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H. C. Dehne and M. Preger: NUMERICAL CALCULATIONS ON THE SMALL ANGLE SCATTERING LUMINOSITY MONITOR USED AT ADONE. -

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I. - INTRODUCTION. -

During the luminosity measurements performed at Adone it appeared necessary to study in more detail the dependence on the source displacements of the small angle e^+e^- scattering cross section, integrated over the solid angle subtended by the apparatus described in ref. (1, 2, 3, 4, 5).

We quote here some numerical calculations, the results of which can be used to determine the sensitivity of the luminosity measurement to displacements of the source, and to check the internal consistency of the measurements.

Although the numerical values refer to the particular apparatus we are studying, the results as a whole, as well as what has already been discussed in the above quoted papers, give an idea of the absolute precision obtainable and of the problems of a precise luminosity measurement by small angle e^+e^- scattering.

For experimental details concerning the luminosity measurement we refer to the work of the "μ-π" group[6].

(x) - On leave of absence from Deutsches Elektronen Synchrotron, Desy, Hamburg (Germany).

II. - DESCRIPTION OF THE APPARATUS. -

The geometrical position of the four telescopes, each consisting of counters P, G, S is schematically shown in Fig. 1.

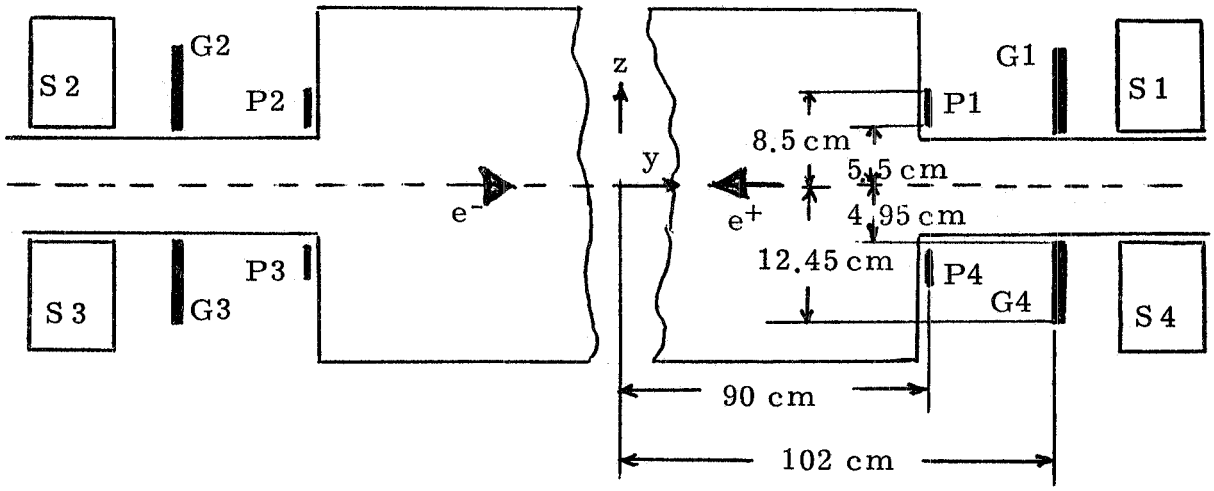

FIG. 1 - Geometrical situation of the four counter telescopes.

The P counters are plastic scintillators with air light guides, 1 mm thick, $3 \times 9\ cm^2$. Their lower edge is assumed to be at 5.5 cm from the axis of the doughnut. Their distance from the centre of the straight section is 90 cm.

The G counters are plastic scintillators with air light guides, 1 cm thick, $7.5 \times 12.5\ cm^2$. Their lower edge is assumed to be at 4.95 cm from the axis of the doughnut. Their distance from the centre of the straight section is 102 cm. All counters are radially centered with respect to the axis of the doughnut.

The S counters are "sandwiches" of scintillator and lead; they are used to reject low energy background. They cover the whole solid angle subtended by the other counters.

Our aim is to find a suitable combination of coincidences, in order to get the best compensation of the counting rate, with respect to possible displacements of the source. Three kinds of coincidences are used.

The fivefold coincidences are defined as follows :

(1)
$$Q1 = (P1.G1.S1.G3.S3) \qquad Q2 = (P2.G2.S2.G4.S4)$$
$$Q3 = (P3.G3.S3.G1.S1) \qquad Q4 = (P4.G4.S4.G2.S2)$$

The sixfold coincidences are defined as :

(2)
$$S_{13} = (P1.G1.S1.P3.G3.S3) \qquad S_{24} = (P2.G2.S2.P4.G4.S4)$$

The OR's are defined as :

(3) $$OR_{13} = (Q1 + Q3) \qquad OR_{24} = (Q2 + Q4).$$

From these definitions it is obvious that the counting rates of the coincidences Q, S and OR are related to each other; for the branch 1-3 (counters labeled by numbers 1 and 3, see Fig. 1) we have:

(4) $$OR_{13} = Q1 + Q3 - S_{13}$$

and for the branch 2-4 :

(5) $$OR_{24} = Q2 + Q4 - S_{24}.$$

If we call $\dot{N}$ the number of counts per hour, we can express the luminosity by the formula:

(6) $$L = \dot{N}_i K_i E^2 \times 10^{29} \ cm^{-2}h^{-1} \qquad (E \text{ in GeV})$$

where the coefficient K_i represents the inverse of the cross section, multiplied by E^2 and integrated over the solid angle of the apparatus, and the subscript i refers to the three types of coincidences Q, S, OR; K_q applies to the counting rate of the fivefold coincidences, or to the average of the two fivefold coincidences of the same branch, or to the average of the four fivefold coincidences, K_{or} applies to the case of the OR of the two fivefold coincidences of the same branch, or of the average of OR_{13} and OR_{24}, and K_s to that of the sixfold coincidence respectively.

K_i was obtained integrating the small angle e^+e^- scattering cross section, multiplied by E^2 :

(7) $$E^2 \frac{d\sigma}{d\Omega} = (2r_o m_e c^2)^2 \frac{1}{\theta^4} \qquad cm^2 MeV^2$$

($r_o = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13}$ cm (classical electron radius); E = = energy of one beam; θ = angle of scattering)

over the solid angle subtended by the apparatus, and taking into account the source dimensions.

The value obtained depends on the length of the source. With head-on collision, the time average of the overlap of two bunches, which is the source of the events, has a gaussian distribution with standard deviation σ :

(8) $$\sigma = \frac{\sigma_b}{\sqrt{2}}$$

where σ_b is the standard deviation of the azimuthal distribution of the particles in a bunch.

Possible changes in the closed orbit or in the voltage or frequency of the RF cavities or instabilities, will cause displacements of the interaction region from the centre of the experimental straight sections. We have calculated the effect of each possible displacement on the counting rate of Q, S, OR, namely the variation

$$(9) \qquad \varepsilon(\vec{\delta}) = \frac{\Delta \dot{N}}{\dot{N}}(\vec{\delta}) = \frac{\dot{N}(\vec{\delta}) - \dot{N}(0)}{\dot{N}(0)}$$

where $\vec{\delta}$ is the displacement of the source. ε is related to a variation of K by the following expression:

$$(10) \qquad \frac{\Delta K}{K} = \frac{K(\vec{\delta}) - K(0)}{K(0)} = -\frac{\varepsilon}{1+\varepsilon} .$$

For small values of ε we have:

$$(11) \qquad \frac{\Delta K}{K} \simeq -\varepsilon .$$

The behaviour of K_q, K_{or} and K_s when the source is centered with respect to the doughnut is shown, as a function of σ, in Fig. 2.

III. - DISPLACEMENTS OF THE INTERACTION REGION. -

a) Radial displacements of the source.

Radial displacements of the source are possible in case of variations of the closed orbit or of the frequency of the RF. In the centre of the experimental straight sections, the displacement with RF frequency is about 4 mm/kHz.

In Fig. 3, 4, 5 we show the sensitivity of the counting rate of the sixfold, fivefold and OR coincidences to radial displacements of the source. Because of the symmetry of the apparatus, the variation of the average of the two fivefold coincidences of the same branch and that of a single fivefold coincidence must be the same.

b) Vertical displacements of the source.

Vertical displacements of the source are possible in case of variations of the closed orbit. Fig. 6 shows the variation of the counting rate for the sixfold coincidences: this variation is the same for positive

and negative displacements (by definition, a positive displacement of the source reduces the distance between the centre of the straight section and the telescopes 1 and 2, increasing the distance from telescopes 3 and 4, see Fig. 1).

Fig. 7 shows the variation of the counting rate of the fivefold coincidence Q1 (Q2).

It is interesting to consider the behaviour of the average of the two fivefold coincidences: if we take for example the branch 1-3, because of the symmetry of the telescopes with respect to the centre of the source, a positive displacement gives on Q1 (Q2) the same effect as the one given on Q3 (Q4) by a negative displacement of the same absolute value. In order to obtain the effect on the average of the two fivefold coincidences of the same branch, we must therefore sum the effects corresponding to a displacement $+\Delta z$ and a displacement $-\Delta z$, that is $[\mathcal{E}(\Delta z)+\mathcal{E}(-\Delta z)]/2$; the behaviour of this expression is shown in Fig. 8.

The effect on the OR of the two fivefold coincidences of the same branch is shown in Fig. 9. Here variations are obviously the same for positive and negative displacements.

c) Azimuthal displacements of the source.

Azimuthal displacements of the source can occur if the voltages of the two RF cavities are not equal or if their phases are not perfectly tuned.

The behaviour of the counting rate of the sixfold coincidences with azimuthal displacements is shown in Fig. 10 and is obviously the same for positive and negative displacements (we define as positive a displacement which decreases the distance between the source and telescopes 1 and 4, see Fig. 1).

The behaviour of the fivefold coincidence Q3 (Q2) is shown in Fig. 11. In this case a positive displacement gives on one of the two fivefold coincidences of the same branch the same effect given on the other by a negative displacement of the same absolute value. The variation of the average of the two fivefold coincidences of the same branch and the variation of their OR are therefore the same for positive and negative displacements.

The behaviour of the average ($\bar{Q}$) and of the OR of the fivefold coincidences is shown in Fig. 12 and 13.

d) Angles in the vertical plane.

It is possible to have angles in the vertical plane, due to deformations of the closed orbit. Defining as positive an angle if the path of the beams approaches the line connecting telescopes 1 and 3, a positive

angle increases the counting rate of both fivefold coincidences Q1 and Q3; the same angle decreases the counting rate of the other two fivefold coincidences Q2 and Q4. The effect on Q2 and Q4 is therefore equivalent to that of a negative angle on Q1 and Q3.

The behaviour of the variation $\Delta \dot{N}/\dot{N}$ for the sixfold, fivefold Q1 and Q3, and OR_{13} coincidences is shown in Fig. 14, 15 and 16 respectively.

Fig. 17 shows the behaviour of the average of the four fivefold coincidences, which is obviously symmetrical with respect to positive and negative angles.

In Fig. 18 we show the behaviour of the average of OR_{13} and OR_{24}.

e) Angles in the radial plane.

The effect of angles in the radial plane, due to deformations of the closed orbit, on the counting rate is negligible for all coincidences considered. Its order of magnitude is 10^{-3} for an angle of 2-3 mrad.

IV. - SIMULTANEOUS DISPLACEMENTS. -

In the case of the simultaneous presence of more than one of the considered displacements, the following rule applies with good approximation. If we put:

$$\vec{\delta} = \sum \vec{\delta}_i \qquad i = 1, \ldots, n \tag{12}$$

where $\vec{\delta}_i$ is one of the considered displacements, we have:

$$\dot{N}(\vec{\delta}) = \dot{N}(0) \left[(1 + \varepsilon_1) \ldots\ldots (1 + \varepsilon_n)\right] ; \tag{13}$$

that is the simultaneous displacements can be considered as independent.

This rule, however, fails in the case of the combination between a vertical and an azimuthal displacement. The variation corresponding to this situation has been calculated, and is shown in Fig. 19, 20, 21, 22 for the sixfold coincidences, for σ = 3, 10, 20, 30 cm respectively. In these graphs the vertical displacements Δz are positive: the graphs corresponding to negative Δz are obtained by reflection with respect to the vertical axis. The graphs refer to S_{24}; they are valid also for S_{13} if the sign of Δz is changed.

In Fig. 23, 24, 25, 26 we show the behaviour of the fivefold coincidence Q2; the variation for the other fivefold coincidences is ob-

tained from the same graphs as follows: for Q1 by changing the sign of Δy only, for Q3 of Δz only, for Q4 of both Δy and Δz.

The effect of this particular combination of displacements must be read off graphs 19-26. Once obtained, it can be made to appear as one of the terms of (13).

Rule (13) has been checked in several cases: the difference between the variation, obtained by integration, and that obtained by (13) is of the order of 1-2 % in case of large variations (> 20-30 %) and of the order of 10 % in case of small variations (< 5-10 %).

V. - INTERNAL CONSISTENCY. -

It is convenient to define a fiducial volume within which we are sure the source must be located - e. g. for Adone we have here assumed :

$$|\Delta x| \leq 0.5 \text{ cm} \; ; \; |\Delta z| \leq 0.5 \text{ cm} \; ; \; |\Delta y| \leq 5 \text{ cm} \qquad |\Delta\theta| \leq 0.5 \text{ mrad} \tag{14}$$

If the length of the source is known, a useful check of internal consistency of the luminosity measurement is the ratio of the fivefold coincidences to their OR. Fig. 27 shows the expected ratio $\dot{N}_{or}/\dot{N}_q$ in ideal conditions and when the four maximum displacements within the fourdimensional fiducial volume of Adone are assumed. The ratio $\dot{N}_{or}/\dot{N}_q$ in case of an angle of 0. 5 mrad in the vertical plane is practically identical with that in ideal conditions.

VI. - COUNTERS POSITIONING. -

We have studied the sensitivity of K_q and K_{or} to the vertical positioning of the G counters. Fig. 28 and 29 show the behaviour of K_q and K_{or}, as a function of θ, for different distances d between the lower edge of the G counters and the axis of the beam.

The variation of the counting rate with the distance d between the lower edge of the P counters and the axis of the beam is, with very good approximation, the same as that, produced by an angle in the vertical plane, given by the relation $\Delta d = \ell\theta$, where ℓ is the distance of counters P from the centre of the straight section (see Fig. 14 and 15).

VII. - CONCLUSIONS. -

- The average of the four fivefold coincidences achieves a good compensation of the effects of displacement for all source lengths.

- The average of OR_{13} and OR_{24} achieves a good compensation for source lengths larger than about 10 cm; for $\sigma < 5$ cm the compensation does not work, because of the strong sensitivity of the sixfold coincidences to source displacements.

- The comparison of the average of the four fivefold coincidences with the average of OR_{13} and OR_{24} shows that in the region of $\sigma > 10$ cm the latter is less sensitive than the former.

- Table I shows the effect on the counting rate of the average of the two fivefold coincidences, of the average of the four fivefold coincidences, of the OR of the two fivefold coincidences of the same branch, and of the average of OR_{13} and OR_{24} of the maximum assumed displacements in Adone. It appears that in presence of three simultaneous displacements (radial, vertical and azimuthal), and for $\sigma > 10$ cm, the average of OR_{13} and OR_{24} gives the best compensation. The maximum effect on this average is of about - 7 % in the region of $\sigma \sim 15$ cm.

- It should be noted that, for σ larger than $\sim$10 cm, radial, vertical and azimuthal displacements decrease the counting rate, both of the average of OR_{13} and OR_{24} and of the average of the four fivefold coincidences. Vertical angular displacements give a negligible increase of the counting rate in both cases.

ACKNOWLEDGEMENTS. -

We are indebted to Prof. F. Amman and Dr. S. Tazzari for helpful discussion during the present work. One of us (H. C. D.) whishes to thank Prof. F. Amman and the Adone group for their kind hospitality.

TABLE I - Variation of counting rates in per cent.

Displacements of source	σ (cm)	$(\frac{\Delta \dot{N}}{\dot{N}})_{Q1,Q3}$	$(\frac{\Delta \dot{N}}{\dot{N}})_{Q1,Q2,Q3,Q4}$	$(\frac{\Delta \dot{N}}{\dot{N}})_{OR13}$	$(\frac{\Delta \dot{N}}{\dot{N}})_{OR13,OR24}$
radial $\pm$ 0.5 cm	10 20 30	- 1.52 - 1.45 - 1.41	- 1.52 - 1.45 - 1.41	- 0.43 - 0.74 - 0.77	- 0.43 - 0.74 - 0.77
vertical $\pm$ 0.5 cm	10 20 30	- 5.4 - 4.0 - 2.5	- 5.4 - 4.0 - 2.5	- 1.8 - 3.4 - 2.1	- 1.8 - 3.4 - 2.1
azimuthal $\pm$ 5 cm	10 20 30	- 3.0 - 2.0 - 1.1	- 3.0 - 2.0 - 1.1	- 0.9 - 1.4 - 0.9	- 0.9 - 1.4 - 0.9
angular $\pm$ 0.5 mrad in vertical	10 20 30	+ 2.6 + 2.8 + 3.0	+ 0.04 + 0.04 + 0.05	+ 2.4 + 2.7 + 2.9	+ 0.03 + 0.04 + 0.05
vertical + 0.5 cm and azimuthal + 5 cm	10 20 30	- 16.3 - 10.8 - 6.3	- 8.4 - 5.7 - 3.4	- 8.5 - 9.6 - 5.8	- 4.1 - 4.9 - 3.0
vertical + 0.5 cm radial + 0.5 cm azimuthal + 5 cm	10 20 30	- 17.6 - 12.2 - 7.7	- 9.9 - 7.1 - 4.8	- 9.3 - 10.5 - 6.8	- 4.9 - 5.9 - 4.0

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(2) - G. Barbiellini, B. Borgia, M. Conversi and R. Santonico, A monitoring system to measure the absolute luminosity of a machine operating with e^+-e^- colliding beams, Atti dell'Accademia Nazionale dei Lincei, Serie VIII, Vol. XLIV, Fasc. 2 (1968).

