) seems however hardly to be feasible because it requires a triple-coincidence measurement and the cross-section is very small.

The theoretical studies of such reactions are also far from being complete. The main difficulty which makes the interpretation of the experiments complicated is in the « final-state interactions ». The wave function of the emitted nucleon is distorted by the nuclear potential. The nucleon may knock out other nucleons as it traverses the nucleus. The process may leave the nucleus in a highly excited state whose lifetime is comparable to the ejection time of the nucleons. Finally, the interaction between the nucleons of a pair after its emission may mask interesting initial correlation inside the nucleus [11]. All these interactions in the final state should be well understood in order to have a complete description and clear-cut interpretation of the data.

Let us consider now the single-nucleon emission process which dominates in the inelastic electron scattering cross-section. It can be considered as the result of direct collisions with the individual nucleons; the struck nucleon recoils and is thus ejected from the nucleus. The cross-section for such a quasi-elastic process depends only on the initial state of the nucleon, namely its momentum distribution $W(p)$ and binding energy B in the nucleus:

$$\sigma \sim \sigma_{\mathrm{M}} |\varphi_{\mathrm{in}}|^2 W(p)\, \delta\left(\omega - \frac{(\boldsymbol{p}+\boldsymbol{q})^2}{2M} - B\right). \tag{8}$$

This gives a powerful method for investigating the structure of nuclei in their ground states. In the double-coincidence (e, e′p) experiment the single-particle binding energies and momentum distributions in various shells can be obtained. The same applies, of course, also to the (p, 2p) experiments. In this case, however, the strong absorption of the proton projectile by the nucleus prevents the investigation of the inner shells. The larger mean free path of electrons in nuclear matter makes them preferable as projectiles. Although the electron experiments are difficult to perform because of the smallness of the electromagnetic cross-sections, the inner shell structure of light nuclei like ^{12}C or ^{27}Al has recently been resolved. It was done, for the first time, by the Sanità group [5] working at the Frascati electrosynchrotron.

The first three components of the above-mentioned final-state interactions tend to blur the simpler image of quasi-elastic scattering. If the energy of the emitted nucleon is large the final-state corrections are, however, not expected to cause any serious problem in the interpretation of the process [12]. The optical-potential formalism may be used to approximate the interactions. For high-energy nucleons the real part of this potential is small and therefore the main effect of final-state interactions is the reduction of the flux of emitted nucleons. Although some modifications are necessary, the simple quasi-free picture does contain then the essential physics. It motivates the use of the quasi-elastic scattering for the study of nuclear structure.

As we have seen, the quasi-elastic process measures the single-particle properties. However, by taking advantage of the kinematic restrictions on the one-nucleon emission, one can hope to extract from certain regions of the cross-section valuable information about two-particle correlations. This applies even to ordinary scattering experiment where the emitted nucleon is not observed (in this case we integrate over $\boldsymbol{p}$ in eq. (8) and obtain the well-known quasi-elastic peak in the energy spectrum).

To understand this possibility we consider the Fermi gas model with the effective nucleon mass M^* and the Fermi momentum p_F. The nucleon are described by plane waves. We have for the quasi-free scattering

$$\omega = \frac{(\boldsymbol{p}+\boldsymbol{q})^2}{2M^*} - \frac{p^2}{2M^*}\,. \tag{9}$$

The total width of the quasi-elastic peak is then given by

$$\begin{cases} \omega'_c = \dfrac{q^2}{2M^*} - \dfrac{qp_F}{M^*} < \omega < \dfrac{qp_F}{M^*} + \dfrac{q^2}{2M^*} \equiv \omega_c & \text{if } q > 2p_F\,, \\ 0 < \omega < \omega_c & \text{if } q < 2p_F\,. \end{cases} \tag{10}$$

The most important point is that in a noncorrelated Fermi gas model there is no scattering for $\omega > \omega_c$. If $q > 2p_F$ the cross-section vanishes also for

$\omega < \omega_c$. In a real, finite nucleus the above restriction is somewhat modified. The nucleon momentum distribution is no longer a step function but has a tail that extends from p_F to $p_F + \delta p$. The spread is due to the localization of the particle in the finite nucleus; one has $\delta p \approx 1/R$, R being the nuclear radius.

We conclude, following CZYŻ and GOTTFRIED [13], that for

$$\omega > \omega_c + \frac{q}{M^*R} \tag{11}$$

the scattering must be primarily due to short-range correlations between nucleons. The correlations introduce high-momentum components into the single-particle wave function.

In order to extract the cross-section due to nucleon correlations one must, however, be able to calculate quite accurately the basic quasi-elastic contribution. Quasi-elastic cross-section depends on the momentum distribution that we choose as the zeroth-order approximation. Short-range correlations are expected to change this distribution by increasing its high-momentum part. One may fear that the increase will be spread over too large a range of momenta, giving thus a small effect. Moreover, the nucleon momenta where the effect of correlations is considerable may turn out to be so high that corresponding to them ω_c occurs in the region of intensive meson production.

The situation could be improved by performing a double-coincidence experiment according to suggestion of DE FOREST [9]. We see from eq. (9) that due to the kinematics of one-nucleon emission the nucleon in the quasi-elastic picture is restricted to the region defined by

$$p_z = \frac{\boldsymbol{p}\cdot\boldsymbol{q}}{q} = -\frac{1}{2}q + \frac{M^*\omega}{q}. \tag{12}$$

If nucleon-nucleon correlations are effective the restrictions are not so stringent. Thus the cross-section in the region $p_z \neq -q/2 + M^*\omega/q$ comes entirely from correlations. For a real nucleus the situation is somewhat modified but there is still a large domain where quasi-elastic scattering is absent. The restriction comes only from the kinematics of the one-nucleon emission process and does not depend on the momentum distribution. Therefore we do not need to make any assumptions about the momentum distribution (as for noncoincidence experiments) to obtain a cross-section which comes only from nucleon-nucleon correlations. DE FOREST [9] suggests that the double-coincidence experiment offers the best approach for studying the correlations since it is easier to perform than a triple-coincidence experiment while it still eliminates rather serious interpretation problems which are present in the noncoincidence approach.

3. – Methods of theoretical analysis.

In the preceding Section we considered possible experiments induced by electrons by which the short-range structure of nuclear interaction can be studied. Now we shall see how the short-range correlations reflect in a theoretical analysis.

We start by explaining the concept of the correlations. The long-range part of nucleon-nucleon interaction implies that each particle in the nucleus moves in a certain average field of forces coming from other nucleons. Average field correlations are responsible for the size and shape of the mass distribution. The antisymmetry condition for the nuclear wave function provides another restriction on the motion of nucleons. Long-range correlations (including the correction [14] due to proper description of the nuclear c.m. motion) together with statistical correlations constitute the zeroth-order approximation of nuclear interactions. Such a single-particle (or shell) model is expected to reproduce well the overall properties of the nucleus provided the average potential is properly chosen. Short-range forces should, however, cause some fluctuations of the average potential. The stronger the forces the greater seem to be the deviations from the simple image of nucleons moving independently in the average field. The nucleon-nucleon interaction appears, in fact, to be strongly repulsive at short separations; moreover the repulsive core is followed by a strongly attractive part of the interaction. Despite these features the single-particle concept of the nucleus turns out to be not so far from the truth, mainly due to the powerful Pauli principle. On the other hand, the existing fluctuations of nucleons around their average shell-model orbits should finally show up if the wavelength of the particles probing the nucleus is small enough.

The methods which take into account the strong interaction at short inter-nucleon distances fall into three main categories.

