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E. Etim[x] : ADLER'S NEUTRINO SUM RULES IN THE SCALING LIMIT AND THE POSSIBILITY OF A CHIRAL POMERON[o]. -

Recently Drell, Lavy and Yan (DLY)[1] have made an experimentally challenging prediction consisting in the statement that the ratio of neutrino - to antineutrino - nucleon total and differential cross-sections with respect to the Bjorken variable[2] $\omega = q^2/2M\nu$ is 3 to 1. If confirmed such a prediction would imply according to the prevalent ideas about dual Pomeron-dominated scale invariance[3] that the Pomeron is chiral in that it distinguishes between left and right-handed neutrinos. To dismiss such a repugnant possibility and any other of its kind that may embarrass the Pomeron exchange model of scaling Harari[4] has argued that Adler's neutrino sum rules[5] should be either trivially satisfied and therefore useless or badly violated and therefore meaningless.

In this letter we show that the 3 to 1 ratio can be accommodated in a diffractive model of scaling and that it in no way implies that the Pomeron is chiral. To this end we have studied the implications of the three neutrino sum rules of Adler[5]

$$(1a) \qquad 1 = g_A^2(q^2) + \int_{M+m_\pi}^{\infty} \frac{K\,dK}{M} (W_{2A}^- - W_{2A}^+)$$

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(1b) $$1 = F_{1v}^2(q^2) + q^2\left(\frac{\mu_v}{2M}\right)^2 F_{2v}^2(q^2) + \int_{M+m_\pi}^{\infty} \frac{K\,dK}{M}\,(W_{2v}^- - W_{2v}^+)$$

(2) $$2 = \left(1 + \frac{q^2}{4M^2}\right) g_A^2(q^2) + \frac{q^2}{4M^2}(F_{1v}(q^2) + \mu_v F_{2v}(q^2))^2 + \int_{M+m_\pi}^{\infty} \frac{K\,dK}{M}\,(W_1^- - W_1^+)$$

(3) $$-g_A(q^2)\,(F_{1v}(q^2) + \mu_v F_{2v}(q^2)) = \frac{1}{2}\int_{M+m_\pi}^{\infty} \frac{K\,dK}{M}\,(W_3^- - W_3^+)$$

bearing in mind the following important features of scaling:

a) Although theoretically scaling should set in for asymptotic energy and momentum transfer ν, $q^2 \to \infty$, experimentally the universal behaviour is already remarkably well verified for values of q^2 that cannot respectably be considered asymptotic[6, 7, 8].

b) The phenomenological analysis of the SLAC-MIT data[6] by Nauenberg[9] aimed at critically testing the validity of various theoretical models proposed to explain the observed universal behaviour of the inelastic electromagnetic from factors W_1, W_2 has made it clear that Pomeranchuk exchange mechanism is not incompatible with scaling but that the latter is a more stringent requirement containing experimentally testable details that cannot be supplied by the diffractive model alone. In the above equations (1)-(3) the structure functions $W_j^{\pm}$ (j=1, 2, 3) are those which describe neutrino (+) and antineutrino (-) scattering off protons; the double differential $\nu(\bar{\nu})$ - p cross-section is expressed in terms of them as[1]

(4) $$\frac{d^2\sigma(\nu,\bar{\nu})}{d\nu\, dq^2} = \frac{G^2\cos^2\vartheta_c}{2\pi E^2}\left[\frac{q^2}{2}W_1^{\pm} + \left(E^2 - E\nu - \frac{q^2}{4}\right)W_2^{\pm} \pm q^2\left(\frac{2E-\nu}{4M}\right)W_3^{\pm}\right]$$

where G is the weak interaction Fermi coupling, ϑ_c the Cabibbo angle, E the incident neutrino energy in the reaction

$$\nu_1(\bar{\nu}_1) + p \to 1(\bar{1}) + \text{HADRONS}$$

q^2 the invariant momentum transfer and $\nu = -p\cdot q/M$ the energy transferred to the hadrons in the rest frame of the proton whose mass is M. The continuum integrals in eqs. (1)-(3) are taken over the invariant mass

$K = (M^2 + q^2 + 2M\nu)^{1/2}$ of the produced hadrons.