(3) - G. Barbiellini, B. Borgia, M. Conversi, G. Iorio e R. Santonico, Un dispositivo per la misura della luminosità assoluta di macchine a fasci collidenti di elettroni, Istituto di Fisica, Roma, Nota interna n. 55 (1968).

(4) - R. Santonico, Thesis, Istituto di Fisica, Roma.

(5) - M. Preger, Thesis, Istituto di Fisica, Roma.

(6) - G. Barbiellini et al., to be published.

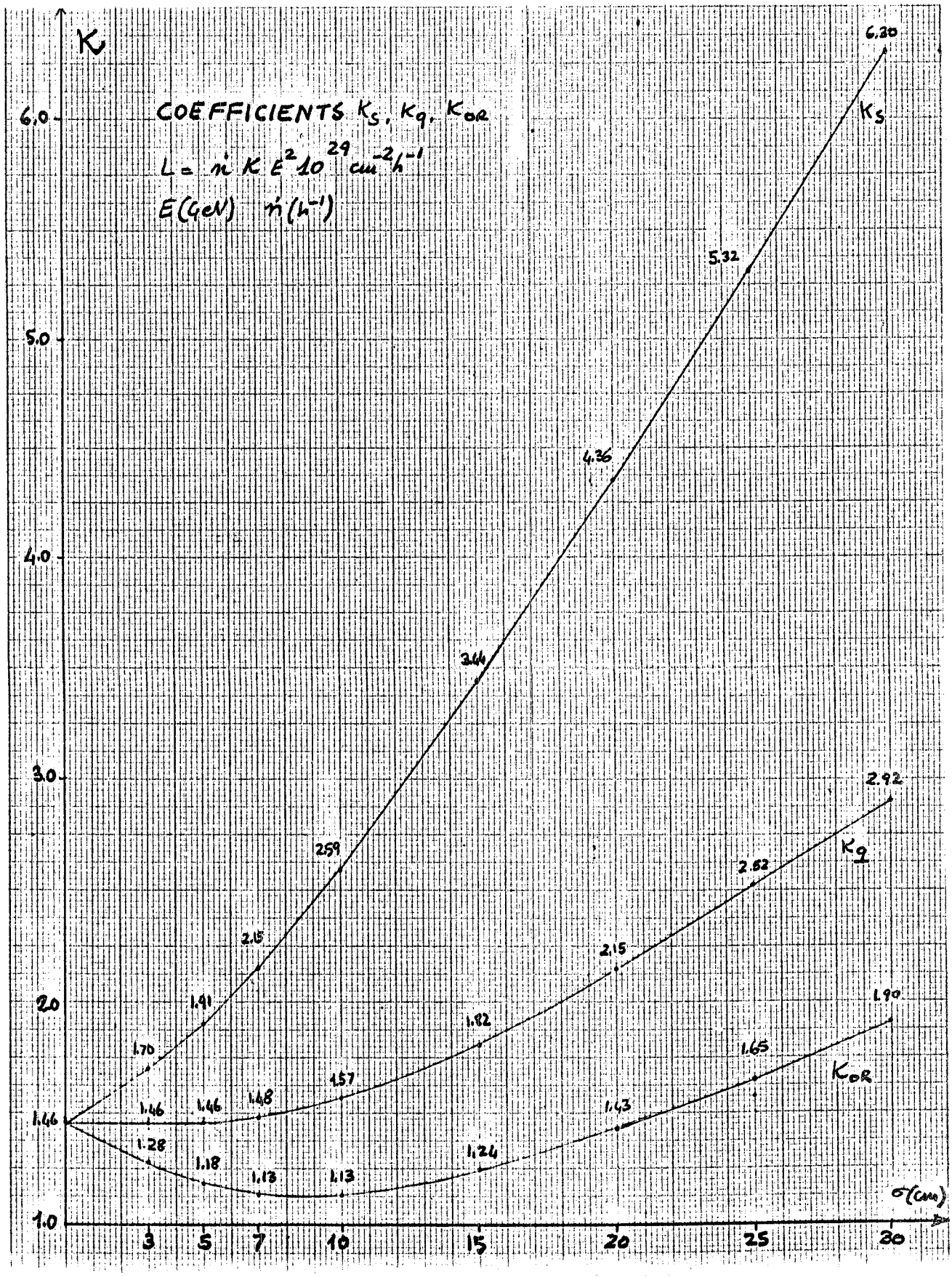

K
COEFFICIENTS K_S, K_q, K_{oR}
$L = n K E^2 10^{29}\ cm^{-2} h^{-1}$
E(GeV) n(h⁻¹)
6.0
5.0
4.0
3.0
2.0
1.46
1.0
K_S
K_q
K_{oR}
6.20
5.32
4.36
3.44
2.59
2.15
1.91
1.70
2.92
2.52
2.15
1.82
1.57
1.48
1.46
1.46
1.90
1.65
1.43
1.24
1.13
1.13
1.18
1.28
σ(cm)
3
5
7
10
15
20
25
30

FIG. 2

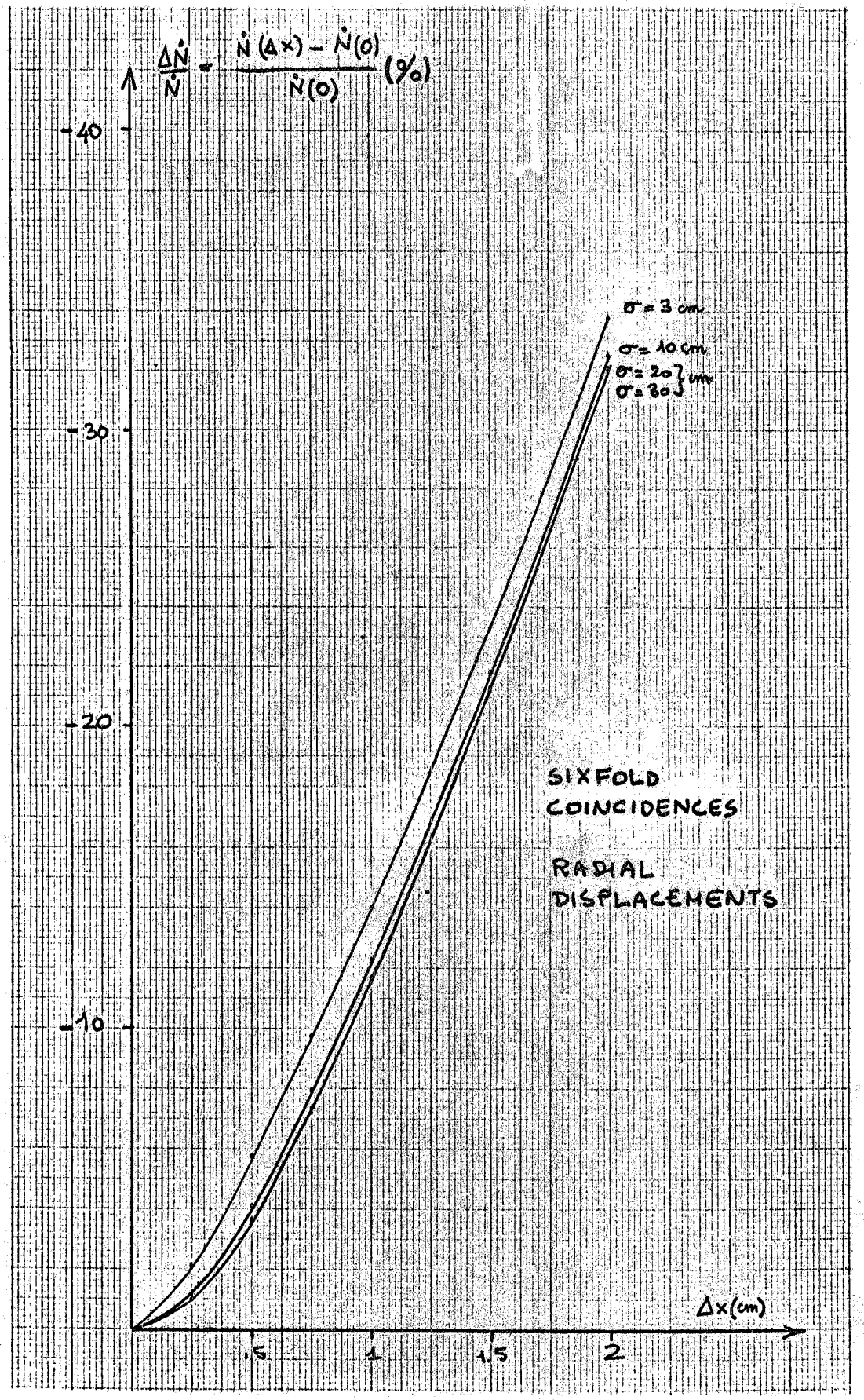

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta x) - \dot{N}(0)}{\dot{N}(0)}$ (%)
-40
-30
-20
-10
σ = 3 cm
σ = 10 cm
σ = 20 } cm
σ = 30 }
SIXFOLD
COINCIDENCES
RADIAL
DISPLACEMENTS
Δx(cm)
.5
1
1.5
2

FIG. 3

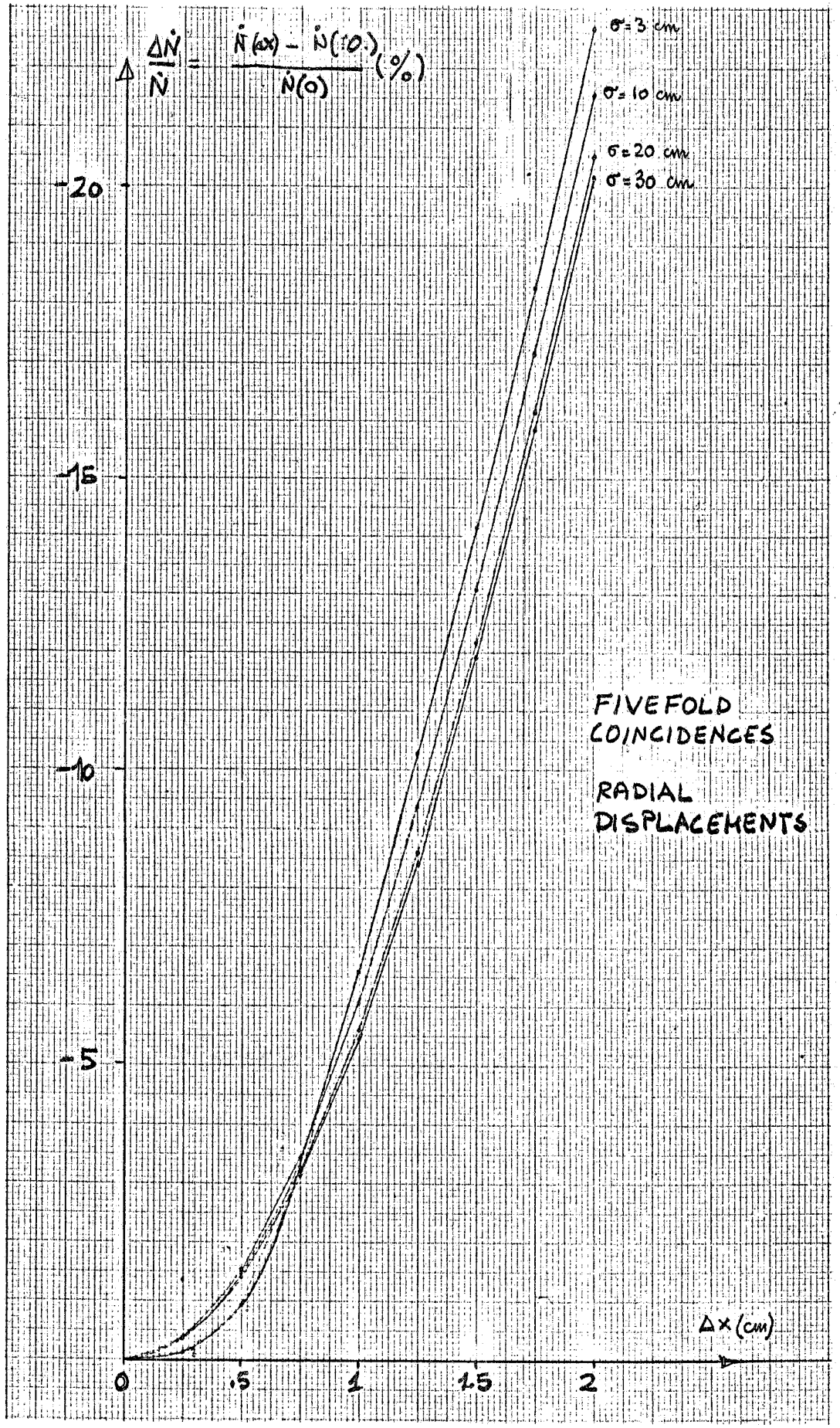

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta x) - \dot{N}(\infty)}{\dot{N}(0)}$ (%)
σ = 3 cm
σ = 10 cm
σ = 20 cm
σ = 30 cm
-20
-15
-10
-5
FIVEFOLD COINCIDENCES
RADIAL DISPLACEMENTS
Δx (cm)
0
.5
1
1.5
2

FIG. 4

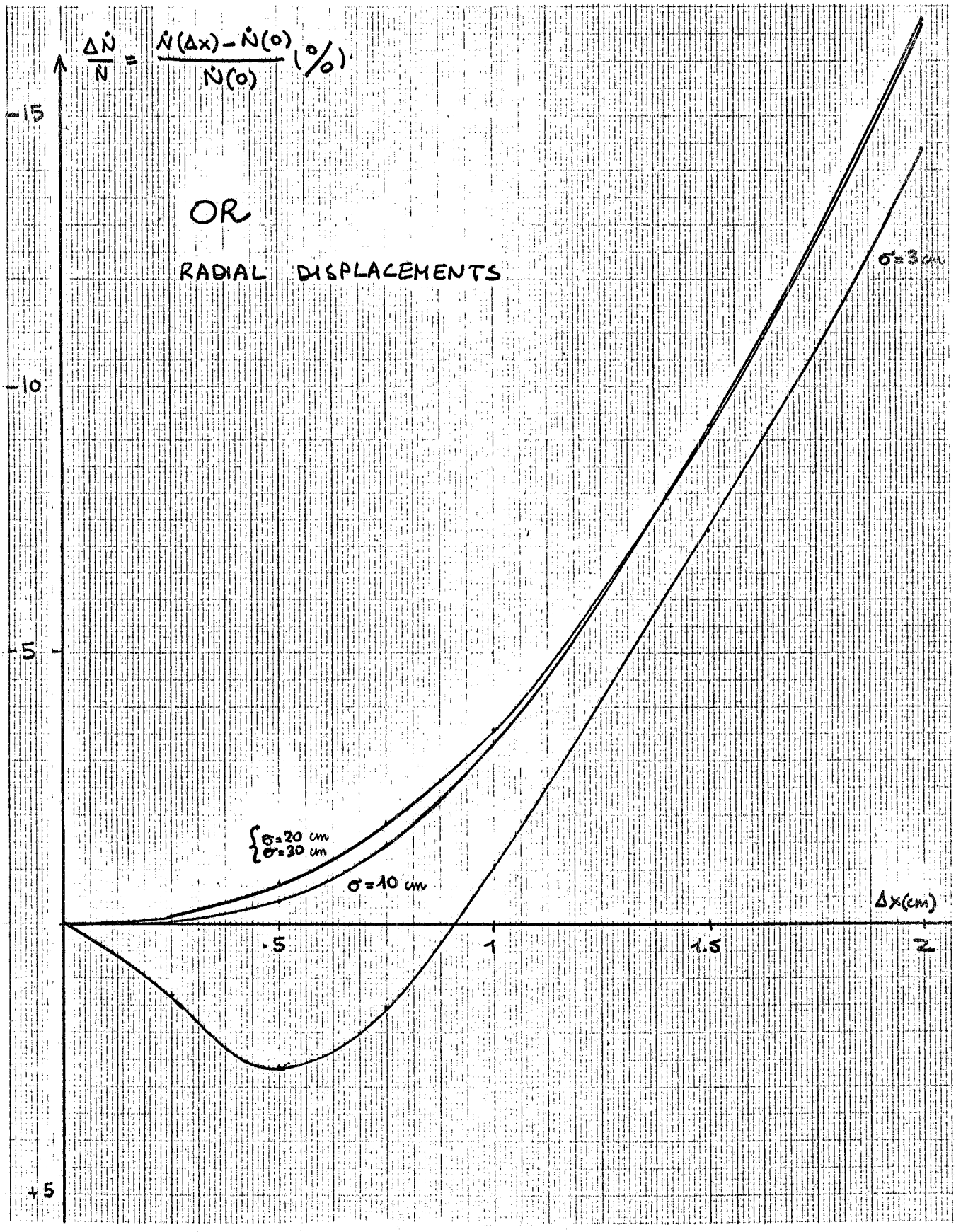

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta x) - \dot{N}(0)}{\dot{N}(0)} (\%)$
OR
RADIAL DISPLACEMENTS
-15
-10
-5
+5
σ=3 cm
σ=20 cm
σ=30 cm
σ=10 cm
Δx(cm)
.5
1
1.5
2

