

COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-70/20
19 Maggio 1970

S. Ferrara and A. F. Grillo: ON T-INVARIANCE IN
ELECTRON-DEUTERON ELASTIC SCATTERING. -

Nota interna: n. 476
19 Maggio 1970

S. Ferrara and A. F. Grillo: ON T-INVARIANCE IN ELECTRON-
DEUTERON ELASTIC SCATTERING. -

It has been suggested by many authors⁽¹⁾ that a possible violation of T-invariance in the electromagnetic interactions of hadrons can be detected by a measurement of the recoil deuteron polarization in electron deuteron elastic scattering. In fact, if T-reversal invariance is assumed for the matrix element of the electromagnetic current (in the one photon exchange approximation), the vector polarization of the recoil deuteron must be zero if the initial state is unpolarized.

However, in a recent paper⁽²⁾, G. Ramachandran suggested that an effect of polarization can be present also in absence of T-violating electromagnetic interaction if a phase between the S and D states of the deuteron is present.

In this note we show, in the impulse approximation, that time-reversal invariance in the electromagnetic transition of the deuteron implies that the phase is 0° or 180° so that there is no discrepancy between the result of ref. (2) and the general conclusion obtained from Lorentz invariance requirement on the electromagnetic vertex of a spin one particle.

Following the notations of M. Gourdin⁽³⁾ we write for the electromagnetic transition of the deuteron (see fig. 1) the following expression

$$(1) \quad Q^{m'm} = A \langle m' | e^{i\vec{q}\vec{r}/2} | m \rangle + B_J \langle m' | e^{i\vec{q}\vec{r}/2} S_J | m \rangle = Q_S^{m'm} + Q_V^{m'm}$$

2.

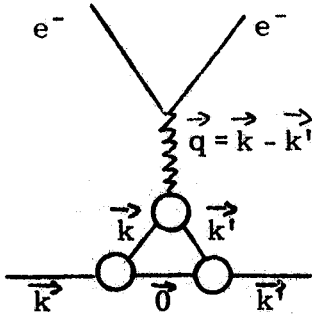


FIG. 1

where m', m are the deuteron helicities, the functions A, B_J are functions of q^2 and of the spin of the electrons but they do not depend on the spin of the deuteron. The matrix elements in (1) are radial integrals involving the wave function of the deuteron and $S_J = S_J^P + S_J^N$ is the deuteron spin operator.

The deuteron wave function can be written as :

$$\begin{aligned} \Psi_m(r) &= (10, m0|1m) \chi_m Y_{00}(\theta, \varphi) u_0(r) + \\ (2) \quad &+ \sum_{m_1} (12, m_1 m - m_1|1m) \chi_{m_1} Y_{2m-m_1}(\theta, \varphi) u_2(r) = \Psi_m^S + \Psi_m^D \end{aligned}$$

where $(J_1 J_2, m_1 m_2 | J m_1 + m_2)$ are the usual Clebsh-Gordon coefficients of $SU(2)$, the Y_{Jm} functions are spherical harmonics, $u_0(r), u_2(r)$ are the radial wave functions of the S and D states and χ_m is the spinor associated to the triplet state. In terms of this wave function we have :

$$\begin{aligned} Q_S^{m'm} &= A \left[(\Psi_{m'}^S, e^{i\vec{q}\vec{r}/2} \Psi_m^S) + (\Psi_{m'}^D, e^{i\vec{q}\vec{r}/2} \Psi_m^D) + \right. \\ (3) \quad &+ \left. (\Psi_{m'}^S, e^{i\vec{q}\vec{r}/2} \Psi_m^D) + (\Psi_{m'}^D, e^{i\vec{q}\vec{r}/2} \Psi_m^S) \right] \end{aligned}$$

where(4) :

