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M. Spadoni : DYNAMIC EFFECTS IN HARD SUPERCONDUCTORS.

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I. - INTRODUCTION. -

As the technological applications of superconductivity become more and more important, more and more people become interested in this field. One of the more important contributions to the development of advanced technology is related with the different applications of superconducting magnets which either have been done or will be presumably achieved in the near future[1-6]. But to reach a full use of the promised advantages in high magnetic fields, some considerations about the behaviour of superconductors carrying currents in the presence of a magnetic field are necessary.

After an introductory talk about the differences between the three classes of known superconductors (i. e. type I, type II and type III or hard), a panoramic account of the dissipative phenomena in type II and hard superconductors will be given and we will be dealing with flux flow resistivity, flux creep and flux jumps. The aforesaid phenomena are normally called dynamic phenomena and accordingly we are impelled to be "dynamic", because the available time is quite short. As a consequence we will give only a schematic survey and there will be a bibliographic aid for people which eventually after these lessons would like to go deeper onto the subject.

II. - DIFFERENT TYPES OF SUPERCONDUCTORS. -

A superconductor is a material which, below a certain critical temperature T_c, has zero electrical resistivity, $\rho = 0$; then the electric field is zero as well, $\vec{E} = 0$.

This property alone seems to indicate that the superconductor is only a perfect conductor, but there is another fundamental property, which makes a striking difference with a perfect conductor: the superconductor is a perfect diamagnet too, i. e. his magnetic induction $\vec{B}$ is zero when in presence of an applied external field. This phenomenon, the Meissner effect, is indipendent of the fact that the magnetic field has been applied either before undergoing the superconducting transition or after, i. e. the flux ($\vec{B} = 0$) is expelled upon cooling even when the magnetic field has been switched on before. From the Maxwell equations it can be seen that this does not happen in the case of a perfect conductor.

This is the reason of the additional equation, introduced by London[7], which, together with the Maxwell equations can account for the behaviour of a superconductors. The London equation is

(1) $$\left(4\pi \frac{\lambda_L^2}{c}\right) \vec{\nabla} \times \vec{J} + \vec{B} = 0$$

where J is the current density into the superconductor.

In solving eq. (1) one obtains that the induction inside a superconductor has an exponential decay

$$B(x) = B_o \, e^{-\frac{\lambda_L}{x}} ,$$

with a characteristic length λ_L of the order of $10^{-5} - 10^{-6}$ cm; thus from a macroscopic point of view, eq. (1) accounts for the perfect diamagnetism of a superconductor.

The length λ_L, the London penetration depth, is given by:

(2) $$\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2}$$

where n_s is the density of superconducting electrons, m their effective mass and e the electron charge.

We have now to introduce another characteristic length of a superconductor, the coherence length ξ. The microscopic theory of superconductivity, worked out by Bardeen, Cooper and Schrieffer[8] (B. C. S. theory) shows that the velocities of two electrons are correlated if their mutual distance is smaller than a certain length ξ. For pure metals the coherence length, called ξ_o, is given by:

(3) $$\xi_o = \frac{\hbar V_F}{\pi \Delta}$$

V_F is the Fermi velocity, Δ the superconducting energy gap. Normally for a pure metal $\xi_o \gg \lambda_L$, as an example in the case of Sn $\xi_o \simeq 2300$ Å, $\lambda_L = 355$ Å. All the superconducting materials for which $\xi_o \gg \lambda_L$ are called type I superconductors. It must be noted that by addition of impurities or by alloying both the coherence and the penetration depth are modified by mean free path effect in such a way that $\xi_o' < \xi_o$ and $\lambda_L' > \lambda_L$.

Thus it may occur that the ultimate material has now $\lambda_L \gg \xi$. The materials such that $\lambda_L > \xi_o$ are known as type II superconductors. There are only two pure metals for which $\lambda_L > \xi_o$, Nb and V, thus referred as intrinsic type II superconductors.

It must be shown[9] that London equation applies only if $\lambda_L \gg \xi_o$, i. e. in the case of type II superconductor; furthermore it is inadequate to treat situations implying variations of superconducting parameters over distances smaller than ξ_o, such as the behaviour of interphase boundaries between superconducting regions, surface phenomena, etc. Nevertheless in view of its simplicity it is a very useful theory, describing correctly Meissner effect, flux quantisation and many other important properties.

A phenomenological theory, due to Ginzburg and Landau[10], allows us to treat a considerable number of problems and applies either to type I or type II superconductors. In itself the theory is valid only in a temperature range near T_c but some work has been done in order to obtain its generalization to a larger temperature range[11-14]; these efforts are justified by the fact that it is much more easy to use than the microscopic B. C. S. theory, which in turn confirms the validity of G. L. equations in the a forementioned temperature range.

The G. L. 's theory is based on Landau's second order phase transition theory[15].

The two basic equations obtained are :

(4) $$\vec{J} = -\frac{e\hbar}{im}(\psi^* \vec{\nabla}\psi - \psi\vec{\nabla}\psi^*) - \frac{4e^2}{mc}\psi^*\psi\vec{A}$$

(5) $$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\vec{\nabla} - \frac{2e\vec{A}}{c})^2\psi = 0$$

The first equation describes the current distribution, i. e. the diamagnetic response of the superconductor to the external field; the second equation describes the equilibrium spatial variation of the order parameter $\psi(r)$, which is in practice the wave function of the superconducting electrons.

For a pure metal in the free electron approximation, the B. C. S. theory gives:

$$\lambda(T) = \frac{1}{\sqrt{2}} \lambda_L(0) \left(\frac{T_c}{T_c - T}\right)^{1/2} ,$$

and $\xi = 0.74\, \xi_o (T_c/T_c - T)^{1/2}$. It can be seen that both $\lambda(T)$ and $\xi(T)$ diverge when $T \rightarrow T_c$, but their ratio is, in the same limit a constant, $K = \lambda/\xi$, and it is called the G. L. parameter of a superconductor.