FIG. 5

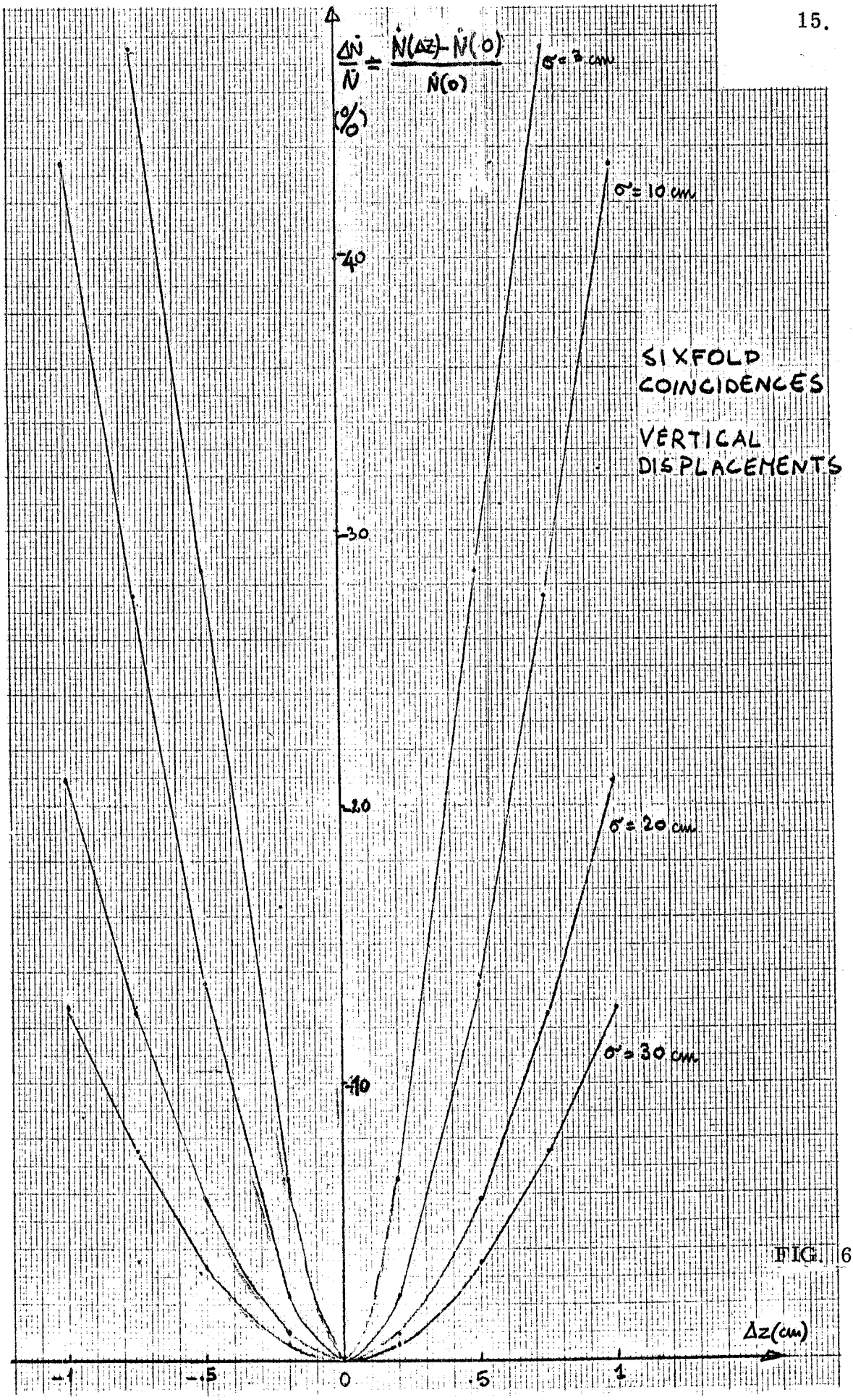

$\frac{\Delta\dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta z) - \dot{N}(0)}{\dot{N}(0)}$
(%)
σ = 3 cm
σ = 10 cm
σ = 20 cm
σ = 30 cm
SIXFOLD
COINCIDENCES
VERTICAL
DISPLACEMENTS
-40
-30
-20
-10
-1
-.5
0
.5
1
Δz(cm)

FIG. 6

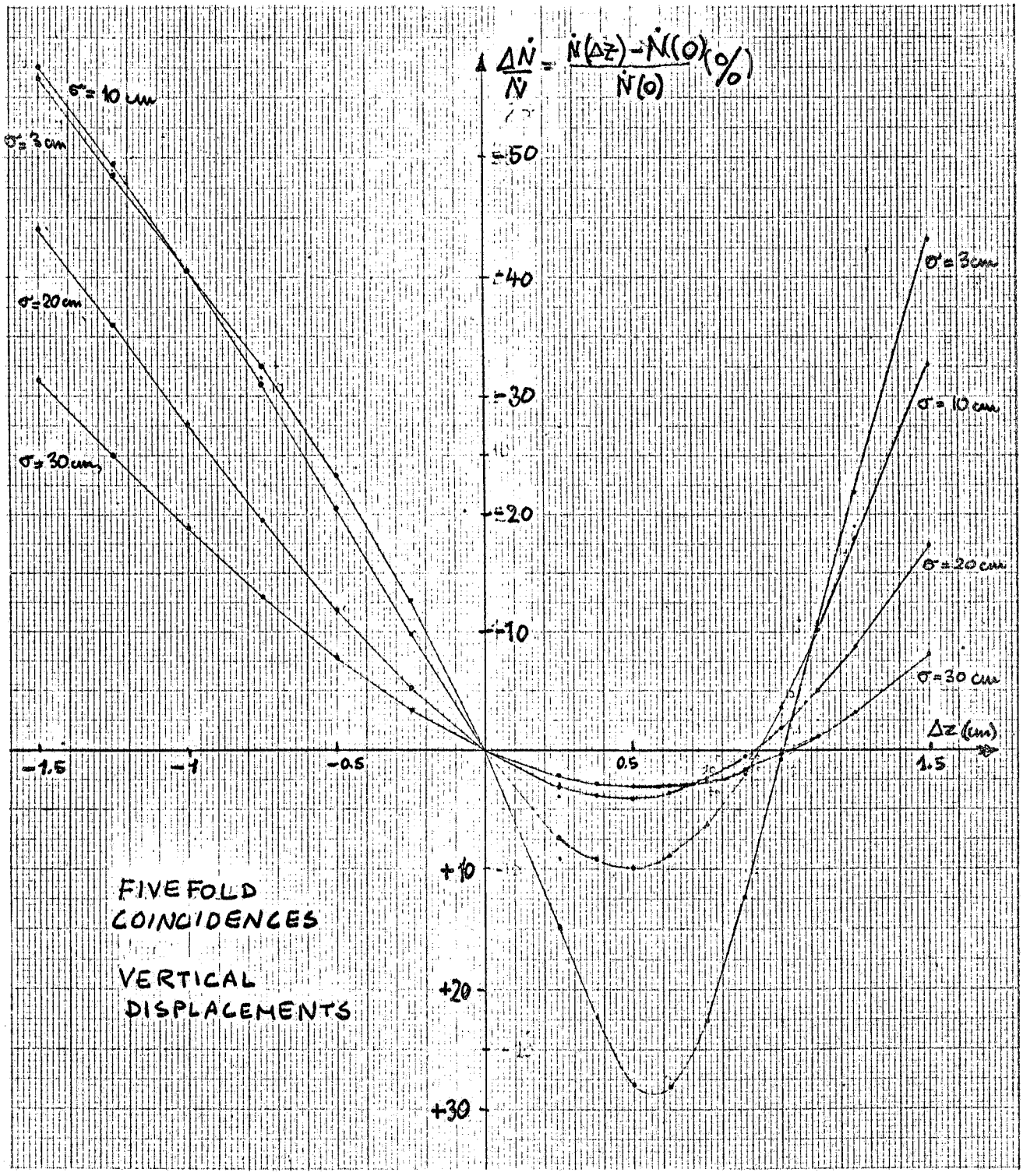

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta z) - \dot{N}(0)}{\dot{N}(0)} (\%)$
σ = 10 cm
σ = 3 cm
σ = 20 cm
σ = 30 cm
-50
-40
-30
-20
-10
+10
+20
+30
-1.5
-1
-0.5
0.5
1.5
Δz (cm)
σ = 3 cm
σ = 10 cm
σ = 20 cm
σ = 30 cm
FIVE FOLD
COINCIDENCES
VERTICAL
DISPLACEMENTS

FIG. 7

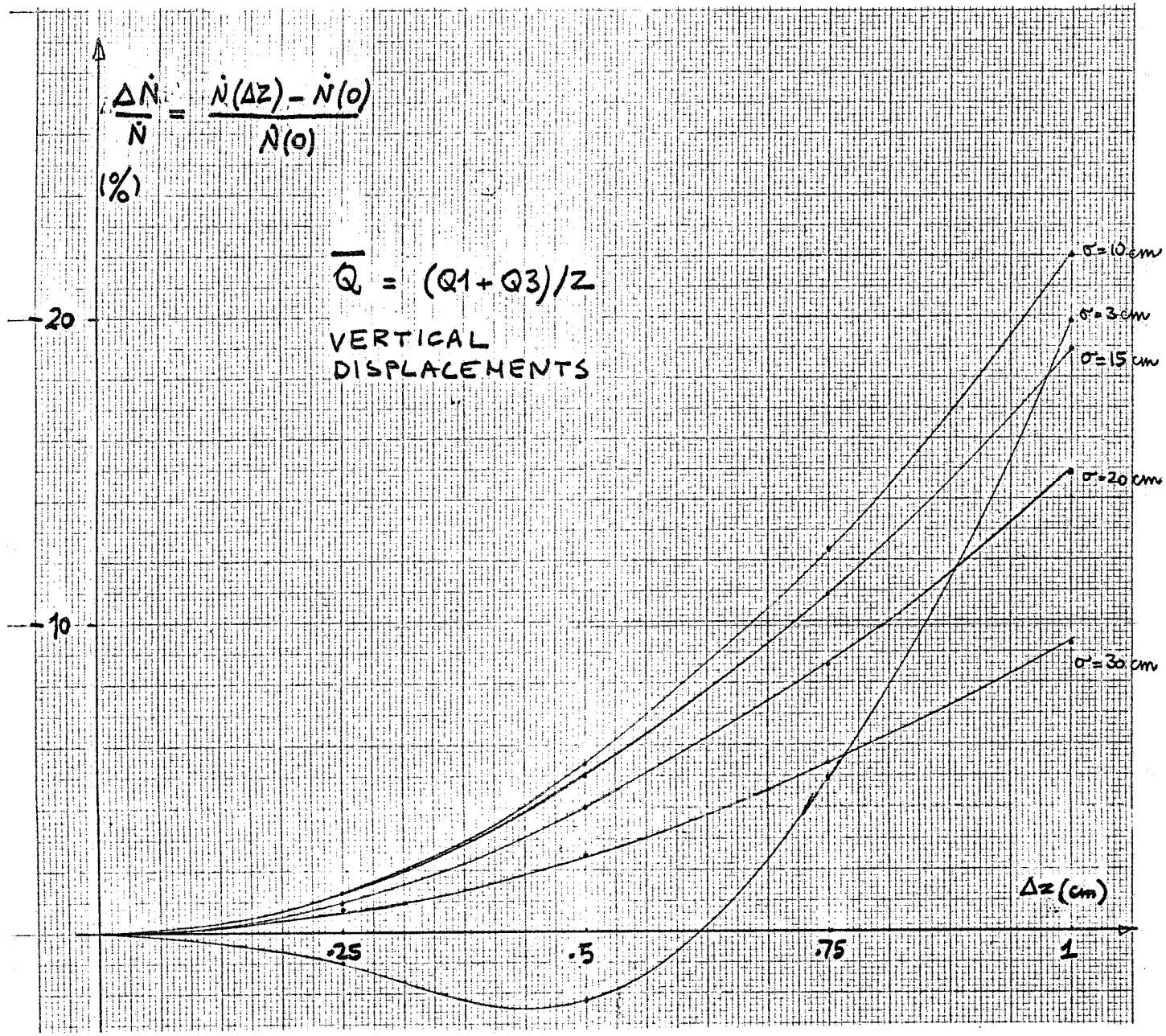

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta z) - \dot{N}(0)}{\dot{N}(0)}$
(%)
$\overline{Q} = (Q1 + Q3)/2$
VERTICAL
DISPLACEMENTS
20
10
σ=10 cm
σ=3 cm
σ=15 cm
σ=20 cm
σ=30 cm
Δz (cm)
.25
.5
.75
1

FIG. 8

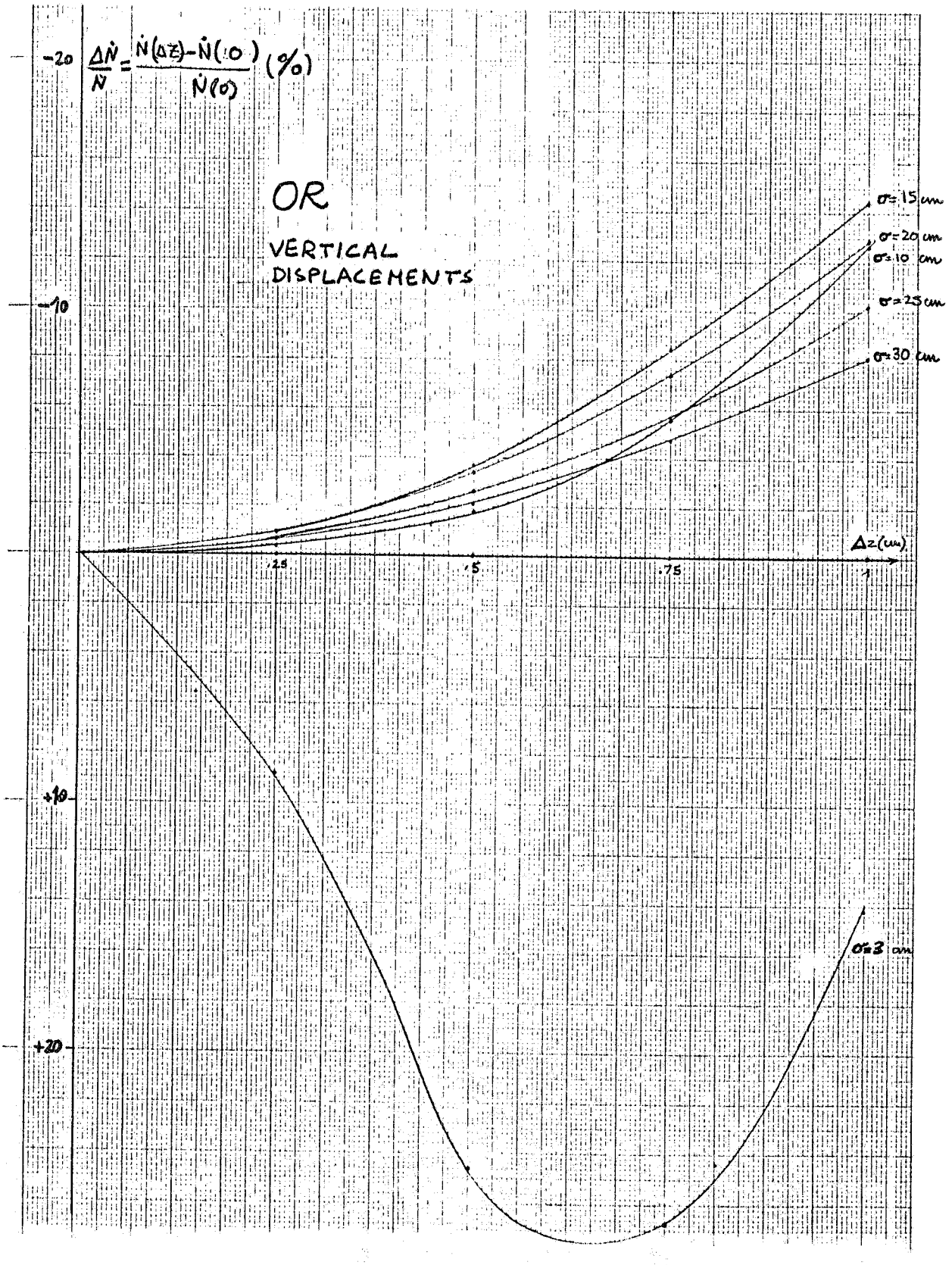

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta z) - \dot{N}(0)}{\dot{N}(0)}$ (%)
OR
VERTICAL
DISPLACEMENTS
-20
-10
+10
+20
σ=15 cm
σ=20 cm
σ=10 cm
σ=25 cm
σ=30 cm
σ=3 cm
Δz(cm)
.25
.5
.75
1

