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On T-Invariance in Electron-Deuteron Elastic Scattering.

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It has been suggested by many authors [1] that a possible violation of T-invariance in the electromagnetic interactions of hadrons can be detected by a measurement of the recoil deuteron polarization in electron-deuteron elastic scattering. In fact, if T-reversal invariance is assumed for the matrix element of the electromagnetic current (in the one-photon exchange approximation), the vector polarization of the recoil deuteron must be zero if the initial state is unpolarized.

However, in a recent paper [2], Ramachandran suggested that an effect of polarization can be present also in the absence of a T-violating electromagnetic interaction if a phase between the S and D states of the deuteron is present.

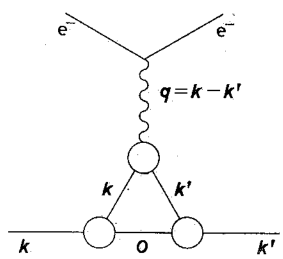

Fig. 1.

In this note we show, in the impulse approximation, that time-reversal invariance in the electromagnetic transition of the deuteron implies that the phase is 0° at 180° so that there is no discrepancy between the result of ref. [2] and the general conclusion obtained from Lorentz-invariance requirement on the electromagnetic vertex of a spin-one particle.

Following the notations of Gourdin [3] we write for the electromagnetic transition of the deuteron (see Fig. 1) the following expression:

$$Q^{m'm} = A\langle m'|\exp[i\boldsymbol{q}\boldsymbol{r}/2]|m\rangle + B_J\langle m'|\exp[i\boldsymbol{q}\boldsymbol{r}/2]\,S_J|m\rangle = Q_S^{m'm} + Q_V^{m'm}\,, \tag{1}$$

where m', m are the deuteron helicities, the functions A, B_J are functions of q^2 and of the spin of the electrons but they do not depend on the spin of the deuteron. The matrix elements in (1) are radial integrals involving the wave function of the deuteron and $S_J = S_J^p + S_J^n$ is the deuteron spin operator.

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(1) I. Kobzarev, L. B. Okun and M. V. Terentov: *JETP Lett.*, **2**, 289 (1965); V. M. Dubovik and A. A. Cheshkov: *Sov. Phys. JETP*, **24**, 111 (1967); V. M. Dubovik, E. P. Lithtman and A. A. Cheshkov: *Sov. Phys. JETP*, **25**, 464 (1967).

(2) G. Ramachandran: *Phys. Rev. Lett.*, **22**, 794 (1969).

(3) M. Gourdin: *Diffusion des électrons de haute énergie* (Paris, 1966).

The deuteron wave function can be written as

$$(2)\qquad \psi_m(r) = (10,\, m0|1m)\,\chi_m\, Y_{00}(\theta,\varphi)\, u_0(r) + \\ + \sum_{m_1} (12,\, m_1 m - m_1|1m)\,\chi_{m_1} Y_{2m-m'}(\theta,\varphi)\, u_2(r) = \psi_m^S + \psi_m^D\,,$$

where $(J_1 J_2,\, m_1 m_2|J m_1 + m_2)$ are the usual Clebsch-Gordan coefficients of SU_2, the Y_{Jm} functions are spherical harmonics, $u_0(r)$, $u_2(r)$ are the radial wave functions of the S and D states and χ_m is the spinor associated to the triplet state. In terms of this wave function we have

$$(3)\qquad Q_S^{m'm} = A\big[(\psi_{m'}^S,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^S) + (\psi_{m'}^D,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^D) + \\ + (\psi_{m'}^S,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^D) + (\psi_{m'}^D,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^S)\big]\,,$$

where [4]

$$(4)\qquad \left\{\begin{aligned}
&(\psi_{m'}^S,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^S) = \delta_{m'm}(10,\, m0|1m)^2 \int r^2 |u_0(r)|^2 J_0\left(\frac{qr}{2}\right) \mathrm{d}r\,,\\
&(\psi_{m'}^S,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^D) = \delta_{m'm}[22,\, 00|00](10,\, m0|1m)(12,\, m0|1m)\cdot\\
&\qquad\qquad \cdot \int r^2 u_0^*(r)\, u_2(r)\, J_2\left(\frac{qr}{2}\right) \mathrm{d}r\,,\\
&(\psi_{m'}^D,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^S) = \delta_{m'm}[20,\, 00|20](10,\, m0|1m)(12,\, m0|1m)\cdot\\
&\qquad\qquad \cdot \int r^2 u_0(r)\, u_2^*(r)\, J_2\left(\frac{qr}{2}\right) \mathrm{d}r\,,\\
&(\psi_{m'}^D,\, \exp[i\boldsymbol{qr}/2]\,\psi_m^D) = \delta_{m'm} \sum_{l=0,2,4} \int r^2 |u_2(r)|^2 J_l\left(\frac{qr}{2}\right) \mathrm{d}r\cdot\\
&\qquad\qquad \cdot \sum_{m_1} [12,\, 0m - m_1|2m - m_1]\,(12,\, m_1 m - m_1|1m)^2\,,
\end{aligned}\right.$$

and similarly for the vector term:

$$(5)\qquad Q_V^{m'm} = B_J\big[(\psi_{m'}^S,\, \exp[i\boldsymbol{qr}/2]\, S_J\,\psi_m^S) + (\psi_{m'}^D,\, \exp[i\boldsymbol{qr}/2]\, S_J\,\psi_m^D) + \\ + (\psi_{m'}^S,\, \exp[i\boldsymbol{qr}/2]\, S_J\,\psi_m^D) + (\psi_{m'}^D,\, \exp[i\boldsymbol{qr}/2]\, S_J\,\psi_m^S)\big]\,,$$