Our analysis commences with the following observations based on the feature (a) above and on generally accepted ideas of the dual diffractive model of scaling[3,4] :

(i) the argument of DLY that the structure functions $W_3^{\pm}$, which in their model must be large in order to get the 3 to 1 ratio of neutrino to antineutrino reactions, are Pomeron-dominated is unacceptable since $W_3^{\pm}$ arise from the interference of the weak vector and axial vector matrix elements and the Pomeranchuk singularity cannot contribute to them if we do not entertain the possibility of G-parity violation[10].

(ii) $W_3^{\pm}$ have the quantum numbers of ω and ϕ ordinary Regge exchanges in the crossed channel[10] and since in the dual scheme the contributions of ordinary exchanges do not scale[4] $W_3^{\pm}$ will always remain negligibly small in the scaling limit.

(iii) from the point of view of the diffractive model with its built in duality the circumstance that $W_3^{\pm}$ are dominated by ordinary exchanges is an attractive and very welcome conspiracy since from the third of Adler's sum rules eq. (3), it follows that the q^2-dependence of the residue functions $\beta_\omega(q^2)$, $\beta_\phi(q^2)$ of these ordinary exchanges is not only different from that of the Pomeranchuk pole $\beta_p(q^2)$ but that this q^2-dependence is at most the same as that of the elastic (and similarly resonant-excitation) form factor $G_v(q^2) = F_{1v}(q^2) + \mu_v F_{2v}(q^2)$[3,4]. If eq. (3) is then understood as an asymptotic statement for large $q^2 \to \infty$ then its validity gives meaning to and provides some justification for the basic assumptions that prop up the difractive model itself.

(iv) making use of the experimental small q^2 feature (a) of scaling for the Pomeron-dominated structure functions[11] $W_{1,2}^{\pm}$ then the second sum rule, eq. (2), is satisfied to within 30% by the pole terms alone if one takes $g_A^2(q^2 \sim 0) \simeq g_A^2(0) = 1.4$; eq. (1b) is saturated by the pole terms alone in the small q^2 limit since $F_{IV}(q^2 \to 0) = 1$[12]. Eq. (1a) is also valid since for small q^2 one can pass into a "parallel configuration" of the leptons $\nu_1(\bar{\nu}_1)$ and $1\,(\bar{1})$[13], use PCAC and get the original Adler-Weisberger sum rule[14].

The overall picture that has emerged from the above considerations is that contrary to Harari's conclusions[4] Adler's sum rules considered in appropriate kinematical regions should not be too flagrantly violated if the basic assumptions of the dual diffractive model are to be accepted and its predictions reconcilable with the experimental evidence of small q^2 manifestation of scaling.

Turning to the second aspect (b) of scale invariance we go on to show that there is some evidence from the combined SLAC-MIT[6] and CERN bubble chamber data[7] that there is a detail, more precisely a 3 to 1 ratio of neutrino to antineutrino events which follows from scaling and Adler's first sum rule but which is difficult to understand with conventional dual Regge theory. Because of the present size of the experimental errors this evidence would have to await the accumulation of more refined data before it can be considered decisive. However as of now this is how matters stand(15); using C V C the isotropy of isotopic spin space allows one to write in the scaling limit

$$\int_{M+m_\pi}^{\infty} \frac{K\,dK}{M} W_{2v}^{+} \simeq \int_0^1 d\omega\, \nu W_{2v}^{+} = 2 \int_0^1 d\omega\, \nu W_2(I = 1) \tag{5}$$

where $\omega = q^2/2M\nu$ is the Bjorken variable and $W_2(I=1)$ and $W_2(I=0)$ are the isovector and isoscalar contributions respectively to the electromagnetic structure function W_2. From the SLAC-MIT data under the assumption that the longitudinal photon-proton cross-section σ_L is negligible with respect to the transverse σ_T one finds(15)