FIG. 9

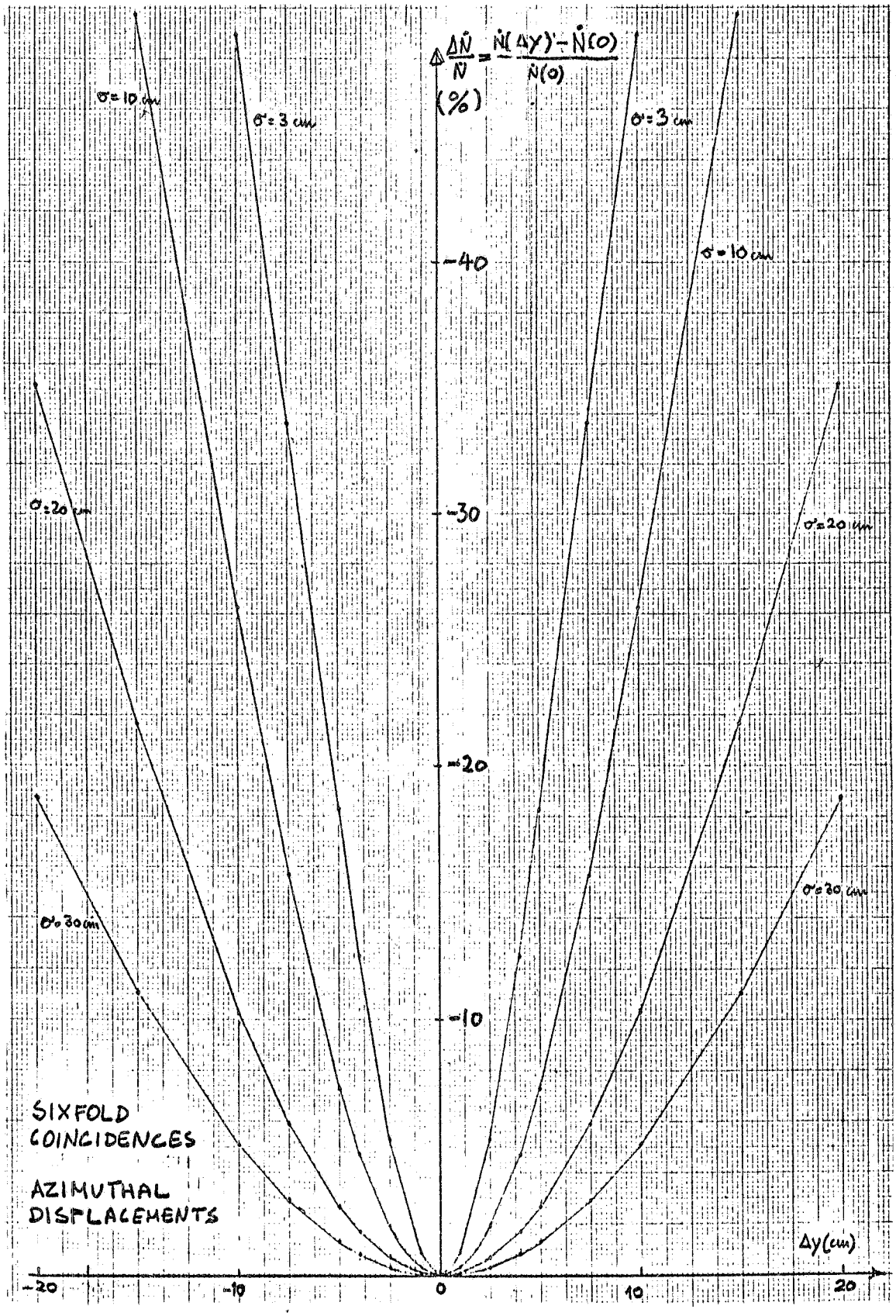

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta y) - \dot{N}(0)}{\dot{N}(0)}$
(%)
σ=10 cm
σ=3 cm
σ=3 cm
σ=10 cm
σ=20 cm
σ=20 cm
σ=30 cm
σ=30 cm
-40
-30
-20
-10
SIXFOLD
COINCIDENCES
AZIMUTHAL
DISPLACEMENTS
Δy(cm)
-20
-10
0
10
20

FIG. 10

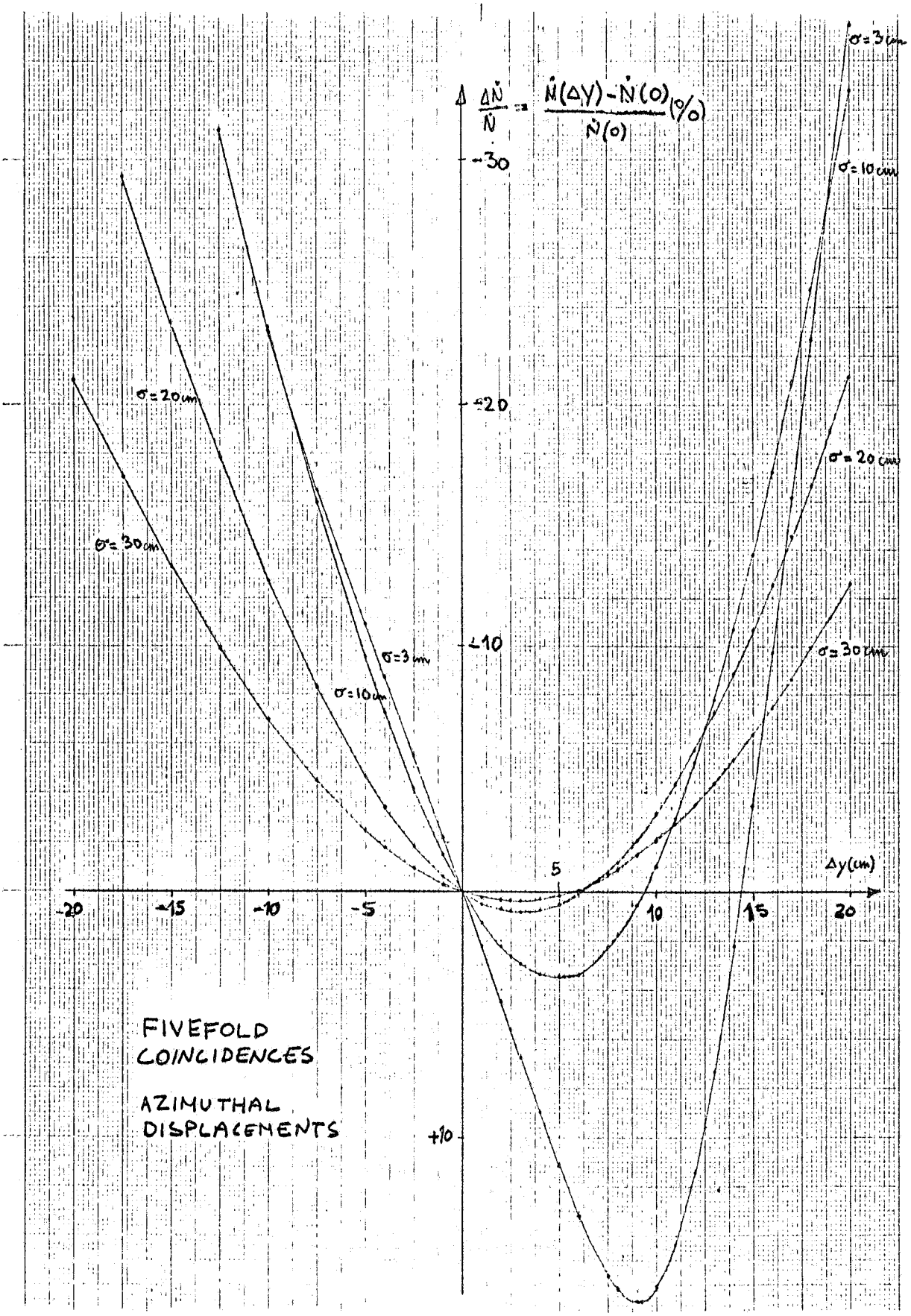

$\frac{\Delta\dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta y) - \dot{N}(0)}{\dot{N}(0)}$ (%)
-30
-20
-10
+10
σ=3cm
σ=10cm
σ=20cm
σ=30cm
Δy(cm)
-20
-15
-10
-5
5
10
15
20
FIVEFOLD
COINCIDENCES
AZIMUTHAL
DISPLACEMENTS

FIG. 11

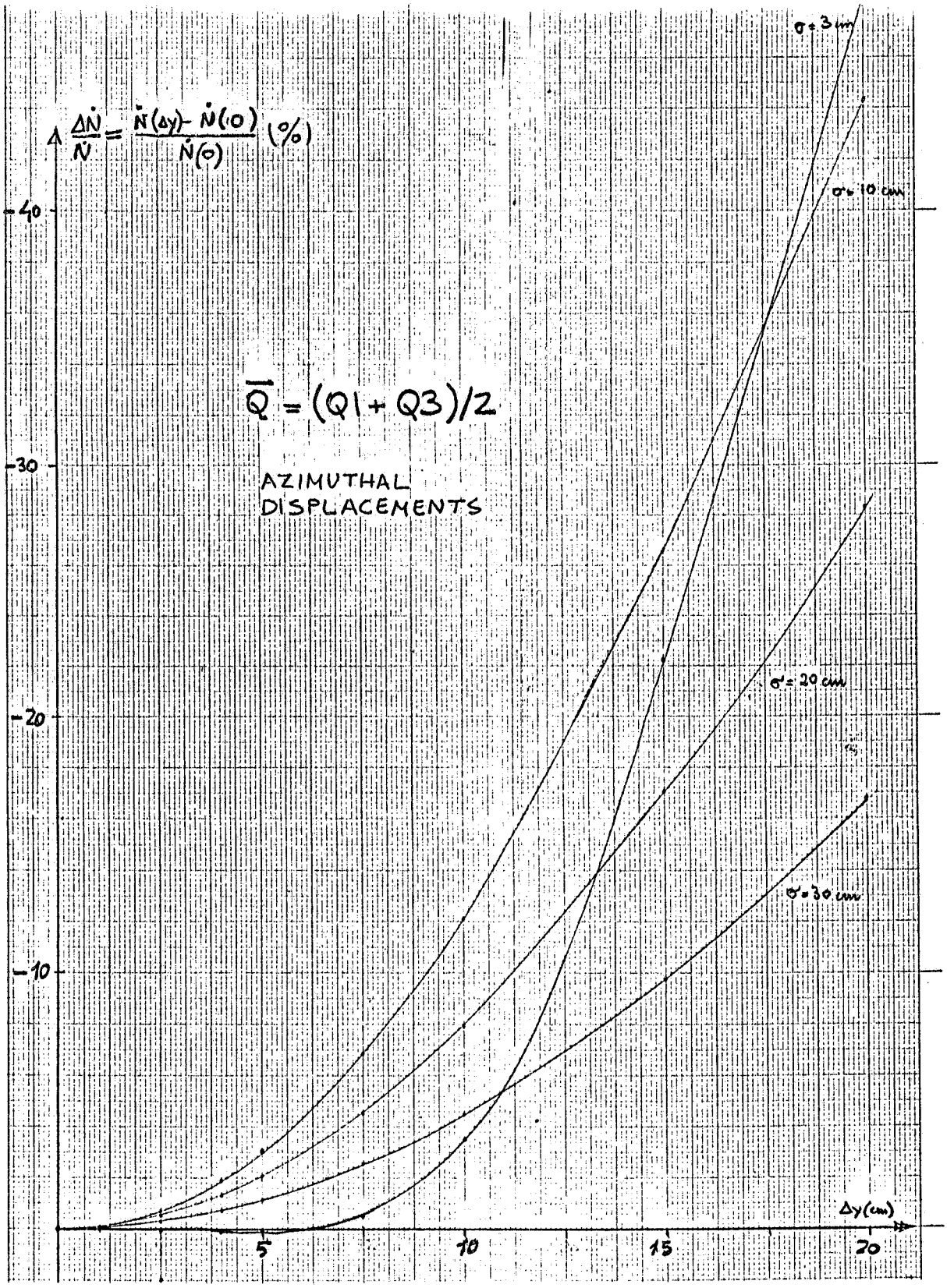

$\frac{\Delta\dot{N}}{\dot{N}} = \frac{\dot{N}(\Delta y) - \dot{N}(0)}{\dot{N}(0)}$ (%)
$\overline{Q} = (Q1 + Q3)/2$
AZIMUTHAL DISPLACEMENTS
σ = 3 mm
σ = 10 mm
σ = 20 mm
σ = 30 mm
-40
-30
-20
-10
5
10
15
20
Δy (cm)

FIG. 12

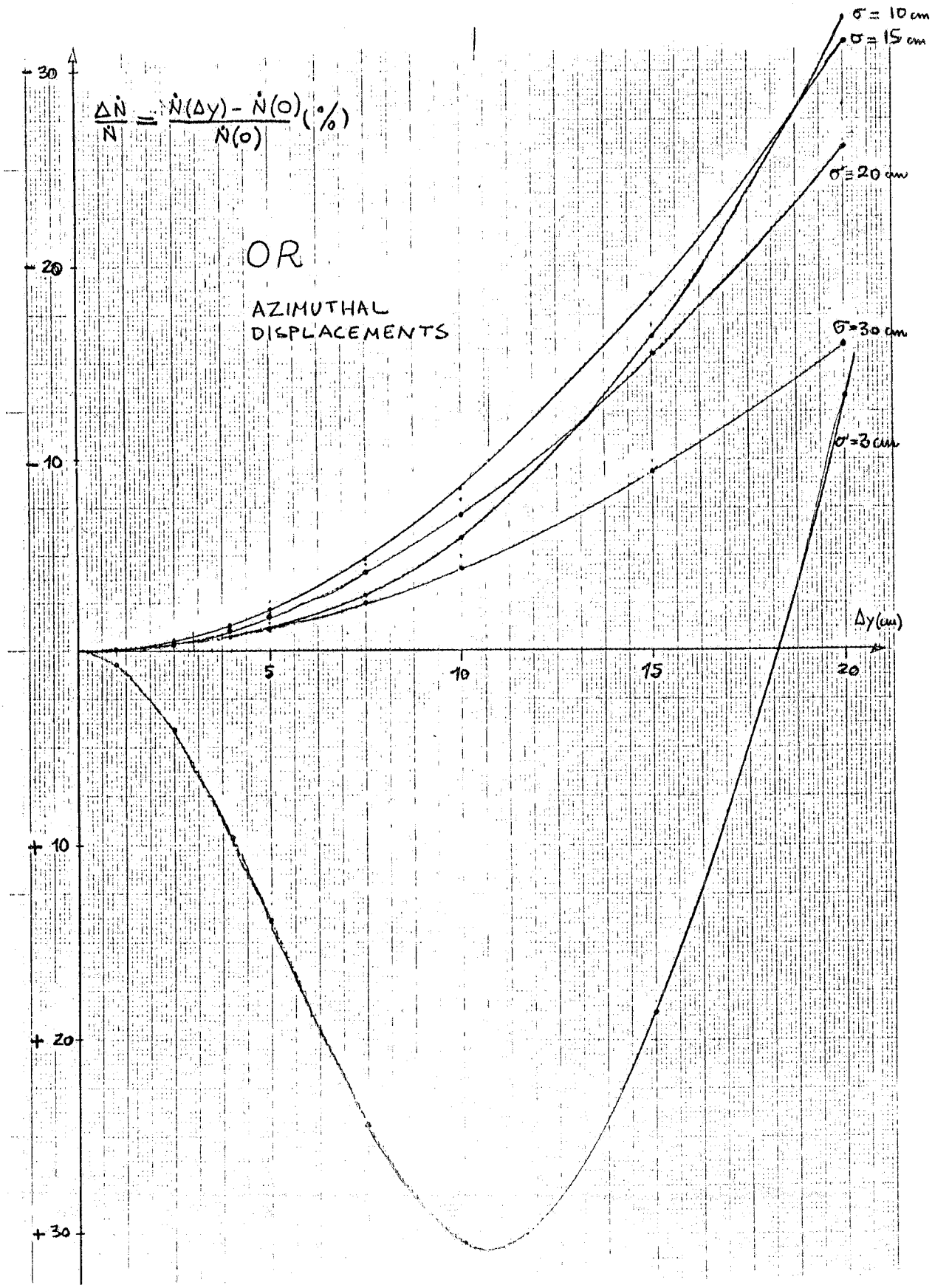

σ = 10 cm
σ = 15 cm
σ = 20 cm
σ = 30 cm
σ = 3 cm
ΔN/N = Ṅ(Δy) - Ṅ(0) / Ṅ(0) (%)
OR
AZIMUTHAL
DISPLACEMENTS
Δy (cm)
5
10
15
20
30
20
10
10
20
30