3'1. *Perturbation approach.* – One approach involves field-theoretic techniques of many-body theory. This is the Brueckner perturbative theory which treats strong forces by replacing the singular potential by a finite reaction matrix for scattering in nuclear medium. It has been remarkably successful in determining the properties of infinite nuclear matter. The Brueckner-type calculations for finite nuclei are very complicated and involve many uncertainties. The heavy nuclei where the size effects should be small can, however, be analysed as a sample of the nuclear matter. The two difficulties which one encounters in such a treatment, namely the necessity of improving the Born approximation, and the fact that in heavy nuclei $Z \neq \frac{1}{2}A$, are expected not to represent serious obstacles.

A few calculations concerning the effect of short-range correlations on inelastic electron scattering from heavy nuclei have been done with this approach [9, 13, 15]. Let us note that the nuclear-matter approach does not seem to be appropriate for elastic electron scattering. As the elastic cross-section for nucleons described as plane waves vanishes the whole scattering would be then ascribed to short-range effects. This conclusion follows from the first Born approximation but its improvement should not change the point.

3'2. *Model-wave-function method.* – The second approach to the short-range correlation problem is that of the model wave function. In its orthodox form the wave function is obtained from the variational principle. The parameters (and possibly the form) of the wave function are so chosen that they give a minimal energy of the system for an applied realistic two-body potential. The variational wave function is then used in calculations concerning electron scattering.

Usually a more liberal, phenomenological procedure is used. The parameters of the model wave function and its form are adjusted to reproduce the experimental scattering data. In this case the nuclear Hamiltonian is not specified although the features of the nuclear interaction are reflected to some extent in the postulated form of the wave function.

The most frequently used form of the nuclear wave function, suitable for calculations with strong repulsion at short separations, is that of Jastrow [16]

$$\tilde{\psi} = \psi_{\text{SD}} \prod_{j>k=1}^{A} g(r_{jk}) , \tag{13}$$

where ψ_{SD} is the Slater determinant wave function, constructed from the single-particle states in a nuclear shell-model potential.

The Jastrow product introduces short-range dynamical correlations besides those already present in the wave function ψ_{SD}. The correlation factor $g(r_{jk})$ has two properties

$$\begin{cases} g(r_{jk}) = 0 & \text{for} \quad r_{jk} < c , \\ g(r_{jk}) \to 0 & \text{for} \quad r_{jk} \to \infty . \end{cases} \tag{14}$$

In this way the presence of the repulsive core (of radius c) and the healing property (see below) of nuclear correlations are properly taken into account.

Because of the complexity $\left(\text{there are } \binom{A}{2} \text{ factors}\right)$ of $\tilde{\psi}$, the matrix elements involving the correlated wave function can be evaluated exactly only for the lightest nuclei [17, 18].

One can, however, try to extract from the Jastrow product the most important terms. If we put $g^2(r_{jk}) = 1 - h(r_{jk})$, $h(r_{jk})$ is different from zero just in the small portion of the nuclear volume. We may approximate [19, 20]

$$\prod_{j>k=1}^{A} g^2(r_{jk}) \approx 1 - \sum_{j>k}^{A} h(r_{jk}) . \tag{15}$$

This is a single-correlated-pair approximation. If two or more pairs were correlated, the above expression would become negative which is not very meaningful. The approximation (15) is expected to fail for heavier nuclei as the number of possible correlated pairs increases with A.

A better approximation may be obtained by taking into account all the two-particle correlations and neglecting higher correlations. Assuming that the probability for three and more particles to interact simultaneously is negligible we have

$$\prod_{j>k=1}^{A} g^2(r_{jk}) \approx 1 + \sum_{l=1}^{A/2} (-1)^l (2^l l!)^{-1} \sum_{j_1 \neq k_1 \neq \ldots j_l \neq k_l} h(j_1 k_1) \ldots h(j_l k_l) . \tag{16}$$

Unfortunately this is a complicated operator.

Let us point out that even if the $g(r_{jk})$ factor approaches unity for large internucleon separations it does not guarantee the correlations to satisfy the healing property. The Jastrow wave function, eq. (13), is, in general, not normalized to unity. After normalization the $\tilde{\psi}$ function will be different from ψ_{sp} also for large r_{jk}'s. In other words, the correlation induced in eqs. (13), (14) contains a long-range component. Therefore, the approximations (15) and (16), which are suitable for short-range correlations, may turn out to be improper.

The Jastrow product requires then a more sophisticated analysis which would assure a convergence of the approximate series. One possibility is to use the cluster expansion procedure as given by IWAMOTO and YAMADA [21]. Although this method has been developed for the nuclear matter it seems to be useful also for light nuclei [22, 23].

Another approach is represented by the unitary-operator method developed by DA PROVIDENCIA and SHAKIN [24, 25]. A slightly different formulation has been given recently by MAŁECKI [26]. In this method the $g(r_{jk})$ factor in the Jastrow product is replaced by the unitary operator $u(r_{jk})$. Because of unitarity the good normalization of the wave function is preserved and the expectation value of the one-body operator takes a very simple form:

$$\langle\tilde{\psi}|\sum_{j}^{A} O(j)|\tilde{\psi}\rangle = \langle\psi_{sp}|\sum_{j(\neq k)} \Big[\prod_{k} u^\dagger(j, k)\, O(j)\, u(j, k)\Big]|\psi_{sp}\rangle . \tag{17}$$

Hence, taking into account only two-particle correlations one obtains

$$\langle\tilde{\psi}|\sum_{j}^{A} O(j)|\tilde{\psi}\rangle \approx \langle\psi_{sp}|\sum_{j\neq k}^{A} u^\dagger(j, k)\, O(j)\, u(j, k)|\psi_{sp}\rangle - (A-2)\langle\psi_{sp}|\sum_{j}^{A} O(j)|\psi_{sp}\rangle , \tag{18}$$

where one has subtracted the contributions from those k's which are not « correlated » to a given j.

The unitary operator $u(j, k)$ is defined through its (normalization preserving) action on the two-particle states [27]:

$$
(19) \qquad \begin{cases} u(1,2)\alpha(1)\beta(2) = \dfrac{g(r_{12})}{\sqrt{N_{\alpha\beta}}}\,\alpha(1)\beta(2)\,, \\ N_{\alpha\beta} = \displaystyle\int d^3r_1\, d^3r_2\, g^2(r_{12})|\alpha(1)\beta(2)|^2\,. \end{cases}
$$

If we impose the $g(r)$ function to satisfy the conditions (14) this reflects the short-range repulsion which is then healed at large separations. In fact, because of the normalization factor $N_{\alpha\beta}$ also a long-range correlation is present in (19). Its effect on the unitary method is, however, much smaller than in the nonunitary approach (16). In turn, the convergence of the theory is significantly improved.

A further step is to remove the long-range correlation induced by normalization. It could be accomplished as follows [28]:

$$
(20) \qquad \begin{cases} u(1,2)\alpha(1)\beta(2) = \sqrt{\dfrac{g^2(r_{12}) + (M_{\alpha\beta}-1)g(r_{12})}{M_{\alpha\beta}}}\,\alpha(1)\beta(2)\,, \\ M_{\alpha\beta} = -\dfrac{\int d^3r_1 d^3r_2 |\alpha(1)\beta(2)|^2 g(1-g)}{\int d^3r_1\, d^3r_2 |\alpha(1)\beta(2)|^2(1-g)}\,. \end{cases}
$$

With the $g(r)$ function having the properties (14) this form of correlation includes three features of realistic nuclear interaction, *viz.* strong short-range repulsion, strong attraction just outside the repulsive core, and healing at large inter-nucleon distances.

The correlation function $g(r)$ contains several parameters which are related to the hard-core radius and healing distance of the nucleon-nucleon interaction. Unfortunately this relation is rather ambiguous since it is influenced by the properties of the model single-particle potential. Recently, an attempt [29] has been given to parametrize the correlation in a manner which would avoid mixing up long- and short-range correlations.