We will discuss now qualitatively how the basic characteristics of superconductors with small or large K values differ in a fundamental way, following essentially the treatment of Lynton and McLean[16]. Let us consider now an unit area of a surface layer separating normal and superconducting material, in a magnetic field H (see Fig. 1). The G. L. theory shows that the order parameter $\psi(\vec{r})$ goes from the equilibrium

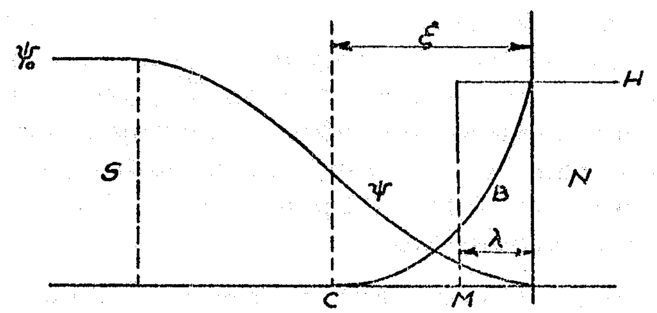

FIG. 1 - Variation of the superconducting order parameter $\psi(r)$ and the flux density at the boundary between normal and superconducting regions.

value in the superconductor to zero over a distance ξ; in this region there is a net increase in the energy given by the volume of the region, ξ, times $Hc^2/8\pi$, the difference of energy between normal and superconducting material. On the other hand the field penetrates with a characteristic length λ: so, in the region of interest, we have also a decrease in energy equal to the volume times the energy due to the expulsion of flux: $\xi \times H^2/8\pi$.

If the field in the normal region is H_c, the net increase in energy per unit area will be $\Delta E = (\xi - \lambda) H_c^2/8\pi$. This elementary treatment leads immediately to conclude that the surface energy will be positive in superconductors with $\xi > \lambda$ (type I) and negative in superconductors with $\xi < \lambda$ (type II). In other words we conclude that in a type I it is energetically unfavorable, for magnetic fields $H < H_c$, the formation of normal inclusions, which in turn can lead to a decrease of the energy of a type II superconductor. So we aspect a magnetic behaviour of the two classes of superconductors showing striking differences.

With the same arguments we can also estimate approximatively at which value of the external magnetic field it is energetically favorable to have normal inclusions in a type II superconductor. Let us suppose a normal inclusion in the form of a thin cylinder, with a radius $r = \xi$; in this region the magnetic field will be H_{ext} and $\psi(r) = 0$. The formation of such a region will be energetically favorable if

$$\pi \xi^2 H_c^2/8\pi \leq \pi \lambda^2 H^2/8\pi \quad (6)$$

i. e. if the energy due to the flux expulsion from the cylinder is equal or less than the decrease in energy due to the superconducting condensation.

From (6) we obtain:

$$H \geq (\xi/\lambda) H_c \quad (7)$$

For a type I superconductor ($\xi \geq \lambda$) $H > H_c$, then the material will be completely diamagnetic if $H \leq H_c$. For type II superconductors the minimum favorable external field will be

$$H_{c_1} \simeq (\xi/\lambda) H_c \leq H_c ;$$

also for fields $H > H_{c_1}$ there will be a persistence of superconducting regions, up to a limiting field H_{c_2}, which, from the G. L. equations is given by:

$$H_{c_2} = \sqrt{2}\, K H_c \quad (8)$$

H_{c_2} is $\gtrless H_c$ if respectively $K \gtrless 1/\sqrt{2}$; this is the limiting value between type I and type II superconductors. Fig. 2 shows the magnetic behaviour of type I and type II superconductors.

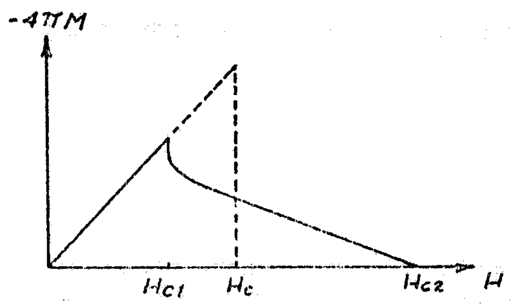

FIG. 2 - The magnetization curves for type I (dashed line) and type II (solid curve) superconductors.

We have now justified the distinction between the two classes of superconductors, obtaining also the result that a type II superconductor if $H_{c_1} < H < H_{c_2}$, is in a state which is neither entirely normal nor entirely superconducting. The detailed characteristics of this "mixed state" can be obtained by solving the G. L. equations under suitable conditions, as done by Abrikosov(17). The result of the calculations was that the field penetration takes place by means of an ordered lattice of quantized vortex lines. A vortex line, consists of a normal core of radius ξ, with supercurrents flowing around; the flux associated with a vortex line is ϕ_o, the flux quantum. Fig. 3 shows the structure of such a vortex line. It can be seen[9] that

(9) $$B(\vec{r}) = \phi_o K_o(r/\lambda_L)/2\pi\lambda_L^2 \ ,$$

with K_o a zero order Bessel function. The asymptotic values are :

(10) $$B(r) = \phi_o \ln \frac{(\lambda_L/r)}{2\pi\lambda_L^2} \qquad \text{for} \quad \xi < r << \lambda_L$$

(11) $$B(r) = \phi_o\left(\frac{\pi\lambda_L}{2r}\right)^{1/2} \exp(-r/\lambda_L)/4\lambda_L^2 \qquad \text{for} \quad r >> \lambda_L$$

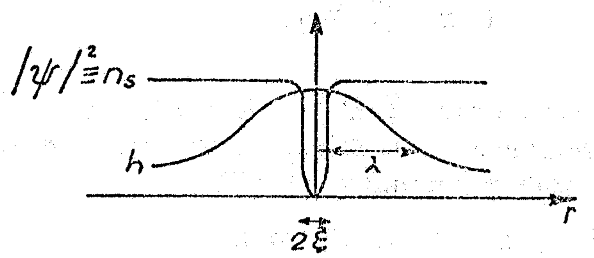

FIG. 3 - Structure of a vortex line in a type II superconductor. Variation of the number of superconducting electrons per cm^3 n_s and the magnetic field B. Note that B is maximum near the center of the line, i. e. in the normal "core region" of radius ξ.

The energy per unit length associated with a line is :

(12) $$F = (\phi_o/4\pi\lambda_L^2)\ln(\lambda_L/\xi).$$

It is interesting to note that F varies as ϕ_o^2 and then it is energetically more favorable to double the number of vortex lines rather than to have two flux quantum in a line.