FIG. 13

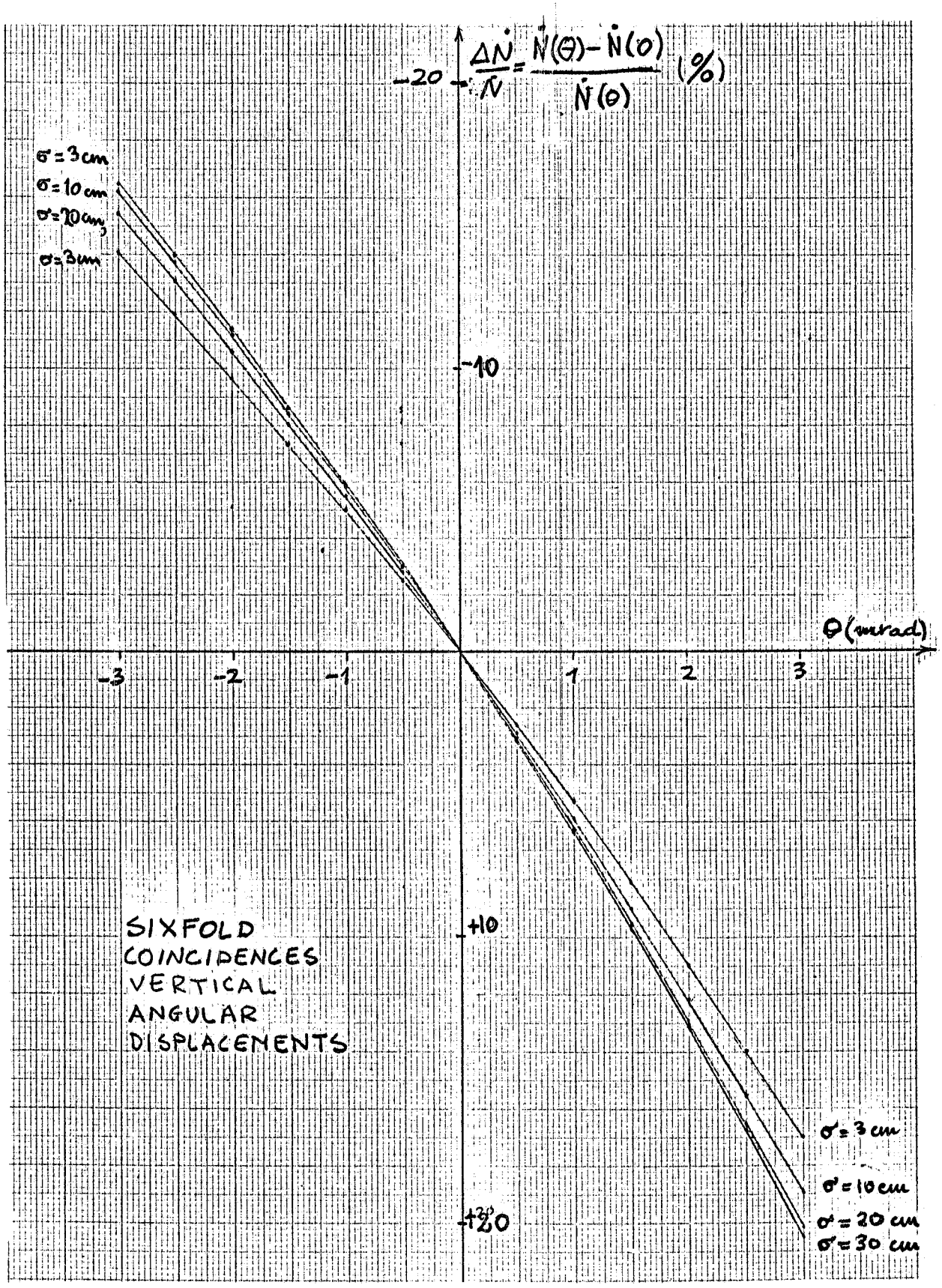

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\theta) - \dot{N}(0)}{\dot{N}(0)}$ (%)
-20
-10
+10
+20
θ (mrad)
-3
-2
-1
1
2
3
σ = 3 cm
σ = 10 cm
σ = 20 cm
σ = 30 cm
SIXFOLD
COINCIDENCES
VERTICAL
ANGULAR
DISPLACEMENTS

FIG. 14

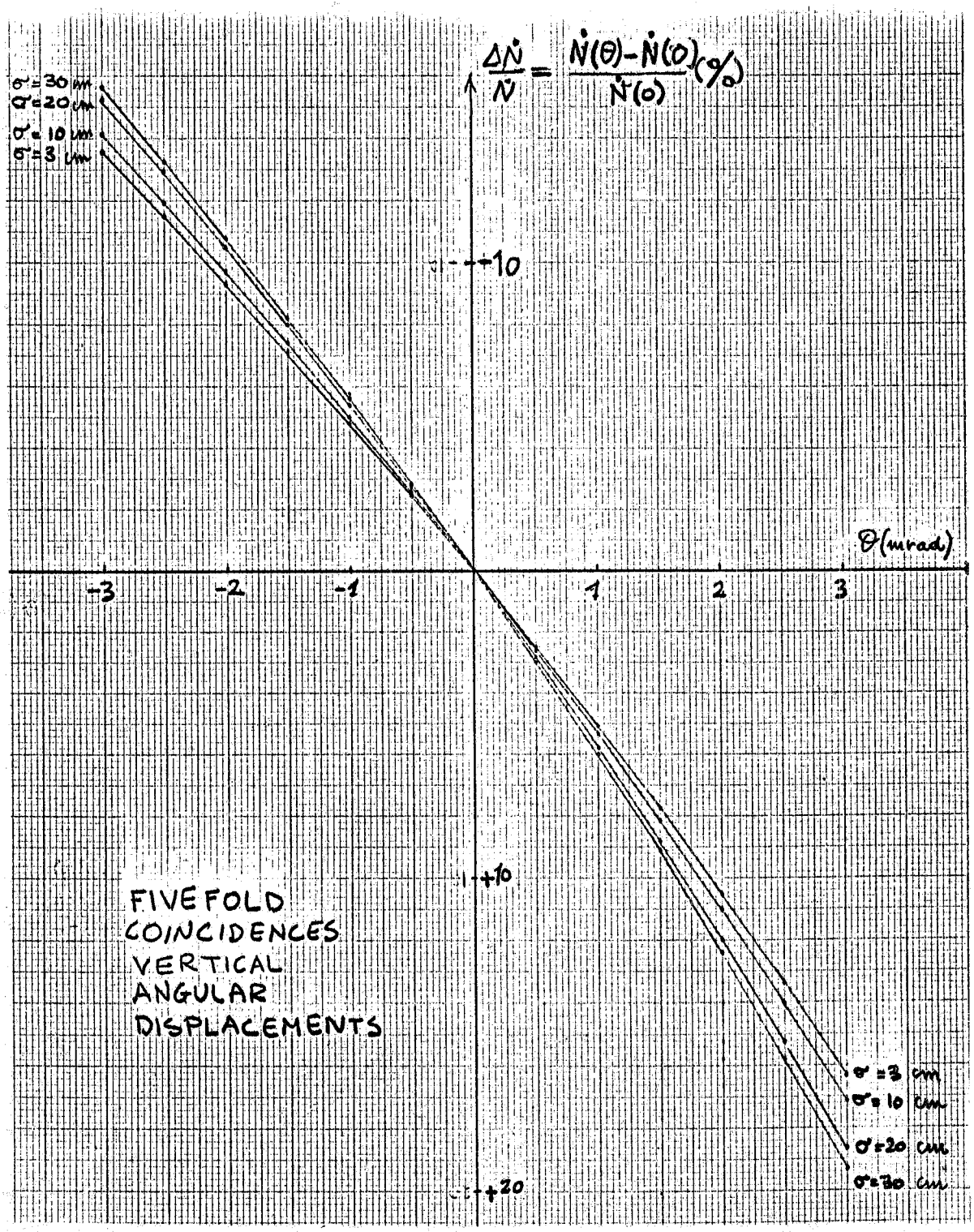

$\frac{\Delta\dot{N}}{\dot{N}} = \frac{\dot{N}(\theta)-\dot{N}(0)}{\dot{N}(0)}(\%)$
σ=30 cm
σ=20 cm
σ=10 cm
σ=3 cm
+10
+10
+20
θ(mrad)
-3
-2
-1
1
2
3
FIVE FOLD
COINCIDENCES
VERTICAL
ANGULAR
DISPLACEMENTS
σ=3 cm
σ=10 cm
σ=20 cm
σ=30 cm

FIG. 15

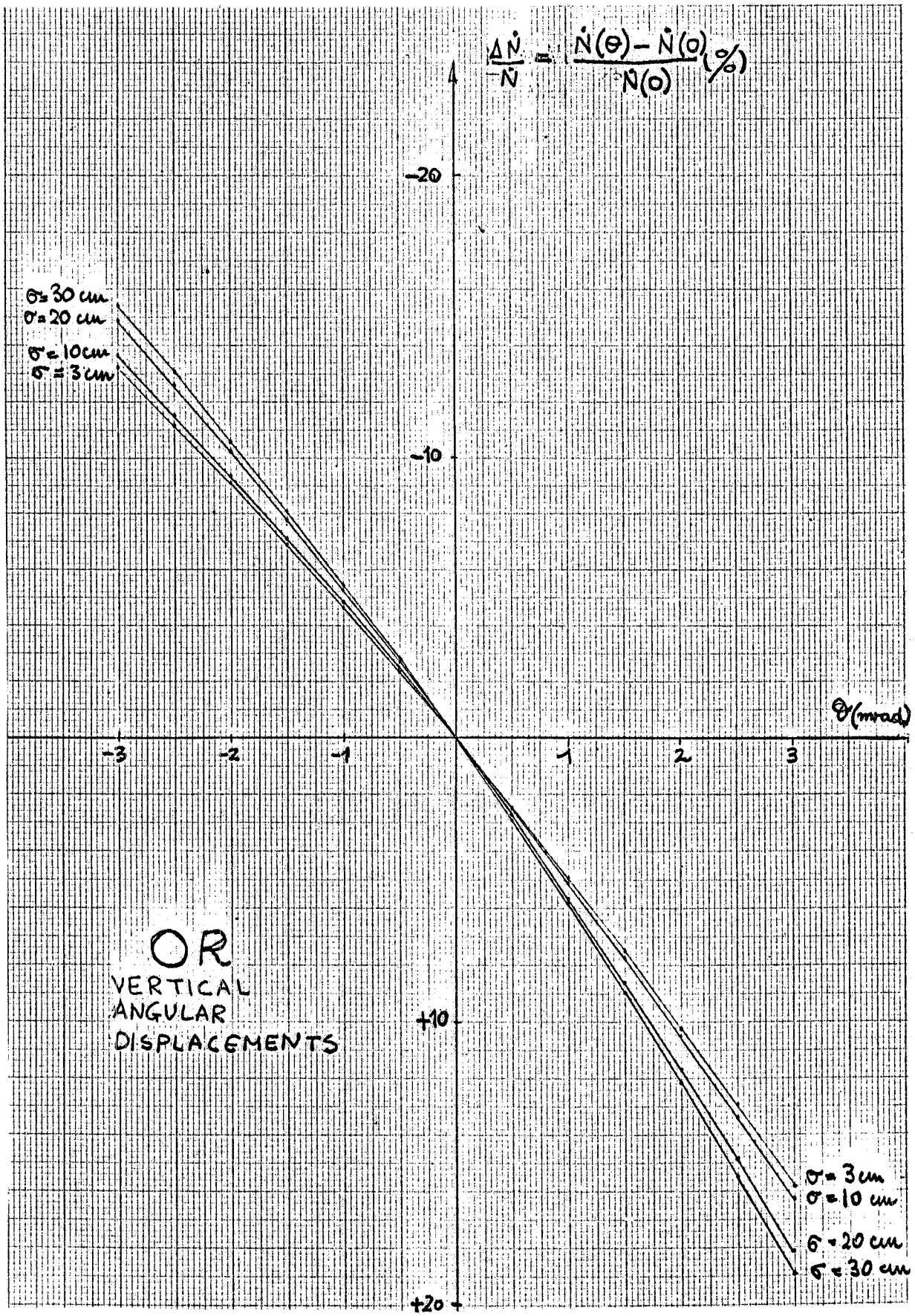

$\frac{\Delta \dot{N}}{\dot{N}} = \frac{\dot{N}(\theta) - \dot{N}(0)}{\dot{N}(0)}$ (%)
-20
-10
+10
+20
σ = 30 cm
σ = 20 cm
σ = 10 cm
σ = 3 cm
-3
-2
-1
1
2
3
θ (mrad)
OR
VERTICAL
ANGULAR
DISPLACEMENTS
σ = 3 cm
σ = 10 cm
σ = 20 cm
σ = 30 cm

FIG. 16

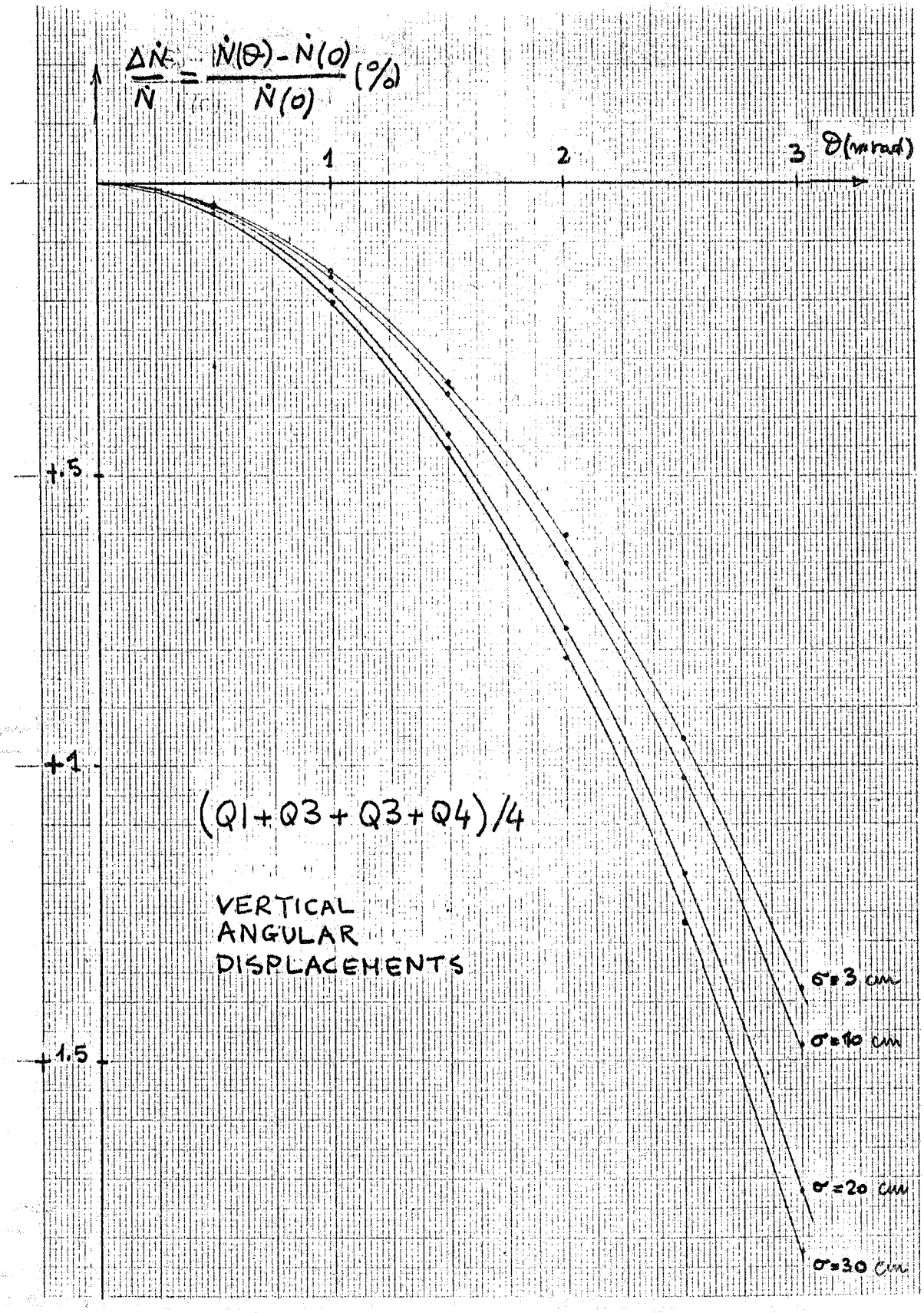

ΔṄ/Ṅ = (Ṅ(θ) - Ṅ(0))/Ṅ(0) (%)
1
2
3
θ(mrad)
+.5
+1
+1.5
(Q1+Q3+Q3+Q4)/4
VERTICAL
ANGULAR
DISPLACEMENTS
σ=3 cm
σ=10 cm
σ=20 cm
σ=30 cm