3'3. *Velocity-dependent potentials.* – Closing this Section let us just recall the third technique for describing the repulsive character of the nucleon-nucleon interaction at short distances. In contrast to the static realistic potentials containing a repulsive core, the dynamic description of the repulsion can be accomplished by introducing nonlocal or velocity-dependent potentials. The dynamic, realistic potential can be directly used in the perturbative method as well as in the model-wave-function procedure. We mention this approach because there exist some calculations regarding electron scattering which are based on the variational principle with velocity-dependent potential [30, 31].

4. – Calculations for elastic electron scattering.

The influence of the short-range nucleon-nucleon correlations on the elastic electron scattering has recently been discussed very extensively. The whole story seems to be started at the 1967 Rehovoth Conference, where the idea of MORPURGO [32] coincided with the experimental results of the Stanford group [2], showing a minimum in the ^{4}He charge form factor. The first theoretical calculations yielding this minimum have been done by CZYŻ and LESNIAK [19] in Krakow, and by TANG and HERNDON [33].

The latter work [33] applied a variational method using a hard-core, central potential. The trial wave function was a totally symmetric function written as a product of $\binom{A}{2}$ functions each depending individually on the inter-particle distance. For each of these functions the solution of the two-body Schrödinger equation is used up to a certain inter-nucleon separation and then it is connected to a variational (four-parameter) function at larger distances. It was found that with a hard-core of radius around $c = 0.6$ fm the experimental feature shown by ^{4}He, *i.e.* that the cross-section has a diffraction minimum at $q^2 \approx 10\ \text{fm}^{-2}$, can be fairly well explained.

In the work of CZYŻ and LESNIAK [19] the Jastrow method has been applied in the form of the single-correlated-pair approximation. An harmonic-oscillator potential has been chosen as the shell-model basis. It gave a good fit to the low-momentum-transfer data. Inclusion of the short-range correlations allowed to reproduce a diffraction dip in the ^{4}He form factor at the right place.

The single-correlated-pair approximation has, however, been questioned by STOVALL and VINCIGUERRA [17], who showed that the exact Jastrow calculation for ^{4}He gives a very different result from that obtained in the approximate way. Including all the terms the minimum shifts to larger momenta and the second maximum is much lowered. This kind of calculation has been also carried out by BASSEL and WILKIN [18].

For heavier nuclei the Jastrow product becomes very complicated. The exact calculation is almost impossible and an approximation must be used. CIOFI DEGLI ATTI [20] has applied the single-correlated-pair approximation in his analysis of the elastic electron scattering from the *s-p* nuclei. A harmonic oscillator was used for the shell-model potential. This potential has a distinctive advantage as it allows one to make an explicit separation of the centre-of-mass motion and the internal dynamics of the system. In particular, a handy separation of the relative and c.m. motions of a nucleon pair, needed in the two-particle correlation calculations, can be carried out. The elastic charge form factor in the oscillator model can be expressed in terms of a one-body operator since now the nuclear c.m. motion correction simply factors out. CIOFI DEGLI ATTI has, however, used the oscillator potential with different strengths for the s- and p-shell where this property is violated. Because of that an approximate centre-of-mass correction was used.

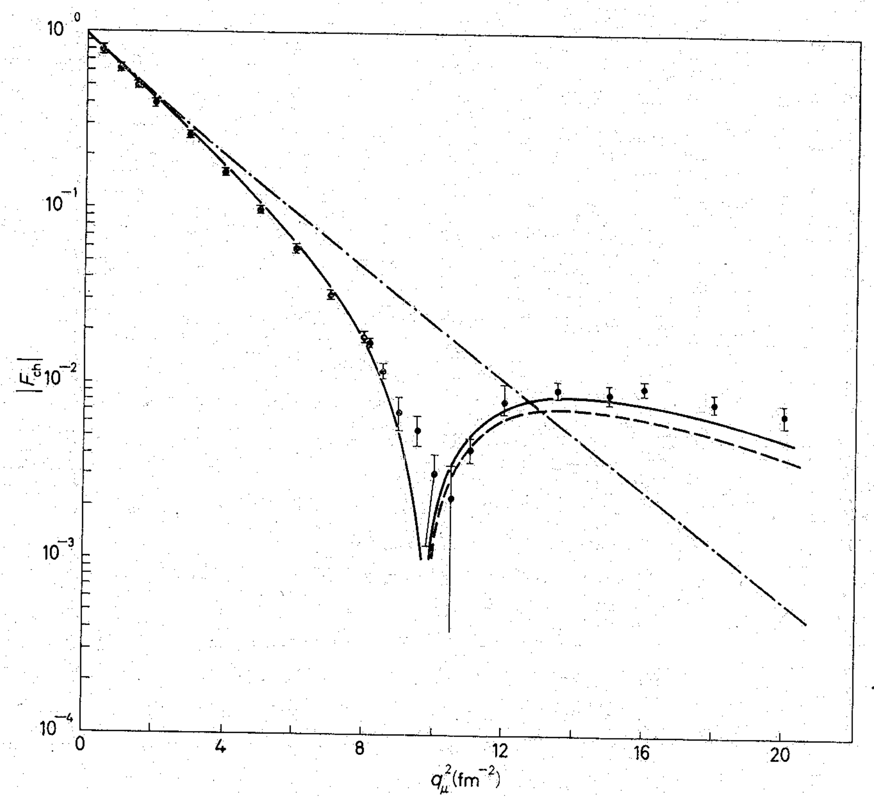

Fig. 1. – Elastic charge form factor of ^{4}He. Oscillator parameter $\alpha = 156$ MeV. Solid line: $C = 0.56$ fm, $B = 0.77$ fm^{-1}. Dashed line: $C = 0$, $B = 0.64$ fm^{-1}. Dash-dotted line: no correlations.

The validity of this correction and the application of the single-correlated-pair approximation has been criticized by MAŁEKI and PICCHI [34, 35]. They applied the unitary version of Jastrow's method. The calculations have been done for the ^{4}He, ^{6}Li, ^{12}C and ^{16}O nuclei and various forms of the correlation function $g(r)$ were checked [34-36]. The calculation of the nuclear c.m. correction has been performed by applying the Gartenhaus-Schwartz transformation [14]. It was found that in the oscillator model with different oscillator strengths for the two shells, there is a substantial difference between the exact result and the approximate c.m. correction, taken by analogy with the model of an oscillator well common to all nucleons. The approximate result turns out to be much bigger, by a factor 4÷10 for large momentum transfers [34]. On the other hand, it appears that the single-correlated-pair approximation considerably over-estimates the effect of the short-range correlations and lowers the cross-section too much [17, 35]. The two effects cancel partly each other making thus the results of both analyses, ref. [20] and [34], very similar.

The unitary-operator method has been also applied by WONG and LIN [37] in their analysis of ^{6}Li. GRYPEOS [38] has recently reported a variational calculation of the ^{4}He charge form factor in which use is made of the unitary-operator formalism.