$$\int_0^1 d\omega\, \nu W_2 = 0.18 \pm 0.02 \tag{6}$$

from which on making use of the octet character of the hadronic electromagnetic current and neglecting any isovector-isoscalar interference gives :

$$\int d\omega\, \nu W_{2v}^{+} = 0.26 \pm 0.04 \,. \tag{7}$$

Now integrating eq. (4) over all ν and q^2 with $W_1^{\pm} = \nu^2/q^2 W_2^{\pm}$ under the assumption (of observation (ii) above) that the contributions of ordinary exchanges (W_3^{+}) are negligible in the scaling limit one finds :

$$\sigma_{tot}(\nu p) = \left(\frac{G^2}{\pi} \cos^2\vartheta_c\, ME\right) \frac{2}{3} \int_0^1 d\omega\, \nu W_2^{+} \tag{8}$$

which on comparison with the CERN bubble chamber result(7) :

$$\sigma_{tot}(\nu p) = (0.6 \pm 0.15)\, \frac{G^2}{\pi}\, ME$$

gives

$$\int_0^1 d\omega\, \nu W_2^{+} = 0.9 \pm 0.2 \tag{10}$$

whence using (7) one finds :

$$\int_0^1 d\omega\, \nu W_{2A}^{+} = 0.64 \pm 0.24 . \qquad (11)$$

Comparing eq. (7) with (11) it may appear a little bit surprising that the axial vector contribution which is dominated by the $A_1(1070)$ should be greater than the vector contribution eq. (7) which is dominated by its lighter chiral partner $\rho^0(760)$[15]. To understand this fact it must be recognised, as pointed out by the CERN experimenters themselves[7], that most of their events, although they verify scaling, do not lie at all in the asymptotic region ν, $q^2 \to \infty$, $\omega \ll 1$ of strict scale invariance and that once one passes from the pair of variables ν, q^2 to the variables ω and the inelasticity $\eta = \nu/E$ used in integrating eq. (4) one automatically loses control over the values of q^2 and as stressed by Nauenberg[19] scaling would set in even for small values of q^2. We are inclined to believe that this is what has happened since the small q^2 manifestation of scale invariance is what is most remarkable about it[6, 7, 8]. For small q^2 we can legitimately pass into a parallel configuration (or one that is nearly so) for neutrino-proton scattering and evaluate eq. (1) in that limit. Under such circumstances the conservation of the weak vector current guarantees that the vector current as apposed to the axial vector current contributes a small part to the inelastic structure functions $W_2^{\pm}$ but dominate the elastic form factors[13, 16]; this accounts for the apparently paradoxical difference between eqs. (7) and (11). There are two other important consequences of this fact; the first is that it leads one to suspect that the sum

$$\nu W_2^{\pm} = \nu W_{2A}^{\pm} + \nu W_{2v}^{\pm} = F_2^{\pm} = \text{const.} \qquad (12)$$

is a constant independent of ω. Eq. (12) seems in fact to be verified by the CERN data which are consistent with[7] :

$$\nu W_2^{+} = F_2^{+} = 0.9 \pm 0.3 \qquad (13)$$

in agreement with (10) and (12). The other consequence is that on applying the small q^2 limit to approximate $g_A^2(q^2)$ by $g_A^2(0) = 1.426$ in eq. (1a) one obtains via scaling :

$$\int_0^1 d\omega\, (\nu W_{2A}^{+} - \nu W_{2A}^{-}) = 0.426 \qquad (14)$$

which together with eq. (11) gives :

$$\int_0^1 d\omega \, \nu W^-_{2A} = 0.214 \tag{15}$$

a value about one-third that in eq. (11); of course since we know what $g_A^2(q^2 \sim 0)$ in eq. (1a) must be from the Adler-Weisberger relation[14] we could have reversed our reasoning and taken the 3 to 1 ratio of DLY[1] in good faith. Then on going to find out the consequences of the DLY prediction we would have found the correct value of $g_A^2(0)$ without having to use CVC and all that. In fact making use of the constancy of $\nu W_2^{\pm}$ from eq. (12), the circumstances that led to it together with eq. (15) one finds that the constant value of νW_2^- is consistent with $\nu W_2^- = 1/3 \, \nu W_2^+$; in other words both total and differential cross-sections with respect to $\omega = q^2/2M\nu$ of neutrino and antineutrino scattering off nucleons are in the ratio of 3 to 1 as predicted by DLY[1]. This then is one of the experimentally testable details which only scale invariance can supply but which are inconceivable in a pure Pomeranchuk exchange model.