FIG. 17

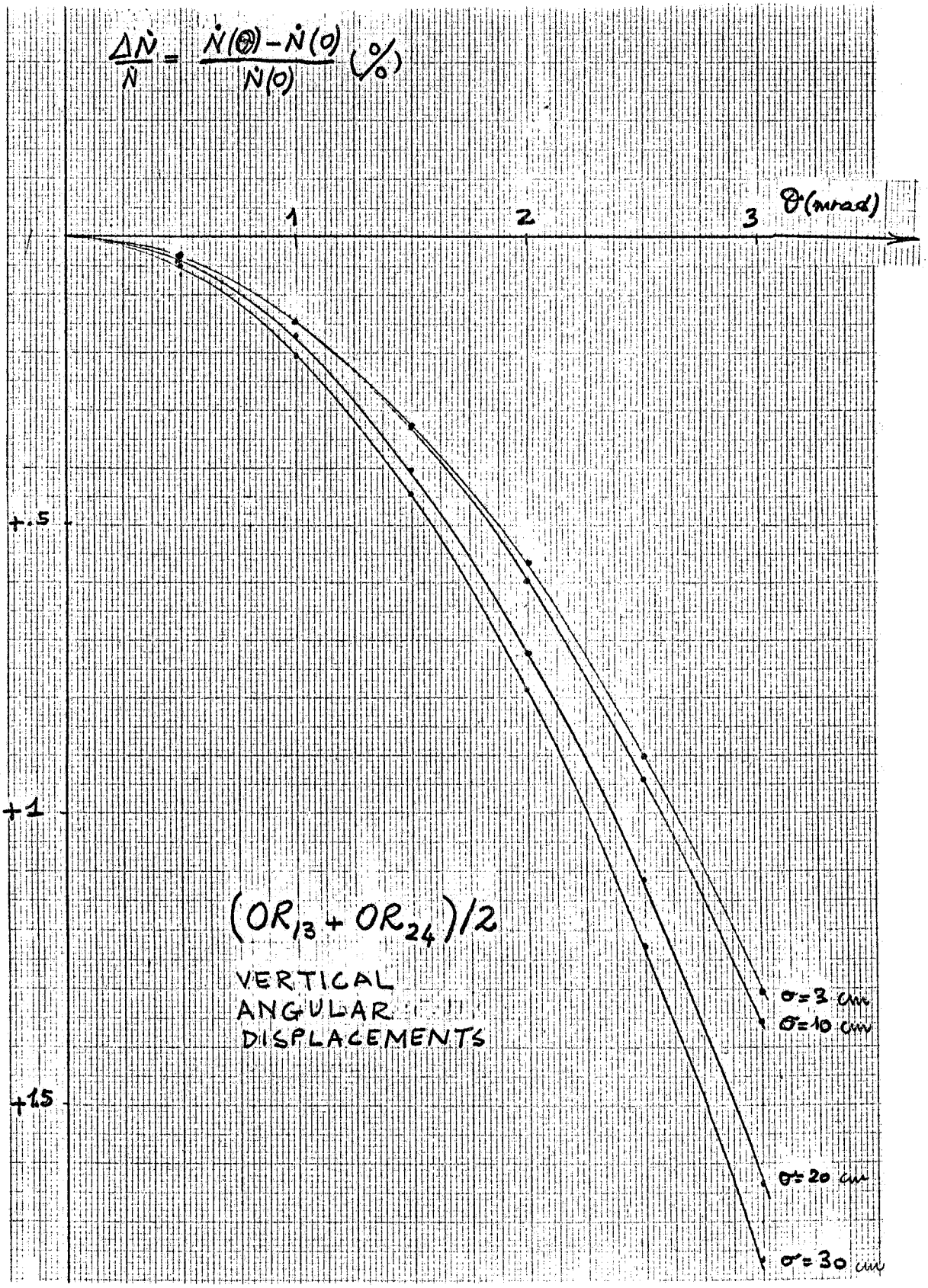

ΔṄ/Ṅ = (Ṅ(θ) − Ṅ(0))/Ṅ(0) (%)
1
2
3
θ(mrad)
+.5
+1
+1.5
(OR13 + OR24)/2
VERTICAL
ANGULAR
DISPLACEMENTS
σ=3 cm
σ=10 cm
σ=20 cm
σ=30 cm

FIG. 18

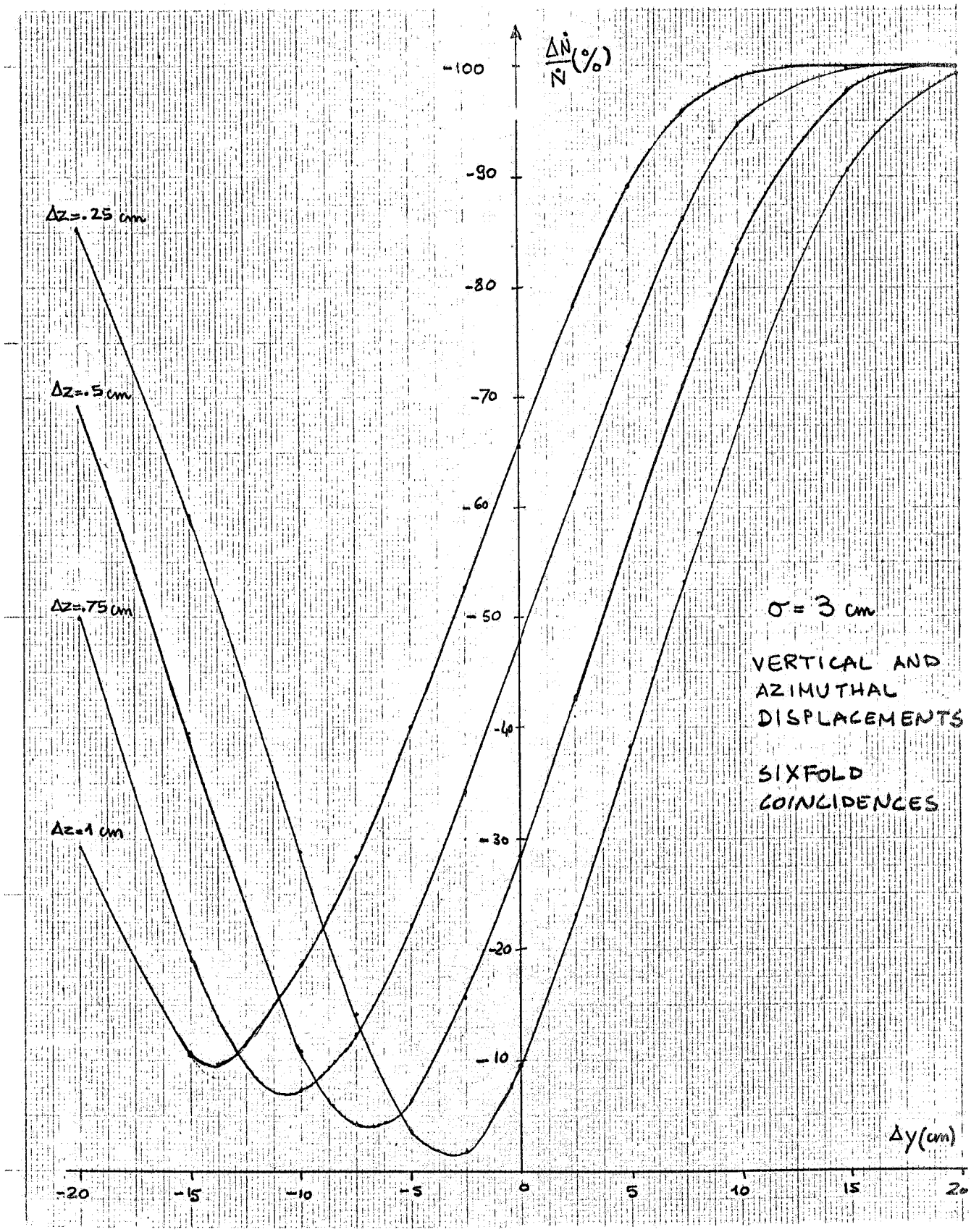

$\frac{\Delta \dot{N}}{\dot{N}}$(%)
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
Δz=.25 cm
Δz=.5 cm
Δz=.75 cm
Δz=1 cm
σ = 3 cm
VERTICAL AND
AZIMUTHAL
DISPLACEMENTS
SIXFOLD
COINCIDENCES
Δy(cm)
-20
-15
-10
-5
0
5
10
15
20

FIG. 19

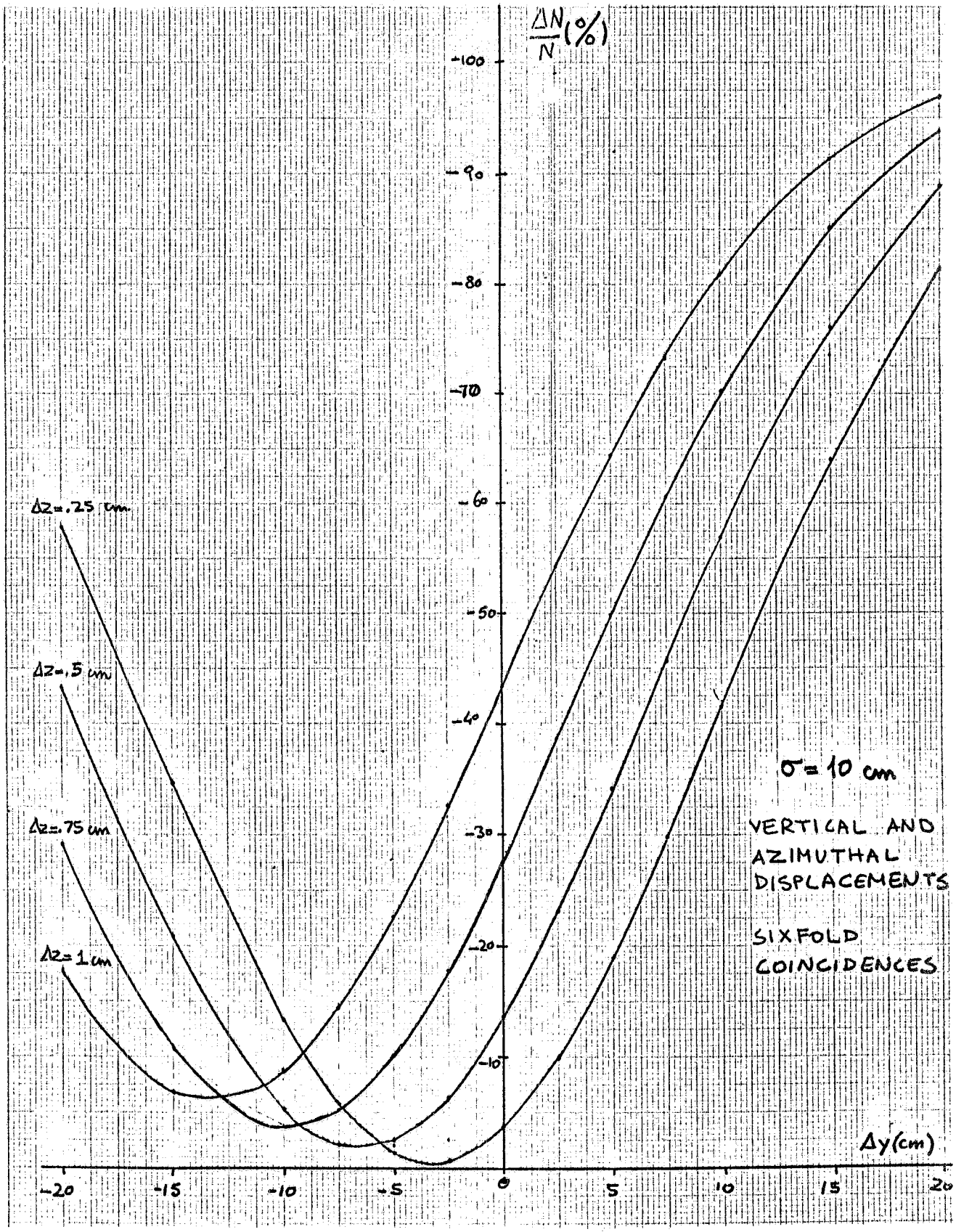

ΔN/N (%)
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
Δz=.25 cm
Δz=.5 cm
Δz=.75 cm
Δz=1 cm
σ=10 cm
VERTICAL AND AZIMUTHAL DISPLACEMENTS
SIXFOLD COINCIDENCES
Δy(cm)
-20
-15
-10
-5
0
5
10
15
20

FIG. 20

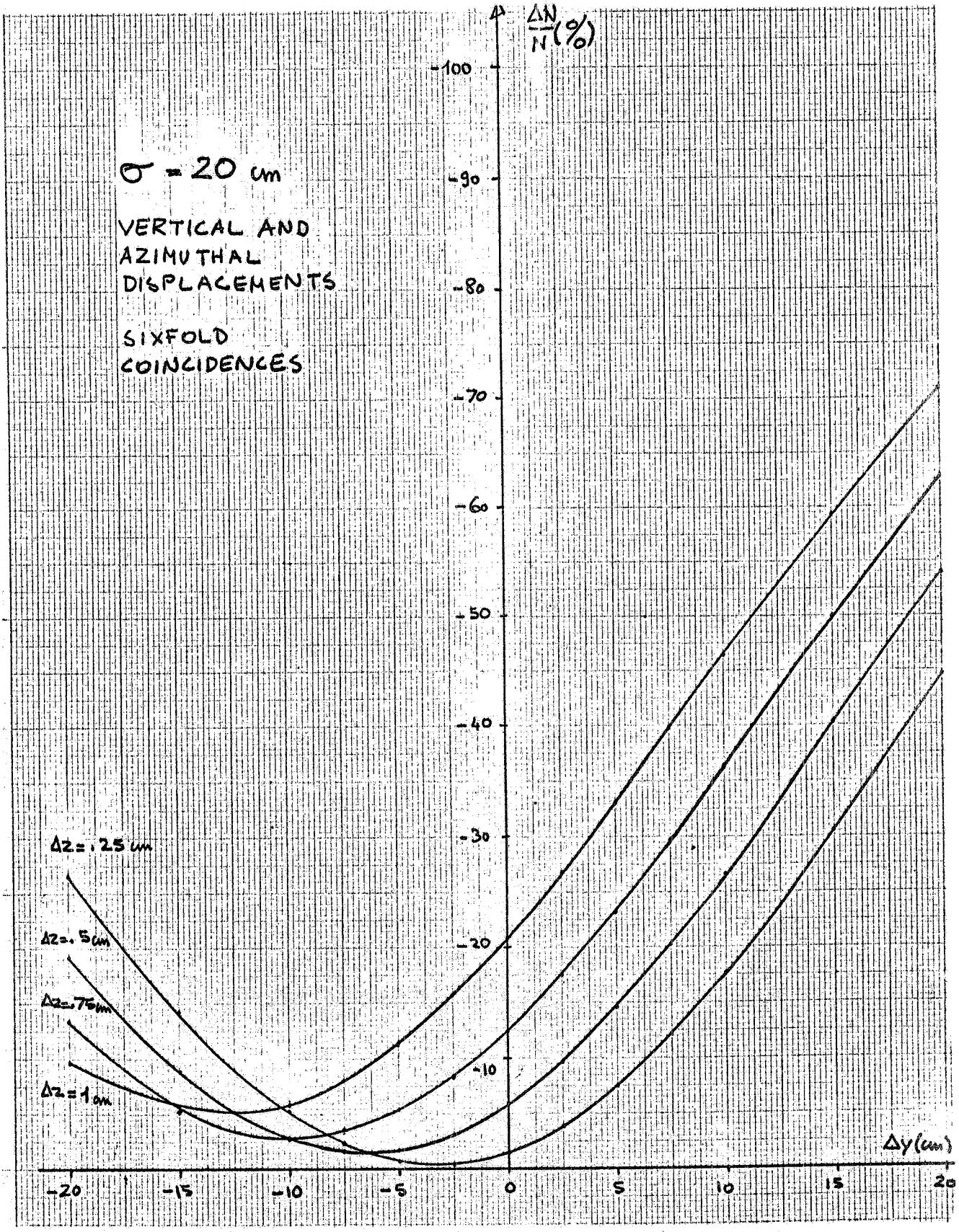

σ = 20 cm
VERTICAL AND AZIMUTHAL DISPLACEMENTS
SIXFOLD COINCIDENCES
ΔN/N (%)
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
Δz = .25 cm
Δz = .5 cm
Δz = .75 cm
Δz = 1 cm
Δy (cm)
-20
-15
-10
-5
0
5
10
15
20