III. - FLUX FLOW RESISTIVITY. -

In the previous discussion on the magnetic behaviour of a type II superconductor, we have seen that there is a maximum limiting field H_{c2} for superconductivity. Since H_{c2} can assume very large values (200-300 kGauss), it is immediate to conclude that these materials can lead to very useful applications, such as the construction of superconducting magnets. Unfortunately this is not true, since in an ideal type II superconductor, in the mixed state, a transport current flowing throught gives rise to a dissipation, i. e. it is surprisingly associated with an electric field E. Having realized that this phenomenon is due to the motion of vortex lines[18] was an important progress in the understanding of the behaviour of type II superconductors and consequently in the development of the technological applications.

The dissipative mechanism of vortex lines, induced by the current flowing throught the sample, is referred as flux flow resistivity. Let us define the flux flow resistivity, following Kim et al.[19]. Suppose that the magnetic field H be in the z direction and the array of vortex lines in the x-y plane. With a sample in the form of a thin foil $B = n\varphi_o \simeq H$, where n is the density of flux lines and φ_o the flux quantum. Obviously the magnetic field H_{ext} is larger than H_{c1}. The transport current J in the x direction creates a gradient in the flux lines density n, such that

(13) $$\frac{\partial B}{\partial y} = \varphi_o \frac{\partial n}{\partial y} = \frac{4\pi}{c} J$$

(14) $$\frac{\partial}{\partial y}\left(\frac{B^2}{8\pi}\right) = \frac{BJ}{c} = n\,\varphi_o \frac{J}{C}$$

From the equation (14) we obtain the "Lorentz force" F_L acting on a single vortex line

(15) $$F_L = \frac{J\varphi_o}{c}$$

The force F_L is responsible for a viscous flow motion of the vortex lines, characterized by

(16) $$\eta V_L = F_L$$

V_L is the velocity of the lines and $\eta(H,T)$ the viscosity coefficient of the material. Since power is required to mantain stationary conditions (V_L and J constants), there must be an observable electric field E_o in the direction of J, Kim consider that this field is due to the motion of the lines, i. e.

(17) $$E_o = n\left(\frac{V_L}{c}\right)\varphi_o = \frac{V_L}{c} B.$$

From (16) and (17) the flux flow resistivity ρ_f can be defined as:

(18) $$\rho_f = \frac{dE_o}{dJ} = \frac{\varphi_o B}{\eta c^2}.$$

A measure of ρ_f gives then the viscosity coefficient η. The experimental results[19] indicate that, for sufficiently small values of reduced temperature $t = T/T_c$, the following relationship is obeyed (see Fig. 4):

$$\frac{\rho_f}{\rho_n} = \frac{H}{H_{c_2}(0)} . \tag{19}$$

An extrapolation of the (19) to $\rho_f/\rho_n = 1$ gives the value of $H_{c_2}(0)$.

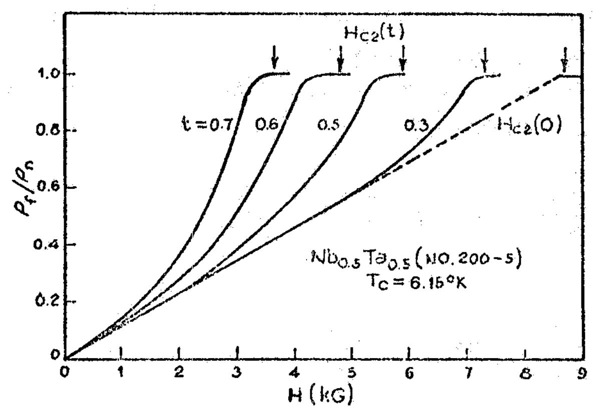

FIG. 4 - Experimental results from ref. (19). Flow resistivity versus H and T/T_c. ρ_f/ρ_n of a Nb-Ta specimen is displayed as a function of H at given values of $t = T/T_c$. Vertical arrows indicate the values of $H_{c2}(t)$ measured resistively. The dashed line indicates the behaviour of ρ_f/ρ_n expected at $t = 0$. The intersection of this line with $\rho_f/\rho_n = 1$ gives $H_{c2} = 8.6$ kG.

But now it must be noted that these experimental results lead to a very suprising conclusion: according to Abrikosov's theory[17], $H \sim B \sim \varphi_0/d^2$, where d is the mean distance between vortex lines, while $H_{c2} \sim \varphi_0/\xi^2$; hence

$$\frac{H}{H_{c_2}} \simeq \frac{\xi^2}{d^2} \tag{20}$$

which is really the fraction of the volume in the normal state. Thus the variation of resistivity with the magnetic field is proportional to the fraction of normal metal and this leads to the conclusion that the current is flowing uniformly either in normal or in superconducting regions; at first sight one would expect a current flowing only in the regions of zero resistivity.

We give now only a brief description of the theories carried out in order to explain these results (20-35). Normally we are dealing with phenomenological theories, based on approximate models of the physical situation.

The starting point is always the vortex line, a cylinder with a core of normal material; it is then quite straightforeward to identify this problem with the hydrodynamical one of a solid cylinder immersed in a uniform flow of fluid. In these theories Eulero's equation

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right] = \text{total density of force} \tag{21}$$

is applied to the superfluid, assuming that London's equation is valid in every point but in the core of the vortex. Normally one studies the motion of a single vortex with a superimposed transport current. The treatment of Bardeen and Stephen(32) based on the assumption of a rigid motion of the vortex and on a local model of the vortex itself, is perhaps the more complete between the hydrodynamical ones. The authors are able to obtain the above surprising result of a uniform transport current inside and outside the normal core. From the analogy with a classical fluid Bardeen and Stephen predict that the Hall angle in a type II superconductor in the mixed state would be the same as in the normal metal, in a magnetic field equal to that of the core. Hovewer measurements have shown a Hall angle smaller than the normal metal value(36, 37).

All these hydrodynamical theories even if they give substantially right previsions and have a great merit on the understanding of the physical phenomena of the flux flow resistivity, suffer the limitations of the phenomenological theories, they are not based on first principles.