$$\begin{aligned} (\Psi_{m'}^S, e^{i\vec{q}\vec{r}/2} \Psi_m^S) &= \delta_{m'm} (10, m0|1m)^2 \int r^2 |u_0(r)|^2 J_0\left(\frac{qr}{2}\right) dr \\ (\Psi_{m'}^S, e^{i\vec{q}\vec{r}/2} \Psi_m^D) &= \delta_{m'm} [22, 00|00] (10, m0|1m) (12, m0|1m) \times \\ &\times \int r^2 u_0^*(r) u_2(r) J_2\left(\frac{qr}{2}\right) dr \\ (4) \quad (\Psi_{m'}^D, e^{i\vec{q}\vec{r}/2} \Psi_m^S) &= \delta_{m'm} [20, 00|20] (10, m0|1m) (12, m0|1m) \times \\ &\times \int r^2 u_0(r) u_2^*(r) J_2\left(\frac{qr}{2}\right) dr \\ (\Psi_{m'}^D, e^{i\vec{q}\vec{r}/2} \Psi_m^D) &= \delta_{m'm} \sum_{l=0, 2, 4} \int r^2 |u_2(r)|^2 J_l\left(\frac{qr}{2}\right) dr \times \end{aligned}$$

$$\times \sum_{m_1} [12, 0m-m_1 | 2m-m_1] (12, m_1 m-m_1 | 1m)^2$$

and similarly for the vector term :

$$Q_V^{m'm} = B_J \left[(\psi_{m'}^S, e^{i\vec{q}\vec{r}/2} S_J \psi_m^S) + (\psi_{m'}^D, e^{i\vec{q}\vec{r}/2} S_J \psi_m^D) + \right. \\ \left. + (\psi_{m'}^S, e^{i\vec{q}\vec{r}/2} S_J \psi_m^D) + (\psi_{m'}^D, e^{i\vec{q}\vec{r}/2} S_J \psi_m^S) \right] \quad (5)$$

with (5):

$$(\psi_{m'}^S, e^{i\vec{q}\vec{r}/2} S_J \psi_m^S) = C(m'm | J) (10, m0 | 1m)^2 \int r^2 |u_0(r)|^2 J_0\left(\frac{qr}{2}\right) dr$$

$$(\psi_{m'}^S, e^{i\vec{q}\vec{r}/2} S_J \psi_m^D) = C(m'm | J) [22, 00 | 00] (10, m'0 | 1m') \times \\ \times (12, m0 | 1m) \int r^2 u_0^*(r) u_2(r) J_2\left(\frac{qr}{2}\right) dr$$

$$(\psi_{m'}^D, e^{i\vec{q}\vec{r}/2} S_J \psi_m^S) = C(m'm | J) [20, 00 | 20] (10, m0 | 1m) \times \\ \times (12, m'0 | 1m') \int r^2 u_0(r) u_2^*(r) J_2\left(\frac{qr}{2}\right) dr \quad (6)$$

$$(\psi_{m'}^D, e^{i\vec{q}\vec{r}/2} S_J \psi_m^D) = \sum_{m_1} C(m, m'-m+m_1 | J) \times \\ \times \sum_{l=0,2,4} [12, 0m-m_1 | 2m-m_1] (12, m_1 m-m_1 | 1m) \times \\ \times (12, m'-m+m, m-m_1 | 1m') \int r^2 |u_2(r)|^2 J_1\left(\frac{qr}{2}\right) dr$$

From these expressions we see that only the real part of the photon wave function $e^{i\vec{q}\vec{r}/2}$ gives contribution to the transition.