We argue nontheless that the dual diffractive model can in fact accommodate such a possibility, and more, if as we have shown previously[17] the two aspects of scaling and Pomeranchuk dominance of the electromagnetic and weak structure functions $W_{1,2}$ and $W^{\pm}_{1,2}$ respectively are decoupled. By this we mean that the Bjorken behaviour[2] of the structure functions is determined exclusively by a combination of polarisation functions which depend both on the polarisation (helicity) states of the incident particle and those of the vector and axial vector mesons which dominate the probing currents. The Regge behaviour of the structure functions on the other hand comes from the Pomeranchuk dominance of the purely hadronic amplitudes of vector and axial vector meson-nucleon scattering. For instance in the case of virtual Compton scattering W_2 is given by[17]:

$$\nu W_2 = \frac{1}{\pi} \sum_{v=\rho^0,\omega,\phi} \left(\frac{m_v^2}{g_v}\right)^2 \sigma_T(v) \, C_v(\nu/q^2) \tag{16}$$

where $\sigma_T(v=\rho^0, \omega, \phi)$ is the total vector meson-proton cross-section and $C_v(\nu/q^2)$ is a combination of a set of polarisation functions $C^{(v)}_{\lambda\lambda'}(\nu/q^2)$

$$C_v(\nu/q^2) = \frac{1}{4} \sum_{\lambda\lambda'} C^{(v)}_{\lambda\lambda'}(\nu/q^2) \tag{17}$$

summed over the four polarisation directions (λ') of the neutral vector mesons and averaged over the polarisation directions (λ) of the virtual photon. As seen from eq. (16), since $\sigma_T(v)$ is practically constant at high energies, Pomeron influence on νW_2 is felt weakly through the constancy of these cross-sections while the burden of scaling is borne by the polarisation functions $C^{(v)}_{\lambda\lambda'}(\nu/q^2)$ which by their very definition contain experimentally interesting and testable details. In probing the nucleon structure in neutrino experiments the situation is similar but the need to specify polarisation details such as are contained only in functions like the $C^{(v)}_{\lambda\lambda}$, becomes even more pressing since the operators $1 \pm \gamma_5$ in the leptonic weak current project out only definite helicity states and there is a further complication in the fact that the weak hadronic current is an admixture of vector and axial vector parts. As in the electromagnetic case scaling is again determined by the behaviour of a definite combination of polarisation functions which, a priori, are different for different helicity configurations but the contributions of the Pomeranchuk pole to $W^{\pm}_{1,2}$, coming from purely hadronic amplitudes, are the same independent of the sign ($\pm$) because of the equal coupling of the Pomeron to the $I_3 = \pm 1$ components of the hadronic weak current. The Pomeron is thus not chiral and its weak coupling cannot be held responsible for any helicity dependence of the structure functions $W^{\pm}_{1,2}$ if the combined effect of scaling and Pomeron dominance turns out to be helicity dependent. The above reasoning is reinforced by the knowledge that helicity arguments provide a unique and absolute way of distinguishing between neutrinos and antineutrinos and experimenters are therefore urged to supply much needed data which besides suggesting new ideas on the nature of scaling will either shoot down or confirm the 3 to 1 ratio of Drell, Levy and Yan.

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(16) - The reason for this is that in the "elastic" weak process $\nu_1 + p \to 1 + p$ we cannot go into the parallel configuration for small q^2 and $K = M$ (the proton mass) and in this case a conserved vector current contributes strongly in the forward direction (see in particular ref. (13) and other references cited there in).
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