A general theory would require a time dependent generalization of the G. L. 's theory, the motion of the vortex lines involving a time variation of the order parameter. A very interesting approach in this direction is due to Kulik(38), but still there is a phenomenological derivation of the starting equations. The theoretical results show that near H_{c_2} $\rho_s \simeq \rho_n$, while $\rho_s \ll \rho_n$ if $H \ll H_{c_2}$; further it gives a linear relations between the voltage and the current, in qualitative agreement with the experimental results. But there is a completely new prediction; the resistive phenomena could be followed by radiofrequency emission, because of the motion of vortex lines. The authors says that a calculation of the intensity of the emitted radiation is quite difficult, because the right conditions to observe it are not fully understood. Further the observation of the effect is complicated by the actual structure of the lattice of vortex lines, not completely regular as supposed in the calculations; the dislocations in the vortex lattice results in a smearing of the emitted frequency bands and in a smaller influence of irradiation on resistivity.

IV. - HARD SUPERCONDUCTORS. -

Let us now come back to the main problem : now we know that neither type I, for the low critical field H_c, neither type II superconductors because of the flux flow resistivity phenomenon, are suitable for technological applications. But we know that superconducting magnets reaching very high fields have been built up and the question now is : what kind of materials have been used? Actually there is a third class of superconductors, "hard" or type III or non ideal type II, which can sustain high current density in the range $H_{c_1} < H < H_{c_2}$; in a superconductor of this kind there must be some mechanism which is able to prevent the motion of the vortex lines up to high values of the transport current; lattice defects of various kind, present in the material, act as pinning centers for the vortex lines. Hard superconductors are then obtained from materials that in a very pure form and in ideal lattice conditions are either type II or type I superconductors.

At present the only known intrinsic type II superconductors are the elements Nb and V and certain intermetallic compounds of these elements, such as the groups Nb_3X and V_3X with the β -tungsten structure.

It must be noted that the flux flow resistivity can be detected in all hard superconductors; the only difference is that the phenomenon starts only above a critical current, when the force F_L on the lines, due to the interaction with the current flowing through the sample, can overcome the prinning force F_p : from this point a hard superconductor behaves as an ideal type II superconductor.

The coupling mechanisms between vortex lines and lattice defects are up to date known in a better way, but not completely, and it is evident the importance of a deeper understanding of the problem; with a detailed knowledge of the pinning mechanisms it could be possible to obtain materials with a predetermined electromagnetic behaviour. At present there is a large amount of experimental work with the aim of obtaining higher critical currents by means of a study of pinning mechanisms.

Now we would introduce the fundamental idea of the "critical state" of a hard superconductor: after there will be a description of two other mechanisms of motion of the vortex lines.

IV. 1. - The critical state.

In an ideal type II superconductor, in the mixed state, the density of vortex lines, $n = B/\varphi_o$, is a constant, with a value determined only by the relation $\vec{B} = \vec{B}(H)$. What are the changes in the magnetic situation of a hard superconductor, where there is a pinning force F_p acting on the vortex lines? The result will be a spatially varying density of vortex lines, i. e. an induction $\vec{B} = \vec{B}(\vec{r})$ which is no more constant. To derive this result, let us suppose a constant pinning force $F_p(\vec{r})$ = cost = α, i. e. an uniform distribution of lattice defects. In the hard superconductor the vortex lines experience the force F_p and a force F_r due to the mutual repulsion between the lines. There will be a static configuration of vortex lines only if the pinning force F_p is larger than F_r :

(22) $$F_p \gtrsim F_r \; .$$

Obviously the real equilibrium situation will be $F_p = F_r$. This state is called critical state, representing a boundary beyond which it is impossible to have a static configuration of vortex lines. It is convenient to emphasize in advance that a really rigid structure of vortex lines can exist, if $F_p > F_r$, only at T = 0; it will be seen after that, at a temperature $T \neq 0$, the vortex line can move, in spite of the (22), because of the thermal activation : this motion of lines is called "flux creep".

Working now in a monodimensional case, and supposing that the equilibrium magnetization in the hard superconductor is B(x), the force F_r (per unit volume) will be :

(23) $$F_r = \frac{B}{4\pi}\frac{\partial B}{\partial X} = \frac{BJ}{c}$$

Hence the critical state is defined by :

(24) $$\left|\frac{B}{4\pi}\frac{\partial B}{\partial X}\right| = \left|\frac{BJ}{c}\right| = \alpha(B)$$

The (23) can be obtained by thermodynamic arguments[9] and it has the same forme as the (14) obtained in the case of a current flowing through a type II superconductor. The physical meaning is indeed the same : to say a spatially varying induction is equivalent to say that there is a macroscopic current flowing into the sample, rot $\vec{B} = \frac{4\pi}{c}\vec{J}$, i. e. in our case $J(x)\frac{c}{4\pi}\frac{\partial B}{\partial X}$. In the critical state idea it is assumed that J is the maximum current density that the sample can carry without dissipation, i. e. the critical current density.

To calculate B(x) in the critical state we must know the relation $\alpha = \alpha(B)$. Bean[39,40,41], who was the first to introduce the critical state, assumed α proportional to B; this is equivalent to assume that the critical current density is independent of the value of the local induction. But Kim's experiments[42] indicate that α is indipendent of B; this is quite a striking result, because normally one would expect a relation between the efficacity of pinning centers and their relative size with respect to the distance between vortex lines (which is obviously dependent on B). Fig. 5 shows B(x) in the Bean and Kim assumptions.

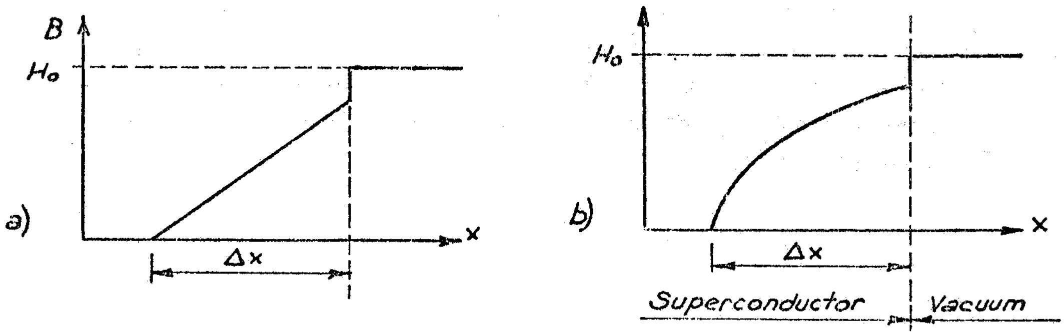

FIG. 5 - Penetration of flux in a hard superconductor.
a) Bean model (J_c = cost);
b) Kim model $J_c = \alpha/(B+B_o)$ (α and β constants).