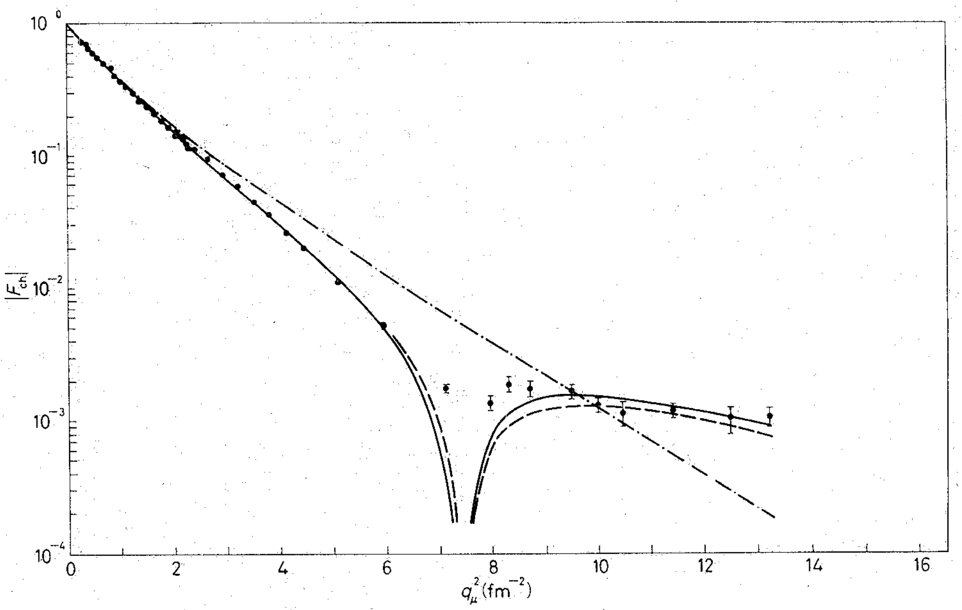

Fig. 2. – Elastic charge form factor of ^{6}Li. Oscillator parameters $\alpha_s = 131$ MeV, $\alpha_p = 99$ MeV. Solid line: $C = 0.55$ fm, $B = 0.64$ fm^{-1}. Dashed line: $C = 0$, $B = 0.57$ fm^{-1}. Dash-dotted line: no correlations.

In Figs. 1, 2, 3 and 4 we present the recent results for the s-p shell nuclei [28], obtained by using the two-particle unitary correlations in the form (20). The experimental data for the ^{4}He nucleus come from FROSCH *et al.* [2], while those for ^{6}Li, ^{12}C and ^{16}O from the very recent measurements carried out at Stanford by MCCARTHY and SICK [3].

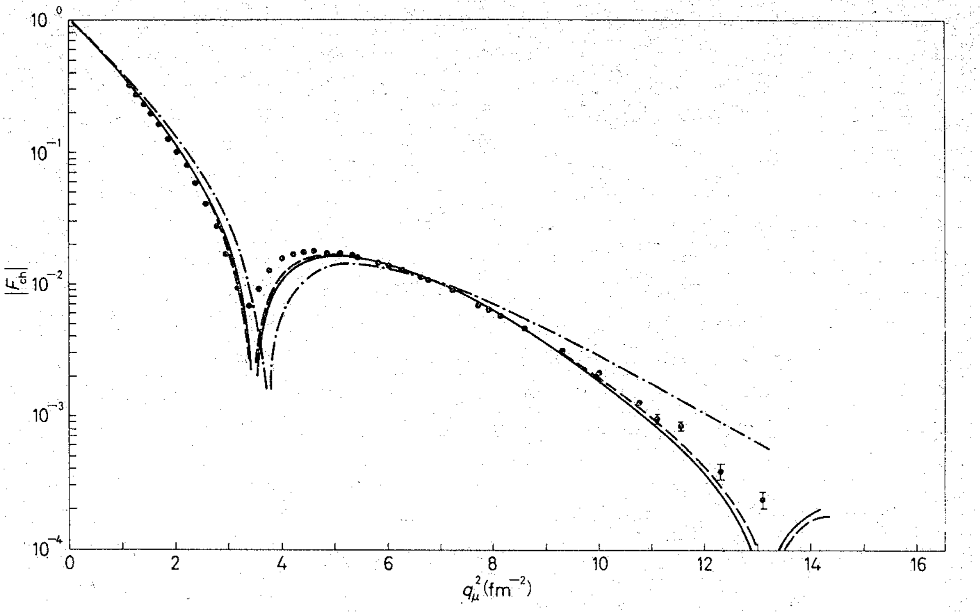

Fig. 3. – Elastic charge form factor of ^{12}C. Oscillator parameter $\alpha = 127$ MeV. Solid line: $C = 0.47$ fm, $B = 0.83$ fm^{-1}. Dashed line: $C = 0$, $B = 0.74$ fm^{-1}. Dash-dotted line: no correlations.

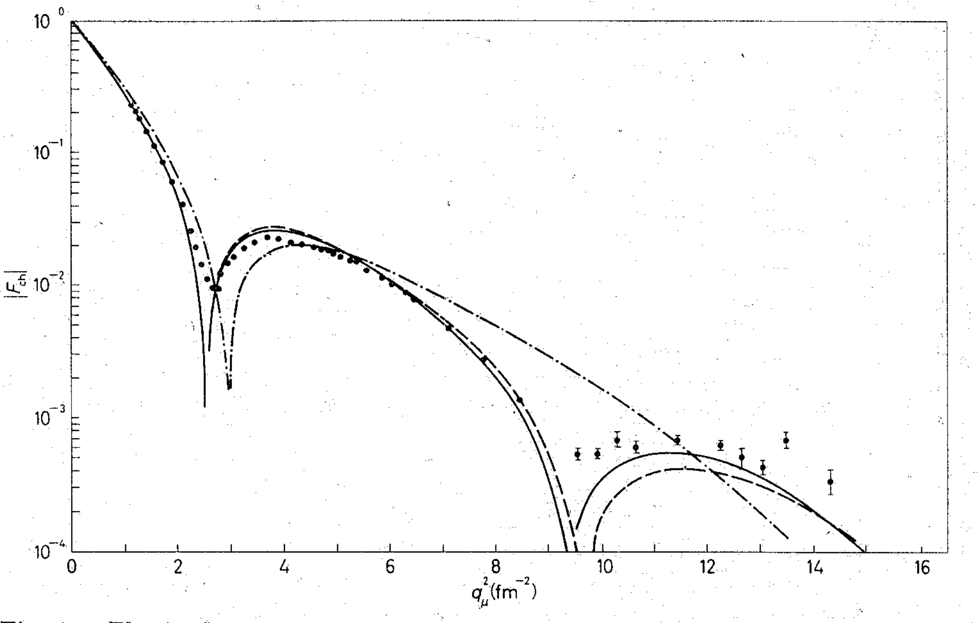

Fig. 4. – Elastic charge form factor of ^{16}O. Oscillator parameter $\alpha = 119$ MeV. Solid line: $C = 0.54$ fm, $B = 0.64$ fm^{-1}. Dashed line: $C = 0$, $B = 0.59$ fm^{-1}. Dash-dotted line: no correlations.

The correlation function has been chosen here in the form

$$g(s) = \begin{cases} 0\,, & s \leqslant c\,, \\ 1 - \exp\left[-B^2(s^2 - s_c^2)\right], & s \geqslant c\,. \end{cases} \tag{21}$$

The full lines represent the hard-core ($c \neq 0$) correlations while the dashed curves are obtained with soft-core ($c = 0$) repulsion. For comparison, the uncorrelated, oscillator shell-model form factors (dash-dotted lines) are also shown. All the three curves for a given nucleus are calculated with the same value of the oscillator parameter α.

According to Fig. 1-4 one could say that elastic electron scattering does not distinguish between the hard- and soft-core repulsion. However, the existence of the short-range repulsion is felt very strongly. Inclusion of the short-range correlations leads to the additional structure of the cross-section.

A good fit for the ^{6}Li nucleus is obtained in a model with the two different oscillator wells ($\alpha_s > \alpha_p$), which simulates that for this nucleus the p-nucleons are bound a good deal less firmly than in heavier nuclei (the alpha-deuteron structure). It turns out, however, that the model with appreciably different oscillator strengths is no more successful for ^{12}C and ^{16}O. Good fits for these nuclei are obtained in the model with the single oscillator potential well.