FIG. 21

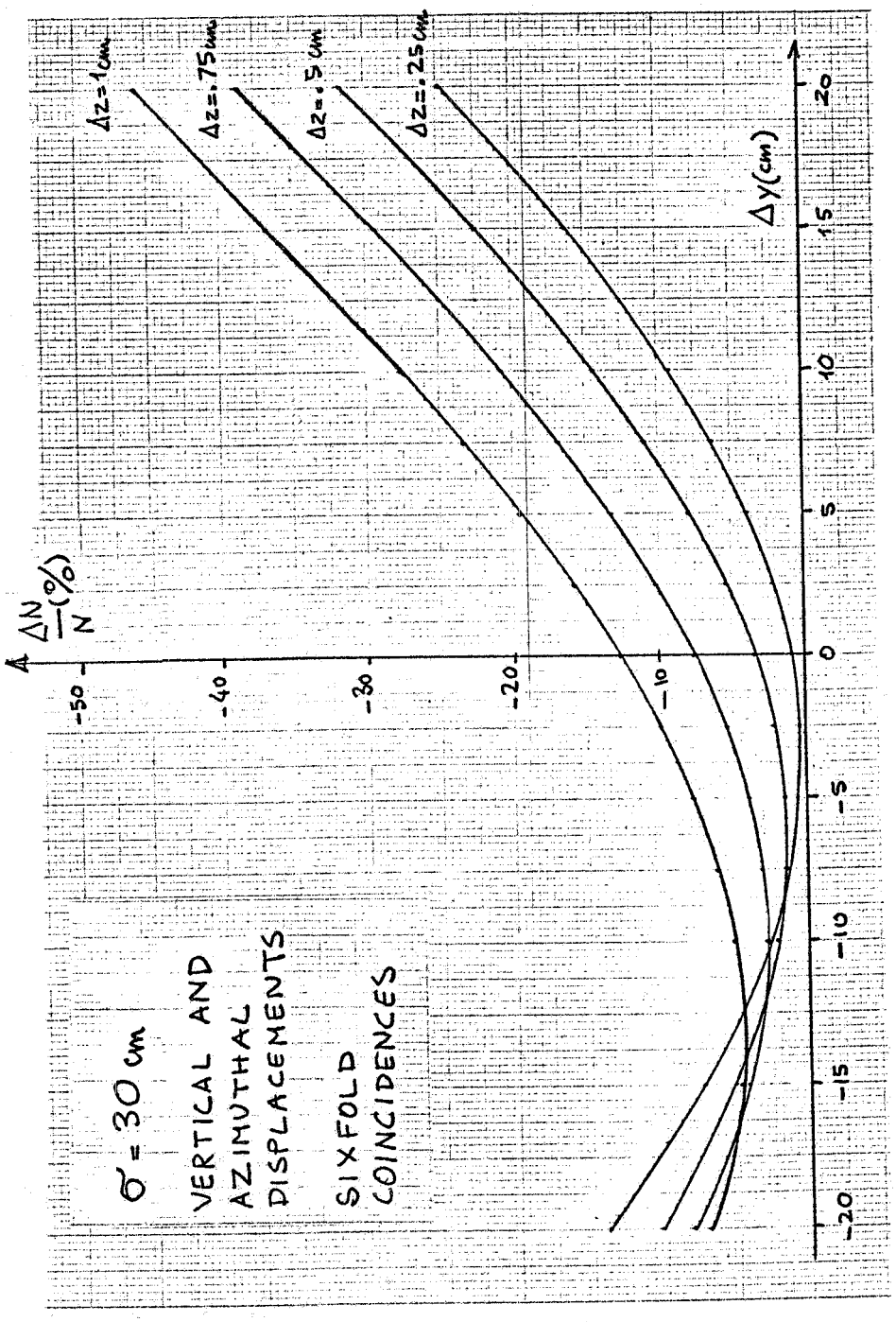

σ = 30 cm
VERTICAL AND
AZIMUTHAL
DISPLACEMENTS
SIXFOLD
COINCIDENCES
Δz=1 cm
Δz=.75 cm
Δz=.5 cm
Δz=.25 cm
Δy(cm)
ΔN/N (%)
-50
-40
-30
-20
-10
0
5
10
15
20
-5
-10
-15
-20

FIG. 22

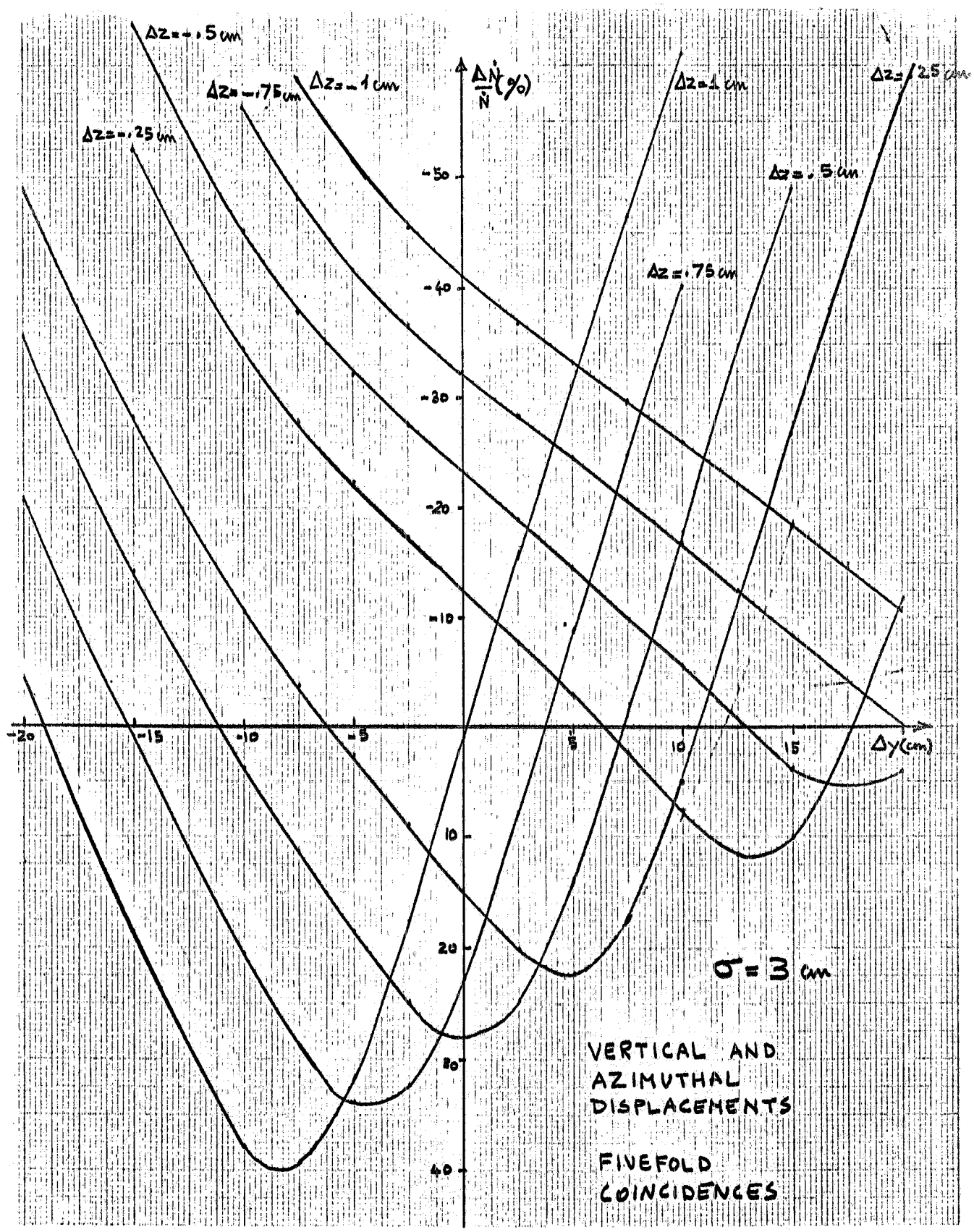

Δz=-.5 cm
Δz=-.75 cm
Δz=-1 cm
Δz=-.25 cm
ΔṄ/Ṅ (%)
Δz=1 cm
Δz=.25 cm
Δz=.5 cm
Δz=.75 cm
-50
-40
-30
-20
-10
-20
-15
-10
-5
5
10
15
Δy(cm)
10
20
30
40
σ=3 cm
VERTICAL AND
AZIMUTHAL
DISPLACEMENTS
FIVEFOLD
COINCIDENCES

FIG. 23

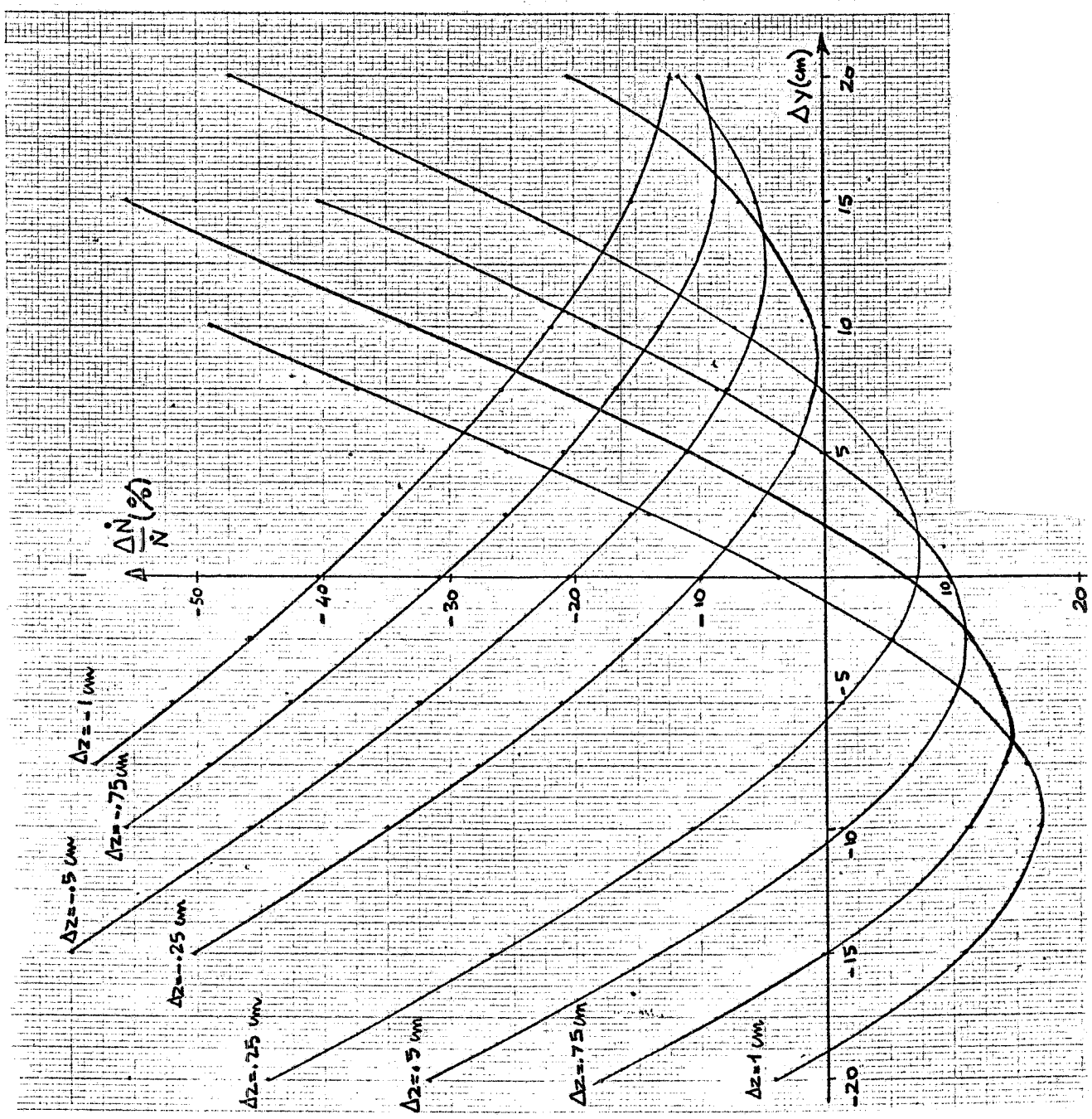

σ = 10 cm
VERTICAL AND
AZIMUTHAL
DISPLACEMENTS
FIVE FOLD
COINCIDENCES
Δ ΔṄ/Ṅ (%)
Δy (cm)
Δz=-.5 cm
Δz=-.75 cm
Δz=-1 cm
Δz=-.25 cm
Δz=.25 cm
Δz=.5 cm
Δz=.75 cm
Δz=1 cm
-50
-40
-30
-20
-10
10
20
-20
-15
-10
-5
5
10
15
20