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(4) We use the following definition:

$$[J_1 J_2,\, m_1 m_2|J m_1 + m_2] = a(J_1 J_2 J m_1 m_2)(J_1 J_2,\, 00|J0)(J_1 J_2,\, m_1 m_2)(J m_1 + m_2)\,,$$

where

$$a(J_1 J_2 J m_1 m_2) = \frac{1}{4}\sqrt{\frac{2}{\pi}\,\frac{(2J_1+1)(2J_2+1)}{2J+1}}\cdot\sqrt{\frac{(J_1-m_1)!}{(J_1+m_1)!}}\sqrt{\frac{(J_2-m_2)!}{(J_2+m_2)!}}\sqrt{\frac{(J+m_1+m_2)!}{(J-m_1-m_2)!}}\,.$$

with [5]

$$
(6)\quad
\begin{cases}
(\psi^S_{m'}, \exp[i\boldsymbol{qr}/2]\,S_J\,\psi^S_m) = C(m'm|J)(10,\, m0|1m)^2\int r^2|u_0(r)|^2 J_0\left(\dfrac{qr}{2}\right)\mathrm{d}r\,,\\[2ex]
(\psi^S_{m'}, \exp[i\boldsymbol{qr}/2]\,S_J\,\psi^D_m) = C(m'm|J)\big[22,\,00|00\big](10,\, m'0|1m')\cdot\\
\qquad\qquad\cdot(12,\, m0|1m)\int r^2 u_0^*(r)\,u_2(r)\,J_2\left(\dfrac{qr}{2}\right)\mathrm{d}r\,,\\[2ex]
(\psi^D_{m'}, \exp[i\boldsymbol{qr}/2]\,S_J\,\psi^S_m) = C(m'm|J)\big[20,\,00|20\big](10,\, m0|1m)\cdot\\
\qquad\qquad\cdot(12,\, m'0|1m')\int r^2 u_0(r)\,u_2^*(r)\,J_2\left(\dfrac{qr}{2}\right)\mathrm{d}r\,,\\[2ex]
(\psi^D_{m'}, \exp[i\boldsymbol{qr}/2]\,S_J\,\psi^D_m) = \sum\limits_{m_1} C(m,\, m'-m+m_1|J)\sum\limits_{l=0,2,4}\big[12,\, 0m-m_1|2m-m_1\big]\cdot\\
\qquad\cdot(12,\, m_1 m-m_1|1m)\cdot(12,\, m'-m+m,\, m-m_1|1m')\int r^2|u_2(r)|^2 J_l\left(\dfrac{qr}{2}\right)\mathrm{d}r\,.
\end{cases}
$$

From these expressions we see that only the real part of the photon wave function $\exp[i\boldsymbol{qr}/2]$ gives contribution to the transition.

Time-reversal invariance implies that

$$(7)\quad A(\psi_{m'}, \exp[i\boldsymbol{qr}/2]\,\psi_m) = A^*(T\psi_{m'}, \exp[i\boldsymbol{qr}/2]\,T\psi_m)^* = A^*(\psi_{m'}, \exp[i\boldsymbol{qr}/2]\,\psi_m)^*\,,$$

$$(8)\quad B_J(\psi_{m'}, \exp[i\boldsymbol{qr}/2]\,S_J\,\psi_m) = B_J^*(T\psi_{m'}, \exp[i\boldsymbol{qr}/2]\,S_J\,T\psi_m)^* = -B_J^*(\psi_{m'}, \exp[i\boldsymbol{qr}/2]\,S_J\,\psi_m)^*\,,$$

where the T-reversal operation on the triplet spinor is defined as

$$(9)\qquad T\chi_m = (\sigma_2^{\mathrm{p}}\otimes\sigma_2^{\mathrm{n}})\,\chi_m = (-1)^m\chi_{-m}\,.$$

We observe from equations (4) that the matrix element $(\psi_{m'}, \exp[i\boldsymbol{qr}/2]\,\psi_m)$ is real, so equation (7) reduces to the condition $A = A^*$ and does not say anything about the phase between the S and D states. However equation (8) implies $B_J = -B_J^*$ (this is obtained for $m = m'$) and the condition

$$(10)\qquad \int \sin\delta(r)\,|u_0(r)\,u_2(r)|\,J_2\left(\frac{qr}{2}\right) r^2\,\mathrm{d}r = 0\,.$$

This is just $\sin\delta_{SD}$, according to the definition in the Ramachandran work [6] where the « transition form factors » are defined as

$$(11)\qquad F_l^{l_1 l_2}(q^2) = \int u_{l_1}(r)\,u_{l_2}^*(r)\,J_l\left(\frac{qr}{2}\right) r^2\,\mathrm{d}r$$

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(5) We put $C(m'm|J) = (\chi_{m'}, S_J\,\chi_m)$.

(6) G. RAMACHANDRAN: *Nucl. Phys.*, 2 B, 565 (1967).

and the phase between the S and D states

$$\text{(12)} \qquad \operatorname{tg}\delta_{SD}=\frac{i(F_2^{02}-F_2^{20})}{F_2^{20}+F_2^{02}}=\frac{\int\sin\delta(r)|u_0(r)\,u_2(r)|J_2(qr/2)\,r^2\,\mathrm{d}r}{\int\cos\delta(r)|u_0(r)\,u_2(r)|J_2(qr/2)\,r^2\,\mathrm{d}r}\,.$$

Then, since the recoil deuteron polarization [2] is proportional to $\operatorname{tg}\delta_{SD}$, we conclude that the presence of the vector polarization already implies a T-violation in the electromagnetic transition.

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