The figures of Małecki and Picchi are supplemented by the result of Khanna [22] for ^{40}Ca (see Fig. 5). Khanna applied the Iwamoto-Yamada expansion [21]

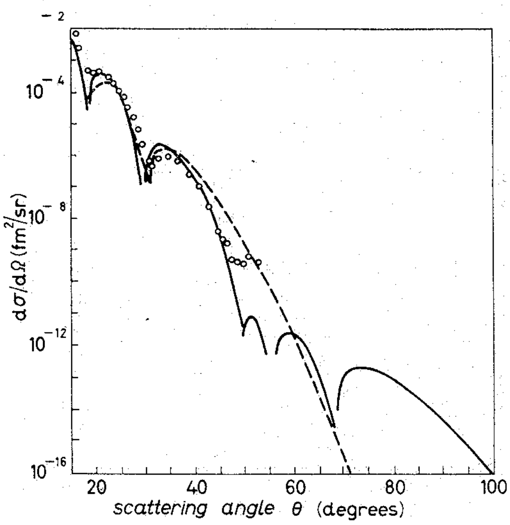

Fig. 5. – Elastic electron scattering cross-section for ^{40}Ca. Oscillator parameter $\alpha = 100$ MeV. Solid line: $C = 0$, $B = 1.39$ fm^{-1}. Dashed line: no correlations.

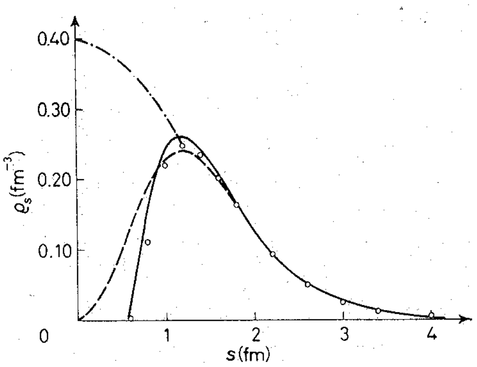

Fig. 6. – Two-particle density as a function of a mutual distance s, for ^{4}He. Same parameters as in Fig. 1. The points are obtained if the influence of other nucleons on a given pair is included.

method and attributed the third minimum found in the data of BELLICARD *et al.* [1] to the short-range correlations. However, his use of the Born approximation and the harmonic-oscillator orbitals seems to be unrealistic for ^{40}Ca.

In order to give an insight of the physical nature of the correlations we used, we present in Fig. 6 the pair distribution functions (integrated over angles) for the ^4He nucleus.

In Fig. 7 the charge density for this nucleus is presented. The central depression of the density due to the short-range repulsion here is remarkable.

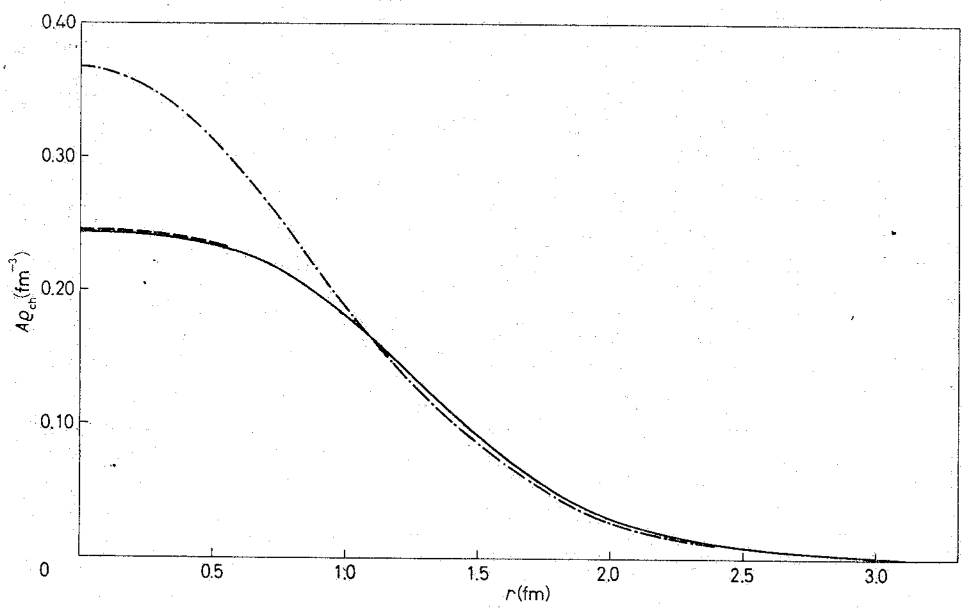

Fig. 7. – Charge density of ^4He. Same parameters as in Fig. 1.

As was stressed in Sect. 2, the evaluation of the short-range correlation influence on elastic electron scattering can be done only in a model-dependent way. The model is affected mainly by the shape of the average nuclear potential; the form of short-range correlations and the way of introducing them into the uncorrelated wave function are expected to be less important. The single-particle potential establishes the Pauli correlations among nucleons. The exclusion principle gives rise to high-momentum components in the wave function, nevertheless the Pauli correlations are nondynamical in the sense that they are not capable of transferring momentum from one nucleon to another. The main difficulty in the detection of dynamical correlations, *i.e.* those induced by inter-particle forces, is that the single-particle potential is not uniquely determined. What is attributed to short-range correlations in one model might be explained with another potential just in terms of the Pauli correlations alone.

In the analyses presented up to now the harmonic-oscillator potential has been used for the description of the uncorrelated, shell-model wave function. It is interesting to see the results also for other potentials.

GIBSON *et al.* [39] have studied the ground state of the ^{4}He nucleus using various single-particle models. Wave functions were generated from a potential whose parameters are chosen to reproduce the correct neutron separation energy. It was found that an infinitely repulsive core must be added to the central potential in order to yield the diffraction dip in the elastic form factor data. *E.g.*, the wave functions generated by the Woods-Saxon potential are much too smooth to give the minimum. The form factor obtained from a Woods-Saxon well including an infinitely repulsive central core exhibits a dip which occurs, however, at too large momentum transfers. A simple square well yields a better fit to the experimental data. If the hard core is included in this potential the fit is quite good (see Fig. 8). From that analysis

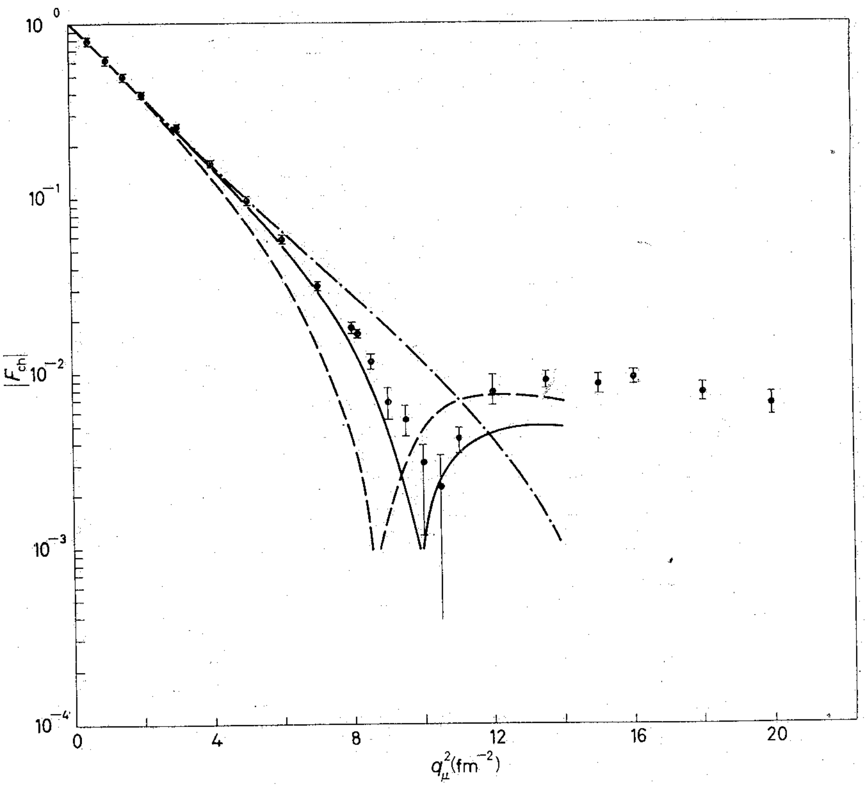

Fig. 8. – Elastic charge form factor of ^{4}He. Wave functions are generated from a square-well potential (depth V_0, radius R_0) with an infinite repulsive core (of a radius c). Full line: $C = 0.5$ fm, $V_0 = 155$ MeV, $R_0 = 1.48$ fm. Dashed line: $C = 0.7$ fm, $V_0 = 170$ MeV, $R_0 = 1.53$ fm. Dash-dotted line: $C = 0$, $V_0 = 55$ MeV, $R_0 = 2.0$ fm.

it appears quite doubtful that the diffraction minimum for ^{4}He could be correctly reproduced by means of a single-particle wave function derived from a central, well-shaped potential. The need of a short but very strongly repulsive core in the origin of the potential indicates very clearly the substantial effect of short-range nucleon-nucleon correlations.