FIG. 24

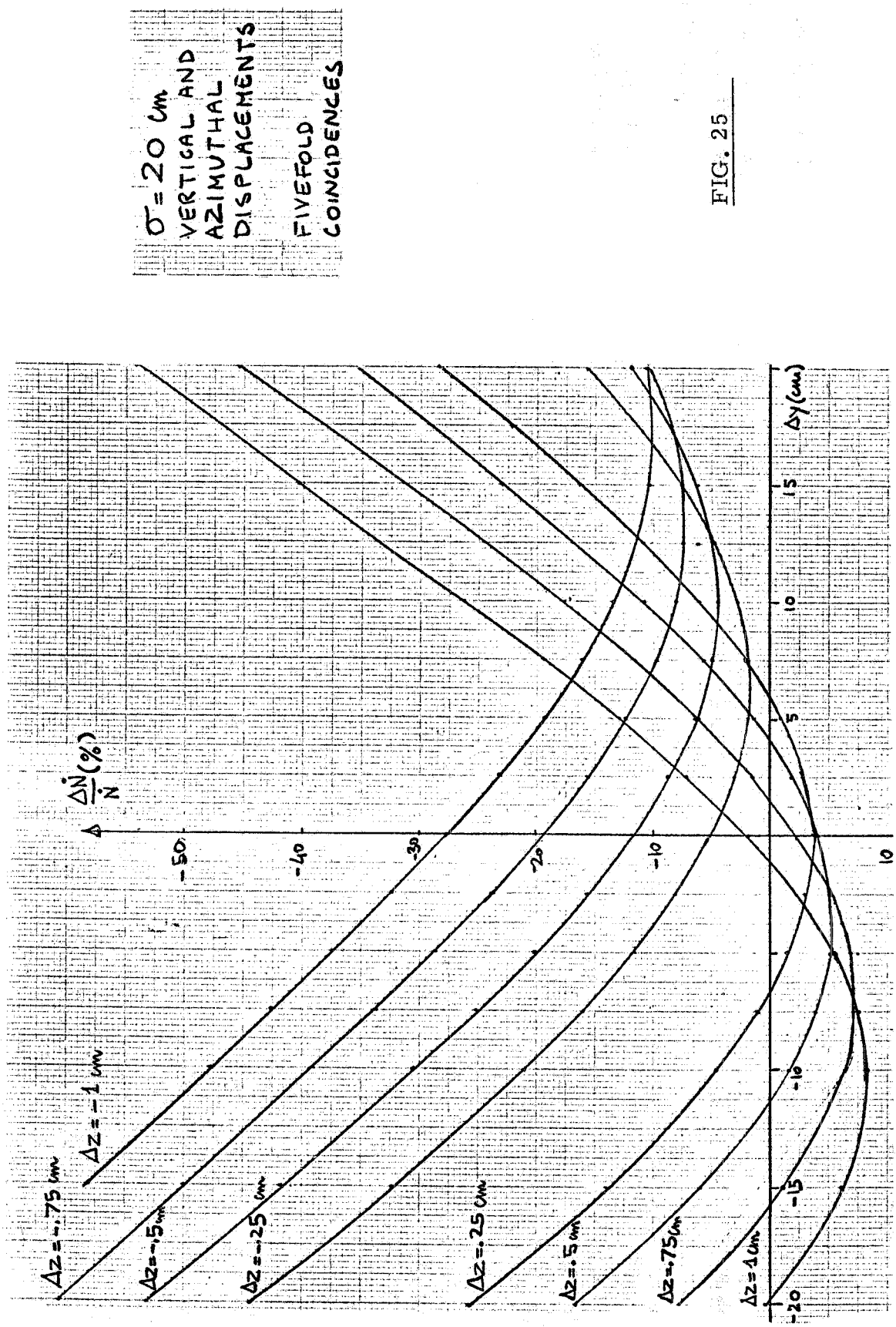
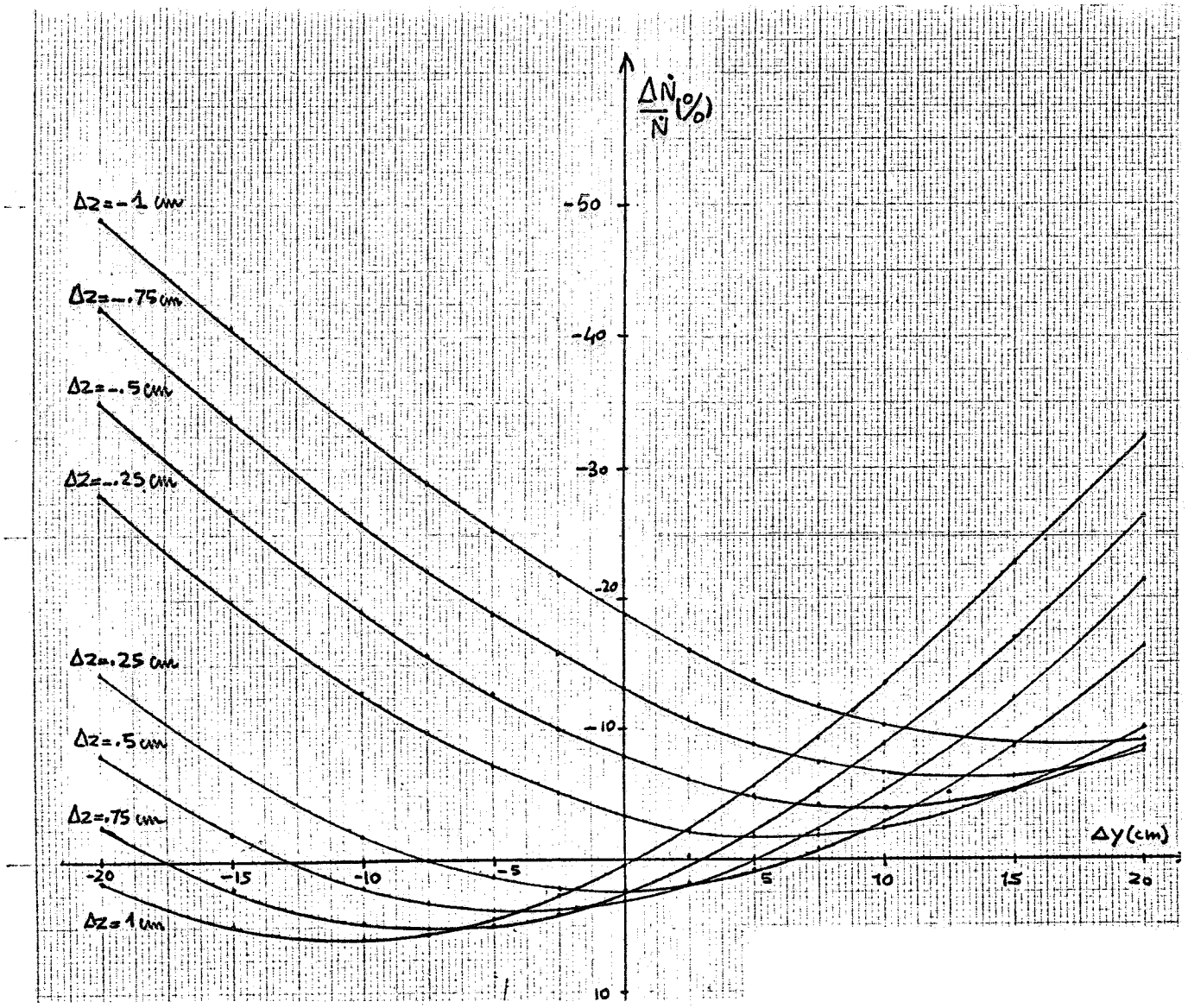

σ = 20 cm
VERTICAL AND
AZIMUTHAL
DISPLACEMENTS
FIVEFOLD
COINCIDENCES
Δz = -.75 cm
Δz = -1 cm
Δz = -.5 cm
Δz = -.25 cm
Δz = .25 cm
Δz = .5 cm
Δz = .75 cm
Δz = 1 cm
ΔN/N (%)
-50
-40
-30
-20
-10
10
Δy (cm)
15
10
5
-10
-15
-20

FIG. 25



δ= 30 cm

VERTICAL AND AZIMUTHAL DISPLACEMENTS

FIVEFOLD COINCIDENCES

FIG. 26

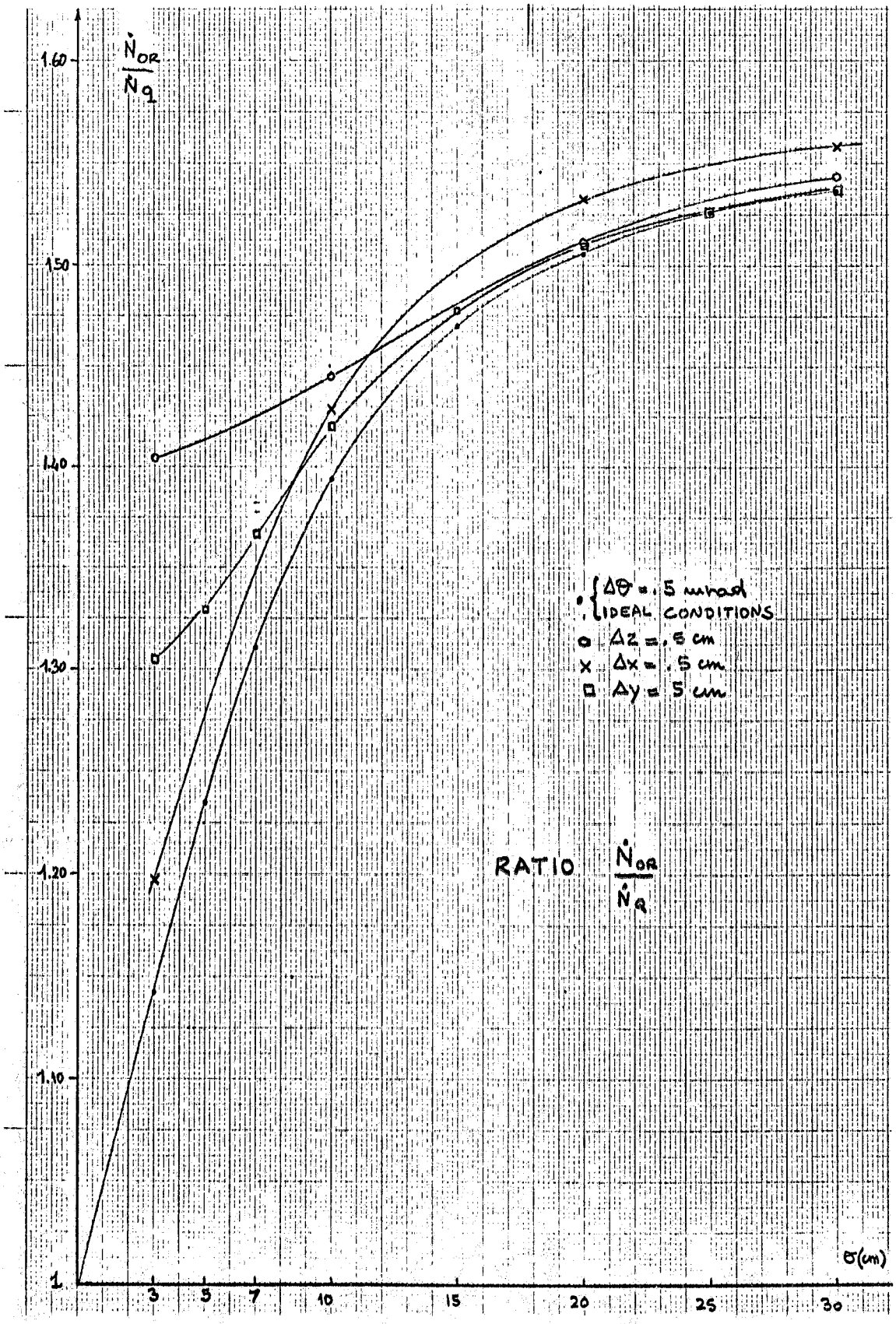

$\dot{N}_{OR}/\dot{N}_Q$
1.60
1.50
1.40
1.30
1.20
1.10
1
3
5
7
10
15
20
25
30
σ(cm)
Δθ = .5 mrad
IDEAL CONDITIONS
Δz = .5 cm
Δx = .5 cm
Δy = 5 cm
RATIO $\dot{N}_{OR}/\dot{N}_Q$

FIG. 27

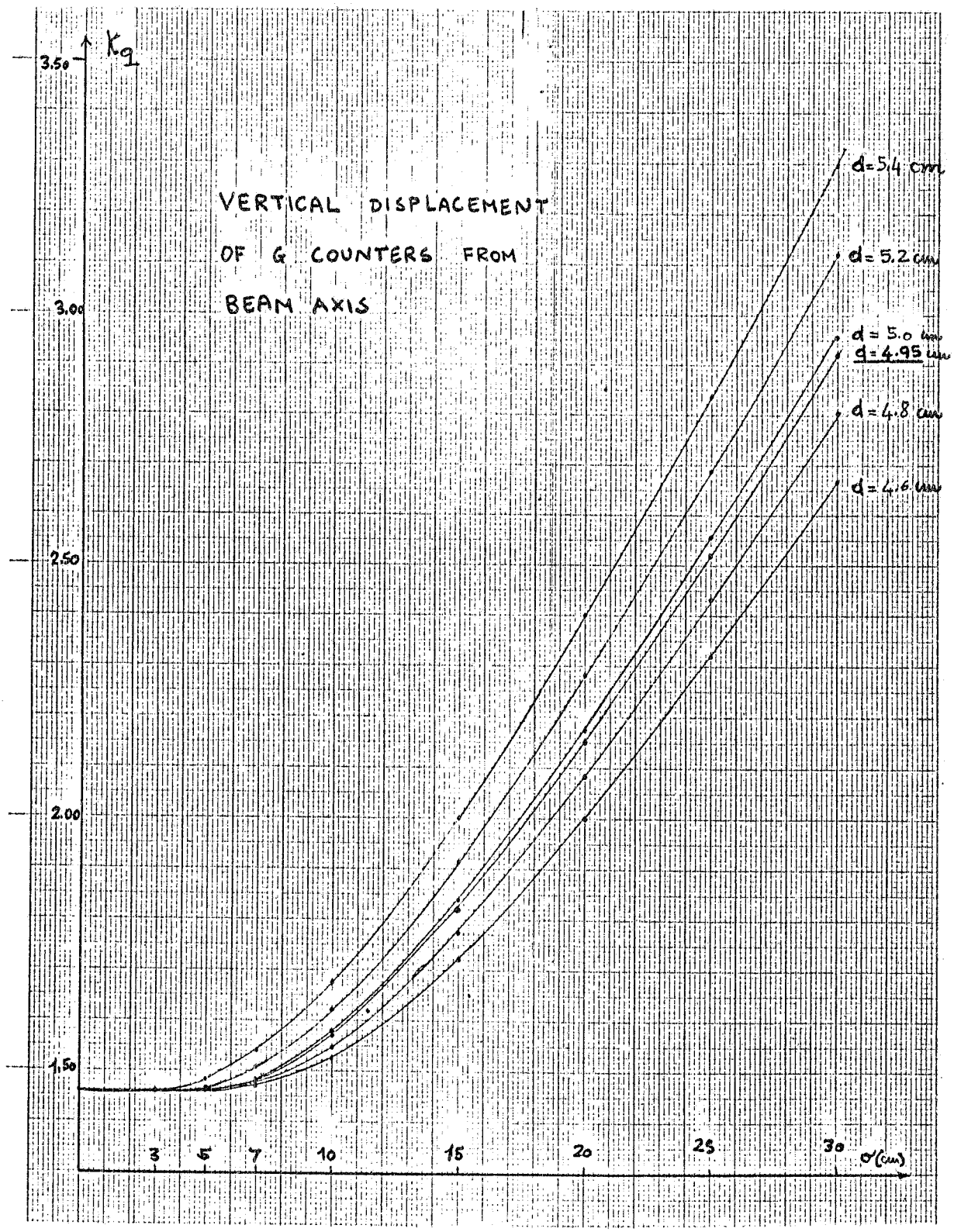

Kg
3.50
3.00
2.50
2.00
1.50
VERTICAL DISPLACEMENT
OF G COUNTERS FROM
BEAM AXIS
d=5.4 cm
d=5.2 cm
d=5.0 cm
d=4.95 cm
d=4.8 cm
d=4.6 cm
3
5
7
10
15
20
25
30
σ(cm)

FIG. 28

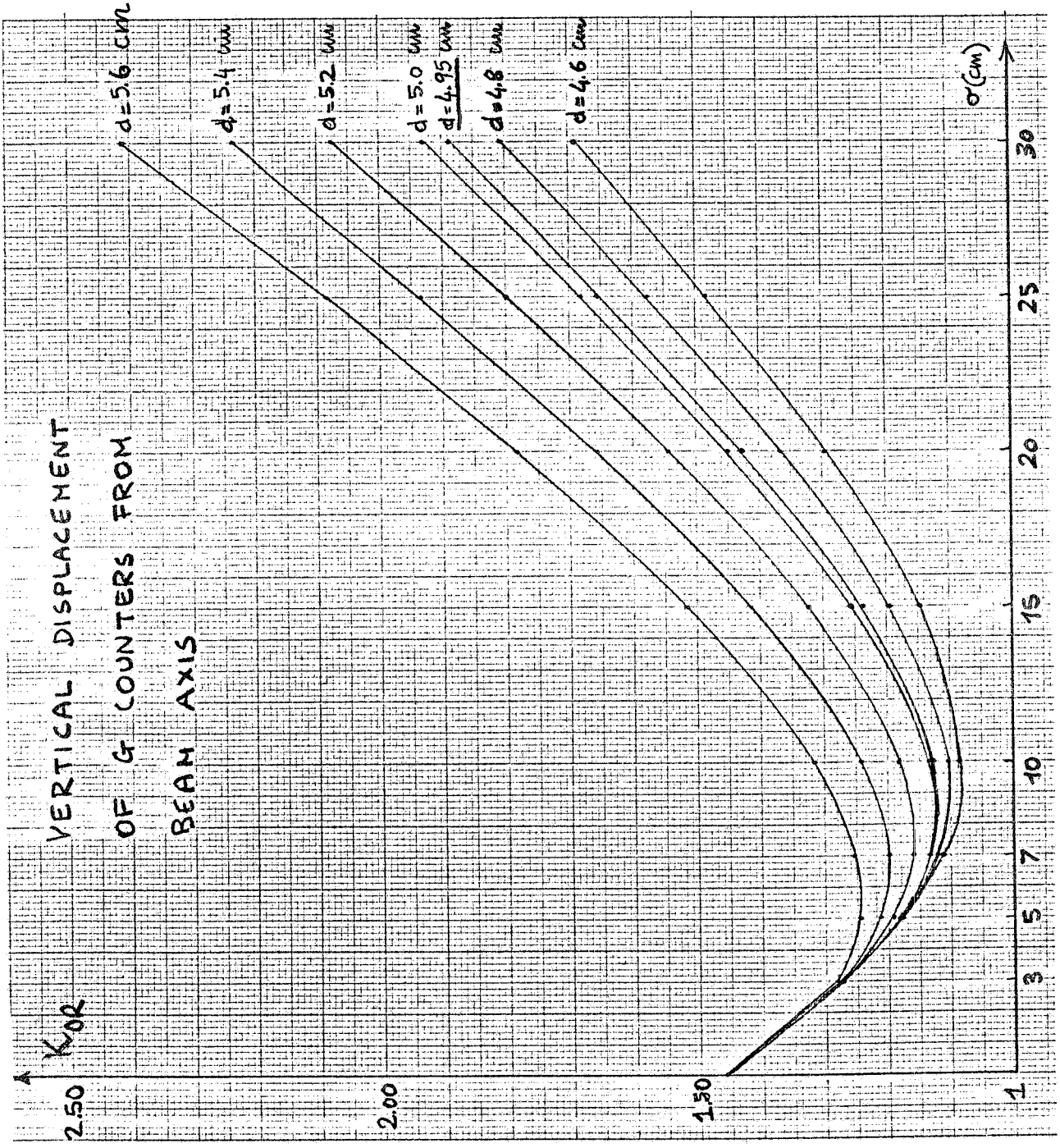

K_OR
VERTICAL DISPLACEMENT
OF G COUNTERS FROM
BEAM AXIS
d=5.6 cm
d=5.4 cm
d=5.2 cm
d=5.0 cm
d=4.95 cm
d=4.8 cm
d=4.6 cm
2.50
2.00
1.50
1
3
5
7
10
15
20
25
30
σ(cm)

FIG. 29