To calculate the internal field distribution there were many other proposals(43-48) for the relation between J and the local induction; all of these are phenomenological and can not give a physical understanding on the behaviour of the critical currents(o).

IV. 2. - Flux creep.

It was emphasized in the description of the critical state that flux lines can move, even in the case $F_p > F_L$, if the temperature T is $T > 0$ and the slow motion is referred as flux creep because of the analogy with the well known phenomenon of mechanical creep.

This characteristic feature of hard superconductors was first observed by Kim et al. (50) in a experiment of magnetization of a hollow cylinder; it was observed that for a fixed external magnetic field H, the field H' in the interior varied according to the relation:

(o) - In a recent paper Mac Inturff and Paskin(49) are able to explain from first principles, the physical meaning of the $J_c(H)$ features. The authors assume that normal pinning defects act as a boundary between two superconducting regions S and S', thus creating microscopic Josephson junctions S-N-S. From known relations about Josephson junctions they obtain

(25) $$J_c = \frac{Im\ \phi_o}{2\pi B}$$

where Im is the maximum Josephson current, ϕ_o the quantum of flux; this relationship closely resembles Kim's empirical formuls. Moreover, since it is known that

(26) $$Im \propto (T_c - T)^2$$

it is possible to predict a dependence of I_c on the temperature in good agreement with the experimental data.

(27) $$\delta H' = - \text{cost} \ln t \; .$$

In addition the strong temperature dependence of the pinning force F_p near 0.1 T_c (where all the other properties of the bulk superconducting state show only a small dependence on T) was unexplained too.

Anderson(51, 52) showed that this can be accounted for by introducing a thermal activation which adds to the Lorentz force F_L, thus giving to a bundle of flux lines the possibility to overcome the free energy barrier created by the pinning force F_p.

The Anderson theory, even though it is based on oversimplified assumptions, had great importance on the physical understanding of the flux creep. We will give here the basic lines of development. First it must be justified that the moving entity is a bundle of vortex lines rather than a single line; it was shown by Abrikosov(17) that the interaction free energy between vortex lines in the case $K \gg 1$, $H_{c_1} < H < H_{c_2}$ can be written in two equivalent ways:

(28) $$F_{int} = \frac{1}{8\pi} \overline{\vec{H} (\vec{H} - \lambda_o^2 \nabla^2 \vec{H})} =$$

(29) $$F_{int} = \frac{H_c^2}{k^2} \sum_{i,j} K_o \left(\frac{|\vec{r}_i - \vec{r}_j|}{\lambda} \right)$$

$k = \lambda_o / \xi_o$ is the Ginzburg-Landau parameter, H_c is the thermodynamical critical field, K_o a Bessel function, and $\vec{r}_{i,j}$ the bidimensional vectors giving the position of the lines. In a normal case $|\vec{r}_i - \vec{r}_j| \ll \lambda_o$, $\nabla^2 \vec{H}$ is relatively small and the interaction energy is the well known magnetic energy $H^2/8\pi$. This means that the force per unit volume will be $\vec{J} \times \vec{H}/c$, the Lorentz force, and on a single line will act the force $\vec{J} \times \vec{\phi}_o/c$ per unit length, the vector $\vec{\phi}_o$ is parallel to the field with a stregth hc/2c.

At the end of Cap. II we have seen that $K_o \to \ln \lambda/r$ as $r \to 0$, while $K_o \to e^{-r/\lambda}$ as $r \to \infty$. This means that we are dealing with a relatively long-range interaction, so that local variations of line density (i. e. motion of a single line) are energetically unfavourable. On the other hand K_o varies slowly near $r = 0$ and the array of vortex lines is not regular from a crystallographic point of view, thus rendering quite easy the motion of a bundle of lines with a radius of the order of λ.

We have now to write the free energy of such a bundle; the force acting on a bundle is about $JH\lambda^2 l/c = J\phi_o n_b l/c$; l represents the effective length of line on which the force is acting and can be seen as a mean distance between pinning centers, n_b is the number of lines in a bundle. As a function of the position x of the bundle the free energy F_{force} due to the force is:

(30) $$F_{force} = JH\lambda^2 lx/c .$$

If we remember that a pinning centre is normally due to a normal inclusion then, supposing a volume ξ_o^3, this gives a free energy F_b':

(31) $$F_b' = H_c^2 \xi_o^3/8\pi .$$

Anderson assumed that only a fraction p of the energy F_b' was effective; in this way the total free energy of the barrier becomes:

(32) $$F_{tot} = F_b - F_{force} = \left[pH_c^2 \xi_o^3/8\pi\right] - \left[JH\lambda_o^2 l\xi_o/c\right] .$$

Supposing a thermal activated motion, a bundle will overcome the barrier with a probability R given by:

(33) $$R = \omega_o e^{-F_{tot}/kT}$$

where ω_o is a vibration frequency of the bundle.

The above formula (33) furnishes the observed time dependence of H'. Moreover it is possible to explain the temperature dependence of the pinning strength, supposing that the critical state corresponds to a situation in which the probability R is too small to be measured (i. e. it is impossible to detect any movement of flux lines); in fact from (33) it can be obtained:

(34) $$\frac{(JH)_{crit}}{c} = \alpha_{crit} = \frac{pH_c^2\xi_o^2}{8\pi\lambda_o^2 l} - \frac{kT}{\lambda_o^2 l\xi_o} \ln \frac{R_c}{\omega_o} .$$

It is quite clear that the above results are not valid near H_{c_1} and H_{c_2}, because of the assumption made in the derivation.

As a conclusion we can say that although the Anderson theory neglect many important factors (e. g. the details of the interaction between vortex lines, free energy variations with current and field, etc.), the obtained results are a primary confirmations of a thermal activated motion of flux structures.