The ^{4}He nucleus represents a particular case since its four nucleons are different from each other so that there are no correlations caused by statistics. For heavier nuclei, however, the Pauli correlations are very important. Since the statistical correlations may give rise to minima in the elastic cross-section, they compete, in a sense, with short-range correlation effects. As a consequence, attention must be paid in drawing conclusions about the dynamical correlations. This was pointed out by DONNELLY and WALKER [40], who calculated the elastic electron scattering form factor for ^{12}C and ^{16}O using the Woods-Saxon potential. In contrast to the situation of the ^{4}He nucleus, the new minima appearing in the experimental data are here explained in terms of single-particle orbitals without any short-range correlation (see Fig. 9). The Woods-Saxon well used in this calculation has a large radius and a small spread in front of the wells obtained from the optical model of nuclear reactions. As shown in Fig. 9 the

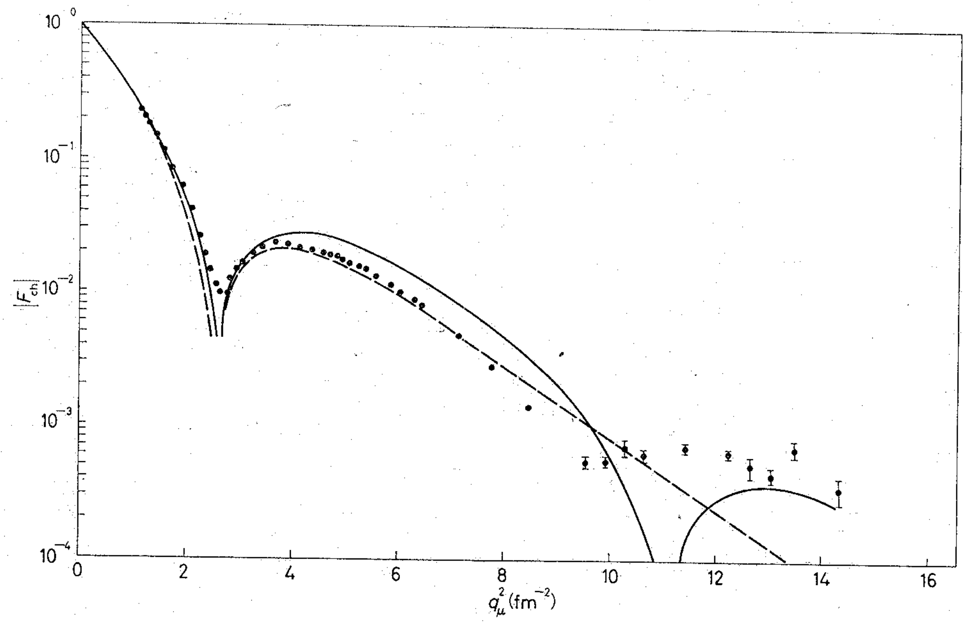

Fig. 9. – Elastic charge form factor of ^{16}O. The solid line is obtained using wave functions generated from a Woods-Saxon potential (including a Coulomb barrier) with radius $R_0 = 3.25$ fm, diffuseness $a = 0.5$ fm, spin-orbit strength $V_s = 6$ MeV and well depth $V_0 = 50.6$ MeV. The dashed curve is obtained using harmonic-oscillator wave functions with $\alpha = 111.5$ MeV.

oscillator fit is better than the Woods-Saxon one in a large range of momentum transfer except for very high q. That makes plausible the interpretation of the elastic electron scattering data with the aid of the oscillator model, corrected for the short-range repulsion between nucleons.

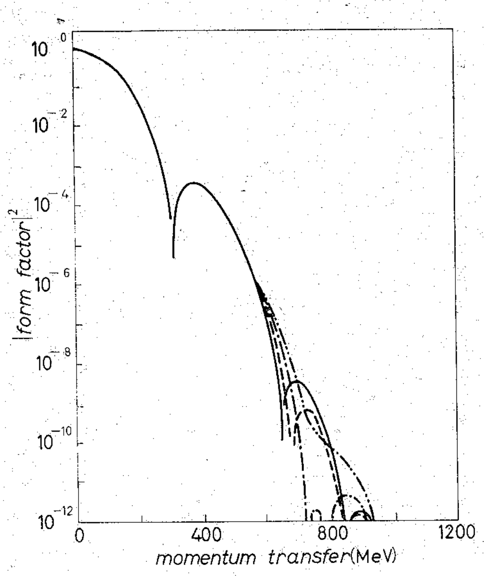

Fig. 10. – Elastic form factor of ^{16}O calculated in the single-particle (Woods-Saxon potential) model without (solid line) and with various correlation functions (broken lines). $\bar{q}$ is a mean momentum exchanged between two otherwise independent nucleons: ——— $\bar{q} = 250$ MeV/c, —·—·— $\bar{q} = 300$ MeV/c, —··—··— $\bar{q} = 350$ MeV/c.

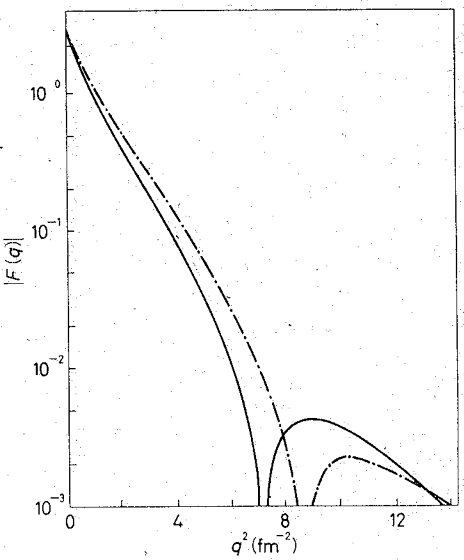

Fig. 11. – Elastic form factor of ^{6}Li, normalized to $F(0) = 3$. Wave functions are generated from a Woods-Saxon potential with radius $R_0 = 2.54$ fm, well depth $V_0 = 55.4$ MeV, diffuseness $a = 0.6$ fm and spin-orbit strength $V_s = 4.73$ MeV. Solid line: the correlation function $g(s) = [1 - \exp[B^2 s^2]]^{\frac{1}{2}}$ with $B = 1.0\,\text{fm}^{-1}$ is used. Dash-dotted line: no correlations. Centre-of-mass and proton finite-size corrections have not been included.

Even if the short-range correlations are not responsible for the second minimum in the elastic form factor of s-p shell nuclei, their presence should be visible at high momentum transfers. The correlations may improve the agreement of the Woods-Saxon predictions with the experimental data. The short-range correlation calculations in the Woods-Saxon single-particle basis have been recently performed by TUAN *et al.* [29], and by GERACE and SPARROW [23]. The representative results of both analyses are presented in Fig. 10 and 11, respectively. The uncertain point in these calculations is the neglect of any correction for the nuclear c.m. motion.

5. – Calculations for inelastic electron scattering.

A search for the short-range correlation effects in inelastic electron scattering from nuclei has a much longer history as compared to the elastic-scattering studies.