Time-reversal invariance implies that :

$$A(\psi_{m'}^S, e^{i\vec{q}\vec{r}/2} \psi_m^S) = A^*(T \psi_{m'}^S, e^{i\vec{q}\vec{r}/2} T \psi_m^S) = \\ = A^*(\psi_{m'}^S, e^{i\vec{q}\vec{r}/2} \psi_m^S) \quad (7)$$

4.

$$\begin{aligned}
 B_J(\psi_{m'}, e^{i\vec{q}\vec{r}/2} S_J \psi_m) &= B_J^*(T \psi_{m'}, e^{i\vec{q}\vec{r}/2} S_J T \psi_m)^* = \\
 (8) \qquad \qquad \qquad &= -B_J^*(\psi_{m'}, e^{i\vec{q}\vec{r}/2} S_J \psi_m)^*
 \end{aligned}$$

where the T-reversal operation on the triplet spinor is defined as :

$$(9) \qquad T \chi_m = (\sigma_2^p \otimes \sigma_2^n) \chi_m = (-1)^m \chi_{-m}$$

We observe from equations (4) that the matrix element $(\psi_{m'}, e^{i\vec{q}\vec{r}/2} \psi_m)$ is real so equation (7) reduces to the condition $A = A^*$ and does not say anything about the phase between the S and D states. However equation (8) implies $B_J = -B_J^*$ (this is obtained for $m = m'$) and the condition :

$$(10) \qquad \int \sin \delta(r) |u_0(r) u_2(r)| J_2\left(\frac{qr}{2}\right) r^2 dr = 0.$$

This is just $\sin \delta_{SD}$, according to the definition of Ramachandran work⁽⁶⁾ where the "transition form factors" are defined as :

$$(11) \qquad F_1^{1112}(q^2) = \int u_{1_1}(r) u_{1_2}^*(r) J_1\left(\frac{qr}{2}\right) r^2 dr$$

and the phase between the S and D states :

$$\begin{aligned}
 \text{tg } \delta_{SD} &= \frac{i(F_2^{02} - F_2^{20})}{F_2^{20} + F_2^{02}} = \\
 (12) \qquad \qquad &= \frac{\int \sin \delta(r) |u_0(r) u_2(r)| J_2\left(\frac{qr}{2}\right) r^2 dr}{\int \cos \delta(r) |u_0(r) u_2(r)| J_2\left(\frac{qr}{2}\right) r^2 dr}
 \end{aligned}$$

Then, since the recoil deuteron polarization⁽²⁾ is proportional to $\text{tg } \delta_{SD}$, we conclude that the presence of the vector polarization already implies a T-violation in the electromagnetic transition.

Acknowledgement.

We thank Prof. A. Reale for having suggested the problem.

REFERENCES AND FOOTNOTES. -

- (1) - I. Kobzarev, L. B. Okun and M. V. Terentov, JEPT Letters 2, 289 (1965); V. M. Dubovik and A. A. Cheshkov, Soviet Phys. -JETP 24, 111 (1967); V. M. Dubovik, E. P. Litman and A. A. Cheshkov, Soviet Phys. -JETP 25, 464 (1967).
 (2) - G. Ramachandran, Phys. Rev. Letters 22, 794 (1969).
 (3) - M. Gourdin, Diffusion des electrons de haute energie, (Manon, Paris, 1966).
 (4) - We use the following definition :

$$\begin{aligned} \left[J_1 J_2 m_1 m_2 \mid J m_1 + m_2 \right] &= \\ &= a(J_1 J_2 J m_1 m_2) (J_1 J_2, 00 \mid J 0) (J_1 J_2, m_1 m_2 \mid J m_1 + m_2) \end{aligned}$$

where :

$$\begin{aligned} a(J_1 J_2 J m_1 m_2) &= \frac{1}{4} \sqrt{\frac{2}{\pi}} \frac{(2J_1 + 1)(2J_2 + 1)}{2J + 1} \times \\ &\times \sqrt{\frac{(J_1 - m_1)!}{(J_1 + m_1)!}} \sqrt{\frac{(J_2 - m_2)!}{(J_2 + m_2)!}} \sqrt{\frac{(J + m_1 + m_2)!}{(J - m_1 - m_2)!}} \end{aligned}$$

- (5) - We put :

$$C(m' m \mid J) = (\chi_{m'}, S_J \chi_m).$$

- (6) - G. Ramachandran, Nuclear Phys. B2, 565 (1967).