IV. 3. - Flux jumps.

Another important motion of flux lines, the more important for technological application, are the flux jumps. A jump is a catastrophic process which makes abruply $B = H_{ext}$ in a hard superconductor, when $H_{ext} < H_{c_2}$.

Fig. 6 (from ref. (42)) shows clearly repeated flux jumps in a measure of the magnetization of hollow cylinder. Before examining

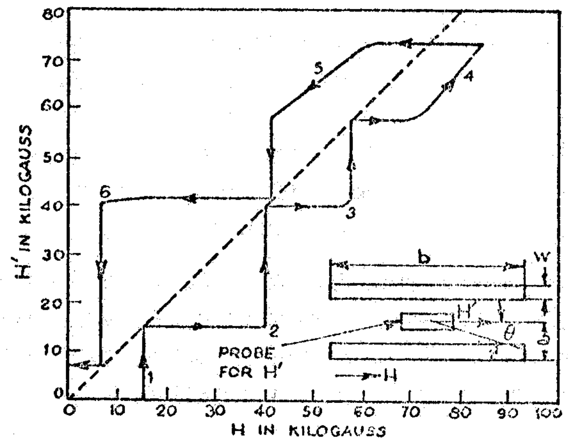

FIG. 6 - Magnetization of a tube sample (from ref. (42)). H' is the magnetic field measured inside the tube, and H the external magnetic field applied parallel to the tube axis. Note repeated flux jumps, after with H' = H.

some detail of this phenomenon it is convenient to spend a short time on its technological importance. Any movement of flux lines is associated with a dissipation, then with a temperature rise of the superconducting sample; the low thermal conductivity of the superconducting state renders quite difficult the recovery of the previous temperature. A creep-type motion gives a small temperature rise which can be dissipated towards the helium bath; on the other hand a jump is an abrupt phenomenon, requiring simultaneous motion of at least 10^8 flux lines(53), thus producing a large amount of heat, which sometimes can induce a local transition to the normal state. Let us imagine such a jump in an energized superconducting magnet : the current flowing through the superconductor cross a normal region, thus producing Joule's heat. This in turn can either induce more jumps in neighbouring regions or the direct resistive transition of a certain amount of the material. It must be added that a coil made only by superconducting material gives a very bad thermal connection between central layers and helium bath. In such a condition a flux jump can cause a abrupt transition to the normal state of an energized superconducting magnet, with all the imaginable related consequences. Up to now these complications are avoided by coating the superconducting strip or ribbon with a normal metal of high thermal and electrical conductivity (normally pure Cu or Ag); in the case of a local transition the normal metal acts both as an electrical and thermal shunt.

This is the very important problem of the stabilization of a superconducting material, which will be treated in detail elsewhere(54). It is important note that in fact the phenomenon of degradation in superconducting solenoids, i. e. the observation that superconducting solenoids return to the resistive state at a transport current level below the critical current of short wire samples can be attributed to local flux jumping in the windings, also in the case of stabilized wires.

In principle the better solution would consist of obtaining a intrinsecally stable superconductor; the stabilization with a normal metal, while reducing the mean current density in the ribbon compels us to a noticeable increase of the dimensions in practical realizations. Intrinsic stabilization is not yet achieved in practice, but some suggestions on the problem will be given later.

From an experimental point of view flux jumps are quite complicated, depending upon the experimental arrangement as well as upon the metallurgical state of the sample. This explains the relatively poor agreement between different experimental works.

It is then convenient to treat a characteristic of flux jumps independent of the geometry and metallurgical state of the sample. It has been observed while measuring magnetization curves that there is a high probability of flux jumps when the external magnetic field has a value near $-H_{c_1}$[55]. The mechanism of jumps in this region does not depend upon pinning and flux motion and must be valid for any material. In fact it depends upon annihilation of positive and negative fluxoids, i.e. flux lines with the field directed in opposite direction. At a field value slightly larger than $-H_{c_1}$ negative fluxoids can enter; in the sample these are positive fluxoids too, trapped during the preceding part of the magnetization cycle. Opposite fluxoids attract and annihilate[9], and their energy F is released as heat. We have seen that F is given by:

(35) $$F = \left(\frac{\phi}{4\pi\lambda_L}\right)^2 \ln\left(\frac{\lambda_L}{\xi}\right) .$$

H_{c_1} can be expressed as a function of λ_L and ξ in the following way:

(36) $$H_{c_1} = \frac{\phi_o}{4\pi\lambda_L^2} \ln\left(\frac{\lambda_L}{\xi}\right) .$$

The heat released (per unit length) Q is equal to 2F, then giving:

(37) $$Q = 2F = 2\phi_o H_{c_1}/4\pi .$$

In a superconductor with $K = \lambda/\xi = 6$, $Q \simeq 2\times10^{-13}$ cal/cm; in the case of a Pb-Bi alloy there is a local temperature rise of $\sim 1^oK$. As a consequence the pinning centres become less effective in that region and there is a large probability of jumps. Further confirmation of the described mechanism has been given by Wischmeyer[56].

Let us now come back to the general problem of flux jumps. The basical idea, taken as a starting point in all the theoretical approaches is very simple: if the thermal energy due to the flux motion moves slowly with respect to velocity of flux lines, a substantially unstable process results. Both heat and flux propagation obey to a diffusion equation; we then expect a large probability of flux jumps if the thermal diffusivity D_{th} is smaller than the electromagnetic diffusivity of the normal state D_{em}. In a large number of alloys thermal conductivity and specific heat are principally due to the conduction electrons; in this case D_{th} is given by (neglecting the difference between normal and mixed state):

$$D_{th} = \frac{L_o}{\gamma \rho_o} . \tag{38}$$

L_o is the Wiedemann-Franz constant, γ the electronic specific heat coefficient per unit volume. Furthermore

$$D_{em} = \frac{\rho_o}{4\pi} . \tag{39}$$

Remembering that in the case of very dirty alloys ($K \gg K_o$) K is given by[57]:

$$K = K_o + 0.167\, N(0)^{1/2}\, e\, \rho_o \tag{40}$$

we obtain from (38), (39) and (40):

$$\frac{D_{th}}{D_{em}} = \frac{0.17}{K^2} = \frac{L_o 4\pi}{\gamma} \frac{1}{\rho_o^2} . \tag{41}$$

These qualitative results, due to Goodman[58], shows that large K values lead to a very favourable situation for the occurrence of flux jumps. Unfortunately all of the technologically important materials have large K values (e. g. for Nb-Zr and Nb_3Sn $K \simeq 25$), then giving rise to complications. Furthermore eq. (41) shows that the normal state resistivity is also very important. We can then qualitatively account for the effect of coating the superconducting material with a normal metal. Supposing that the compound wire has a resistivity given roughly by the parallel of the two materials, the coating results in a large decrease in resistivity; in the case of Nb-Zr coated by Cu we have $\rho_{Nb-Zr} \simeq 30\,\mu\Omega$-cm, while $\rho_{parallel} \simeq 1\,\mu\Omega$-cm.