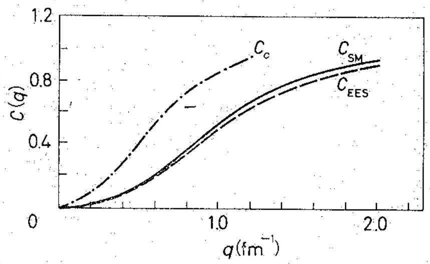

Fig. 12. – The Coulomb sum rule (including only inelastic scattering) for ^{16}O. C_c, classical perfect gas model prediction; C_{SM}, Pauli exclusion principle included; C_{EES}, Coulomb scattering computed from the two-nucleons correlation function of EDEN, EMERY and SAMPANTHAR [49], which includes the effect of a hard core of radius 0.4 fm.

It was realized long ago [41] that the total cross-section $\sigma(q)$ for scattering a weakly interacting projectile through a fixed momentum transfer q gives a direct access to the two-particle correlation function of the target. Electrons being the most readily available weakly interacting probes, they became the natural choice. Thus, quite a few theoretical analyses have been made to compute sum rules for inelastic electron-nucleus scattering [27, 42-48]. Unfortunately, it appears that the sum rules are insensitive to the details of nuclear structure, being thus a rather poor source of new information. MCVOY and VAN HOVE [47] have pointed out that the total inelastic cross-section is mostly influenced by the correlations due to the Pauli exclusion principle. More interesting, short-range nucleon-nucleon correlations have very little influence on it. This is evident from Fig. 12 where the inelastic Coulomb sum rule for ^{16}O is presented. The correlated result $C_{EES}(q)$ does not deviate from the shell-model (harmonic-oscillator) prediction $C_{SM}(q)$ by more than 5% at any value of q. This point has been already explained in Sect. 2.

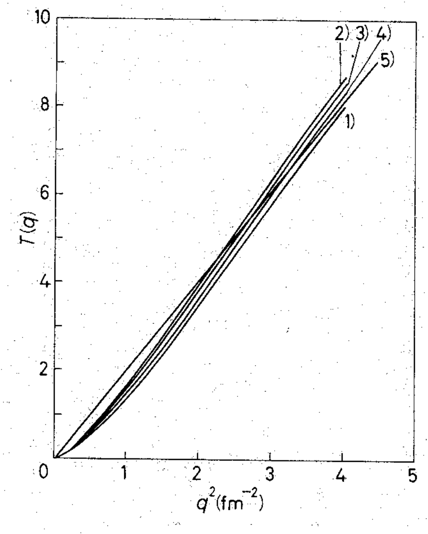

Fig. 13. – The transverse sum rule for ^{16}O. Curve 1) T_{GS}, Pauli exclusion principle included. Curves 2)-4) $\tilde{T}$ ($\gamma = 5, 8, 14$), Pauli principle with repulsive short-range correlations of various strengths: Curve 2) $\gamma = 5$, curve 3) $\gamma = 8$, curve 4) $\gamma = 14$. Curve 5) $(q^2/2M^2)\cdot(Z\mu_p^2 + N\mu_n^2)$, μ_p and μ_n are magnetic moments of proton and neutron, respectively.

The influence of the short-range correlations on the transverse sum rule has been studied by CZYŻ, LESNIAK and MAŁECKI [27] with the aid of the unitary-operator method. The effect is small as shown in Fig. 13. The dominance of the leading term, which depends on the magnetic properties of nucleons, is here so strong that nucleon-nucleon correlations do not matter very much. The significance of this dominance was discussed in ref. [27].

A small sensitivity of the total cross-section to the short-range correlations had led CZYŻ and GOTTFRIED [13] to discuss an experiment where one prescribes not only the electron momentum transfer q, but also its energy loss ω. The interest in such an experiment is due to the fact that for a free, infinitely extended Fermi gas the cross-section $\sigma(q, \omega)$ vanishes in a certain region of the q-ω plane. For an infinite nucleus the nuclear density uniquely determines the momentum distribution of free nucleons, and hence also the single-particle, « uncorrelated » cross-section. Any deviation from it can be ascribed to dynamical correlations beyond those induced by the exclusion principle at this density. For real nuclei the finite size of the system spreads out the boundaries of the above-mentioned domain, but there is still a region where the cross-section $\sigma(q, \omega)$ is strongly affected by the interactions in the nucleus. This point was already discussed in Sect. **2**.

To extract the cross-section due to dynamical correlations one first has to calculate the dominant inelastic contribution, *i.e.* quasi-elastic scattering, quite accurately. This problem has been considered by CZYŻ [50] and others [51-54].

In contrast to the total cross-section, the cross-section $\sigma(q, \omega)$ is no longer related to the conventional pair correlation function, but can be expressed through the Fourier transform of a space-time correlation function. This latter quantity is much harder to visualize than a static correlation function, and it is very difficult to compute it. CZYŻ and GOTTFRIED [13] in their study of short-range effects have applied the field-theoretic method of many-body theory. They considered a heavy nucleus as a homogeneous gas of neutrons and protons interacting via a hard core of radius c. The cross-section was computed to lowest (second) order in c. The result, for two values of q, is presented in Fig. 14. The quantity plotted is proportional to the inelastic cross-section $\sigma(q, \omega)$. It is independent of nuclear size. One must, however, recall that an infinite nucleus with $Z = \frac{1}{2}A$ was assumed. The dashed line is the prediction of the free Fermi gas. The solid line is the response given by the gas of hard spheres. The finite-size effects are expected to distort appreciably the dashed lines. The experiment proposed by CZYŻ and GOTTFRIED concerns values of ω corresponding to the solid lines, where finite-size effects are thought to be relatively unimportant.

The hard-sphere Fermi gas model has been applied also by DE FOREST [9] in his calculations of the one-nucleon emission in a double-coincidence experi-

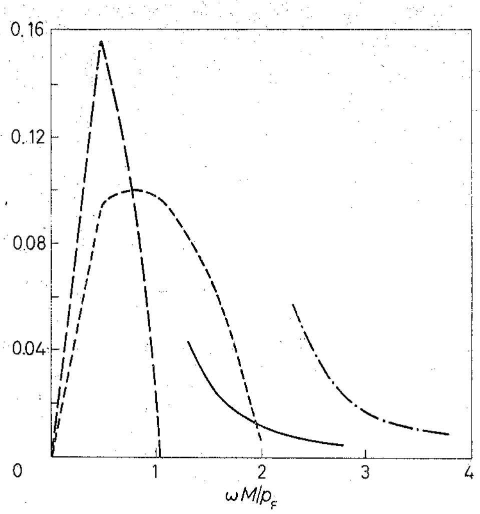

Fig. 14. – The quantity plotted is proportional to the inelastic cross-section $\sigma(q, \omega)$. The dashed lines are the predictions of the free Fermi gas (— — — $q = 0.75p_F$, - - - $q = 1.25p_F$), the solid and dash-dotted lines those of a hard-sphere gas with $Cp_F/\pi = 0.2$ (——— $q = 0.75p_F$, —·—·— $q = 1.25p_F$).

ment. The proton distribution, with respect to the direction of the momentum transfer q, is presented in Fig. 15. The result is given in a dimensionless form and is independent of nuclear size. We see that the protons tend to be emitted parallel to q. Thus this region would probably be the easiest to investigate experimentally. In Fig. 15 we also shown the region where quasi-elastic scattering occurs. By integrating over the momenta of emitted nucleons the noncoincidence cross-section can be obtained. DE FOREST [9] reports that his results are smaller than those of CZYŻ and GOTTFRIED [13] by about a factor of five.

The effects of the nuclear correlations on inelastic electron scattering were discussed also by SITENKO and SIMENOG [55]. They calculated two-nuclear correlation functions for several nuclear models, *viz.* Fermi gas model at nonzero temperature, superfluidity model and Fermi liquid model.

REINER [15] has extended the calculations of inelastic electron scattering from nuclear matter to more realistic forces. He calculated the quasi-elastic scattering from properly dressed particles with well-defined lifetimes. The contribution due to dynamical correlations between those quasi-particles turns out to be rather small.

As for the inelastic electron scattering from light nuclei the correlations were first introduced by DA PROVIDENCIA and SHAKIN [24], who computed the form factor of the 3.56 MeV level in the ^{6}Li nucleus. They pointed

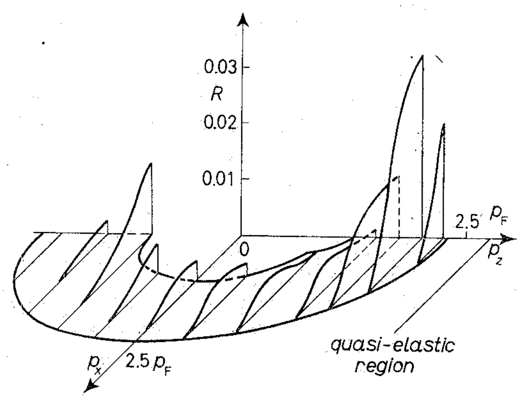

Fig. 15. – The amplitude R for detecting an emitted nucleon of momentum $\boldsymbol{p}$ in electron scattering from a hard-sphere Fermi gas. The momentum transfer $\boldsymbol{q}$ is chosen along the z-axis. $x = \omega M/p_F^2 - q^2/2p_F^2 - q/p_F$, ω being the energy transfer.

out that the short-range correlations can significantly affect the cross-section at large momentum transfers.

KHANNA [55] has investigated the effect of the correlations in the reaction $e+{}^3He \rightarrow e'+p+{}^2H$.

Recently MAŁECKI and PICCHI [28] have considered the quasi-elastic electron scattering from light nuclei. The oscillator-potential shell model with inclusion of short-range correlations was used. The parameters of the potential and those of the correlations were obtained from the elastic-scattering results. The inelastic (noncoincidence) cross-section for ${}^4He$ is presented in Fig. 16. One can see that the correlation cross-section of the type discussed by CZYŻ and GOTTFRIED is quite small. This would be in agreement with the results of DE FOREST [9]. The short-range correlations lower to a very little extent the inelastic cross-section in a large range of energy transfer ω, except for very high ω's where the correlated cross-section considerably exceeds the prediction of the shell model. Such an influence of the correlations might have been anticipated from Fig. 1. The introduction of correlations decreases the Fourier components of the wave function near the Fermi surface. The correlated momentum distribution $\tilde{W}(p)$ has a minimum at about $p = 500$ MeV [28], and then rapidly rises up. Only for $\omega > 400$ MeV the very-high-momentum components, mixed into the wave function by the nucleon-nucleon correlations, begin to play a substantial role.

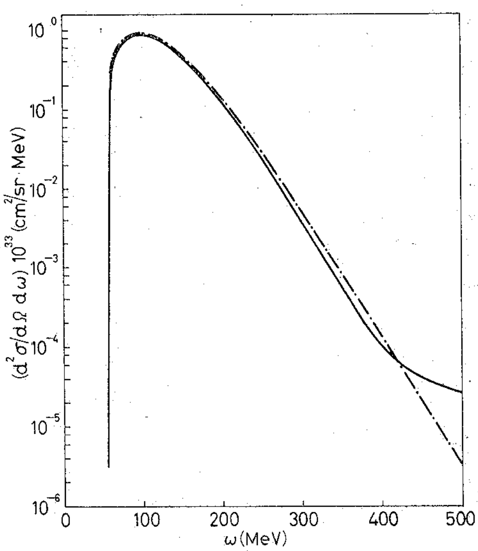

Fig. 16. – The quasi-elastic, noncoincidence cross-section for ${}^4He$. Solid line: hard-core correlations; dash-dotted line: no short-range correlations. Same parameters as in Fig. 1. Nucleon binding energy $B = 20$ MeV, initial electron energy $\varepsilon = 500$ MeV, momentum trasfer $q = 500$ MeV.

This happens far above the meson production threshold. To extract the cross-section due to the correlations one must be able to make a separation between the nuclear-physics processes and meson production. Unfortunately, very little is known about the electroproduction of mesons from nuclei. CZYŻ and WALECKA [56] have estimated the contribution from single-pion electroproduction in the region of energy transfer just above meson threshold. MONIZ [57] has extended these crude estimates up through the first nucleon resonance. Nevertheless, with to-day's understanding of the subject, the consideration of the region where electron's energy loss equals a few hundred MeV seems to be of academic interest. The cross-section is here quite small. Moreover, the higher relativistic corrections to the electron-nucleus interaction may turn out to be important.

SRIVASTAVA [7] and YU [8] have suggested as a direct way of measuring the short-range correlation structure of nuclei an experiment in which a pair of closely separated nucleons are sent forth by an inelastically scattered projectile. The angular and energy correlation of the two ejected nucleons, measured in coincidence with the scattered particle, gives direct access to the Fourier transform of the target's pair correlation function. Actually, this powerful result is partially invalidated by the final-state interactions of the nucleons with each other and with the residual nucleus. It was estimated that these effects are most relevant only for low relative momenta. For high relative momenta, where short-range effects are important, the correlation spectrum is relatively unaffected. For a quantitative measurement of the correlations, however, a more sophisticated treatment of the three-body vertex is required. The triple-coincidence cross-sections is very small, hardly to be seen with the present experimental technique. Therefore nucleons, rather than electrons, seem to be desirable as projectiles.

6. – Summary and conclusions.

This review article might have been entitled « How do the short-range nucleon-nucleon correlations influence the electron scattering cross-section? ». We have seen that correlations lead to an additional structure of the elastic cross-section and to a central depression of the nuclear charge density. Due to dynamical correlations the nucleon emission cross-section does not vanish in certain regions where the response for noninteracting nucleons would be zero.

Unfortunately, from the analysis presented no well-defined conclusion follows as for a more interesting, inverted question: « How could the short-range correlations be measured in electron scattering? ». The experiments which bear upon the two-particle correlations most directly, *i.e.* inelastic sum rules and two-nucleon emission, either are only very weakly influenced by the dynamical correlations, or contain too many theoretical and experimental difficulties. Some hope may be set on elastic and quasi-free scattering although they constitute a model-dependent procedure for the determination of nuclear correlations. In elastic electron scattering the effects of the short-range correlations are dominant in the high-momentum-transfer region. It is a remarkable advantage of elastic process that the correlation effects are here unmasked by problems of reaction mechanism and final-state interactions. Worthy of investigation is the one-nucleon detection cross-section outside the region of quasi-elastic scattering; it is strongly affected by nucleon-nucleon interactions. Noncoincidence cross-section seems to be unsuitable with to-day's understanding of electroproduction processes.

The calculation we present above, regarding recent elastic-scattering experiments, make their interpretation in terms of nucleon-nucleon correlations very plausible. Further studies, both experimental and theoretical, are required to establish the exact nature of short-range correlations. On the experimental side one would welcome more elastic (and discrete inelastic) scattering measurements at high momentum transfers and a complete study of a double-coincidence experiment. Theoretically, it would be desirable to have a better estimate of final-state interactions and meson production. A very promising approach is to compare different processes on the same target, so that various high-energy reactions can be studied in detail. A coherent investigation of different processes induced by electrons alone—elastic and discrete inelastic scattering, noncoincidence, double and eventually triple-coincidence experiments—would be also of great interest. A special stress should be laid on simplest nuclei where the effects of dynamical correlations among the nucleons are not masked by the Pauli correlations. In particular, a deuteron target is very worthy of detailed investigation.

* * *

On of us (P. P.) takes pleasure to thank Dr. R. BARBINI for many valuable discussions.

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