The above very simple calculation is very useful from a heuristic point of view, but is clearly rough and cannot furnish any quantitative prediction. The final aim of a full theoretical treatment is to give H_{fi} and H_{fj} as a function of the temperature and of other important parameters of the materials. H_{fi} and H_{fj} represent the values of excess or shielding field (i. e. the difference between applied and internal field) separating the full stability region respectively from the limited instability region and the complete jumps region. A flux jump is called complete if it makes the external field equal to the internal one; Wipf and Lubell[59] have shown that a complete flux jump will heat the superconductor to the critical temperature appropriate to the applied field.

In spite of the fact that instability conditions are generally not completely reproducible, even for the same sample[53, 59, 60], some essential features of flux jumps are now quite well established.

The spacing between flux jumps is approximately constant for a given sample[61-64], depending on the temperature[64, 65] and rate of field sweep[36, 41, 44, 45, 47]. Near absolute zero and near the critical temperature the spacing between flux jumps is smallest, with a maximum spacing at intermediate temperatures[64, 65]. Furthermore the spacing of flux jumps decreases as the field sweep rate is increased[53, 59, 62, 63, 65].

The constant spacing between flux jumps has a clear physical meaning in the case of complete flux jumps. It is in fact evident that magnetic instabilities depend on the difference between external and internal magnetic field; after a complete flux jump $H_{ext} = H_{int}$ and a subsequent increase of H_{ext} restores the preceding situation, a gradient of induction being present as before. This conclusion follows directly from our knowledge of the critical state in hard superconductor, as introduced in Cap. IV. 1.

In the case of bulk samples, flux jumps are observed without any superimposed transport current. On the other hand in many important applications hard superconductors are in the form of wires or ribbons carrying a transport current. Magnetic instabilities have been observed when the superimposed transport current and/or the applied field are changed[66-70]. Unfortunately the author knows of no present theories of magnetic instabilities in superconductors with a superimposed transport current.

At present many articles have been proposed for a theoretical description of flux jumps[56, 59, 71-79]. All the theories are based on the critical state, where $F_L = F_p$. This equation, neglecting flux creep phenomena, describes the equilibrium state of flux lines. A magnetic disturbance (which can be visualized as an increment of flux $\Delta\phi$ in the interior of the sample), implies a dissipation, then a temperature rise. In this way $\Delta\phi$ involves a decrease ΔF_L of the Lorentz force F_L. Furthermore in all the materials the pinning force decreases when the temperature is raized. It is then clear that there will be a stable equilibrium only if the decrease of the Lorentz force is larger than the corresponding decrease of the pinning force. Once the stability limit $\Delta F_p = \Delta F_L$ is exceeded, then the following equilibrium equation

(42) $$F_L = F_p + F_v$$

has to be considered in order to study the process initiated by a disturbance. The term F_v, the viscous force density, has to be introduced to take into account the viscous flow of vortex lines in the instability regions.

As far as the field sweep rate dH/dt is concerned, theoretical approach consider normally either the case of small dH/dt or the opposite one. The case of small sweep rates is called isothermal condition, because the power transfer from the sample to the helium bath is assumed smaller than 0. 8 W/cm^2, the limit for film boiling.

The adiabatic condition is achieved with a large sweep rate; in this case there is a field pulse ΔH in a small time with respect to the thermal diffusion time and large with respect to the magnetic diffusion time.

It must be noted that for practical applications the intermediate case between isothermal and adiabatic conditions is very important too. It seems however a formidable task to solve, owing to mathematical complications. In the isothermal limit the most complete treatment has been developed by Wipf[77]. With the further assumption of semi-infinite half space (good approximation as long as the shielding layer does not reach the centre of the specimen) he obtained that the stability limit H_{fi} is given by

$$(43) \qquad H_{fi} = \frac{1}{2}\pi\left[\pi C(T_c^2 - T^2)/T\right]^{1/2} .$$

This equation is remarkable as it contains only the specific heat C and the critical temperature T_c and neither the pinning strength parameter α nor the rate of change of the external field dH/dt. The study of the instability regions is more complcated and Wipf could not obtain an explicit expression for H_{fj}, although he was able to perform a very useful discussion, which will be briefly sketched here. Above the stability limit the vortex lines can be accelerated, so that there would be some power dissipation into the sample. There are two different instability regions: limited instability and runaway instability i. e. flux jump. In the limited instability region the thermal conductivity can reduce the temperature rise, thus partially restoring the pinning force. As a result of such a process there will be a large, locally limited disturbance, as distinct from a flux jump. Limited instability is then an acceleration to a maximum speed, v_{max}, of the flux lines followed by a deceleration; v_{max} is given by:

$$(44) \qquad V_{max} = \frac{H}{4\pi F_p(H)} \frac{dH}{dt} \exp\left[\frac{0.015\, a_i\, x_o^2}{\alpha_{th}}\right]$$

here α_{th} is the thermal diffusivity of the material, x_o is the initial thickness of the shielding layer as obtained from the equilibrium critical state and :

(45) $$a_i = f\left(F_p(H,T),\ B(x),\ \frac{\partial F_p}{\partial B},\ \frac{\partial F_p}{\partial T},\ \frac{\partial B}{\partial x},\ \frac{\partial^2 B}{\partial x^2}\right)$$

It is supposed in obtaining (44) that the duration of this process is short enough so that the shielding layer has not grown noticeably compared to x_o. The thickness d at a certain time is generally given by:

(46) $$d = x_o + \int v\, dt\ .$$

If $\int v\, dt$ becomes comparable to x_o during the acceleration process, then the heat conduction becomes less and less effective and the front of advancing flux may outrun the heat conduction, resulting in a complete flux jump. Wipf found that this process can occur only if

(47) $$\frac{\alpha_{th}}{d^2} \leq 0.015\ .$$

From condition (47) he obtained an equation containing H_{fj} in implicit form, which can be solved by an analogic computer. It can be seen that the results cannot be expressed in a simple form. Furthermore the exact nature of the magnetic disturbance plays an essential role in obtaining (43) and (44).

Although in principle the knowledge of H_{fj} is more important than the knowledge of the stability limit H_{fi}, the latter can be used as a very useful indication on the field region where flux jumps cannot occur. Furthermore H_{fi} is more or less proportional to H_{fj}. The influence of the specific heat on the flux jumping field has been experimentally detected[80-82]. The idea of limited instability has been confirmed by Wischmeyer et al. [83], whose experiments clearly show small rushes of flux immediately after the occurrence of flux jumps and above the stability limit. Furthermore these limited instabilities are also localized with regard to the specimen surface[84].

In the adiabatic limit, Swartz and Bean[79] with a treatment analogous to ref. (41), (56) and (63), have obtained:

(48) $$H_{fi} = \left[\pi\, C\, T_c \left(1 - \frac{T}{T_c}\right)\right]^2 .$$

It must be noted that also in the adiabatic case the same dependence of H_{fi} on C as in the isothermal case is obtained. Eq. (48) has a maximum at $T = 0.75\ T_c$ and falls to zero both at $T = 0$ and $T = T_c$. To determine $H_{fj}(T_o)$, the full flux jump field at a temperature T_o Swartz and Bean equated the decrease in field energy of complete penetration to the energy necessary to heat the sample to a higher temperature T_u, at which is not superconducting in the presence of the field in question.

This latter temperature is usually very close to the critical temperature T_c of the sample[59]. The equations that determine H_{fj} are then

(49) $$\frac{H_{fj}^2(T_o)}{8\pi} = \int_{T_o}^{T_u} C(T)\,dT$$

(50) $$H_{fj}(T_o) = H_{c_2}(T_u) .$$

In the case of Nb_3Sn the following approximation are used:

(51) $$C(T) = \beta T^3 ; \qquad H_{c_2}(T) = \gamma(1 - \frac{T}{T_c})$$

where the latter approximation is valid near the critical temperature. From (49), (50) and (51) an equation can be found for T_o in terms of T_u:

(52) $$(\frac{T_o}{T_c})^4 = (\frac{T_u}{T_c})^4 - (\frac{\gamma^2}{2\pi\beta T_c^4})(1 - \frac{T_u}{T_c})^2 .$$

By choosing β, γ and T_c values pertinent to Nb_3Sn, Swartz and Bean are able to evaluate $H_{fj}(T_o)$ for this material. Fig. 7 shows H_{fi} and H_{fj} in the adiabatic limit, as well as H_{fi} obtained by Wipf in the isothermal limit (eq. (43)) and calculated for Nb_3Sn with the same β, γ and T_c values.

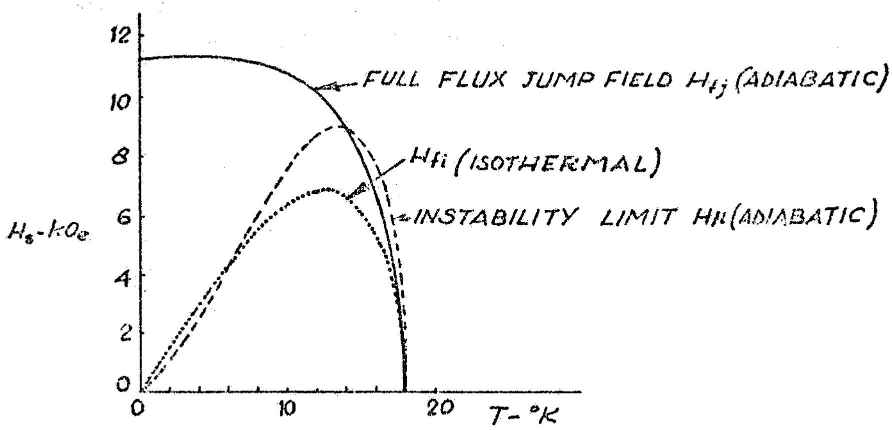

FIG. 7 - The temperature dependence of two flux jump conditions for Nb_3Sn in the isothermal and adiabatic approximation. H_{fi} is the instability limit of the excess or shielding field (dashed and point line curves). The solid curve is the excess field required for the flux jump to be complete in the adiabatic case.

There have been no systematic studies that provide a direct test of the quantitative predictions of the two theories. But is very interesting to note that two experimental papers(85, 86) find a general accordance with the adiabatic limit in the case of large sweep rate, while the ideas of Wipf seem to describe the case of slower sweep rate.

At the beginning we mentioned flux jumps intrinsic stabilization. We now come back to this problem and mention the methods suggested by the previous argument. First of all there would be a method which derives directly from the discussion on the relative variation with temperature of F_L and F_p; it is clear that, a superconductor in which $\partial F_p/\partial T \geqq 0$, is stable against flux jumps. This method has been proposed by Livingston. Although there are some examples of superconductor with this property(87), the author knows of no present application of the method to materials of technological importance.

A second method, suggested by the instability conditions (48) and (43) is to distribute a second phase within the superconductor such that the specific heat of the whole is raised. This is in accord with experimental results on porous Nb_3Sn in a helium bath(88).

A third method(59, 79) which is not evident from our discussion relates the thickness W of the sample to the specific heat C, to the critical current density J_c and its temperature derivative $\partial J_c/\partial T$ in the following way:

(53) $$W \leq \left\{ \frac{-10^2 \pi C}{4 J_c \left(\partial J_c/\partial T\right)} \right\}^{1/2} .$$

This last method applies hardly in a practical case; for Nb_3Sn, with the assumptions $C = 2.3 \times 10^2\ T^3\ {}^oK$(89), $J_c(T) = J_c(0)\,(1 - T/T_c)$, with $J_c(0) \simeq 10^7$ Amp/cm^2, we obtain that W must be less than a few hundred Angstrom, a very small dimension indeed.

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