R. Del Fabbro, G. Matone and L. Satta: A POSSIBLE TEST
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I. - INTRODUCTION. -

In 1964 the discovery of the $K_L^o \rightarrow 2\pi$ has pointed out the CP violation in weak interactions. So the interference between the decay amplitude into 2π of K_S^o and K_L^o (short and long-lived kaon) has been foreseen and experimentally proved. That suggested the idea to search the same interference effect due to a C-violating decay of parapositronium into 3γ.

The positronium system is a bound state of an electron and a positron which, in the ground state, is in S=0 ($^1S_0$ - parapositronium) or in S = 1 ($^3S_1$ - orthopositronium).

However in a microwave magnetic field it is possible to obtain a system which is a quantum mixture of parapositronium and orthopositronium with $S_z = 0$. So the amplitude of the allowed decay $^3S_1 \rightarrow 3\gamma$ and that of the C-violating decay $^1S_0 \rightarrow 3\gamma$ are coherent and interference effects can be observed.

Experimentally Mills and Berko[1] have put for the branching ratio

$$\mathcal{E} = \frac{\text{rate } ^1S_0 \rightarrow 3\gamma}{\text{rate } ^1S_0 \rightarrow 2\gamma}$$

the upper limit of 2.8×10^{-6} with a 68% confidence limit.

They make use of a C-non conserving phenomenological interaction that can cause the decay $^1S_0 \rightarrow 3\gamma$. The simplest nonvanishing form is that suggested by Berends[2]. The parity conserving (C and T non conserving) interaction is given by

(1) $$\mathcal{L}_I = h \left(\frac{1}{2m_e}\right)^8 (\bar{\Psi}\gamma_5\gamma_\mu\Psi) G_{\alpha\beta} F_{\alpha\beta,\gamma\delta} F_{\mu\gamma,\delta}$$

where $\qquad G_{\mu\nu} = \frac{1}{2} \xi_{\mu\nu\varrho\sigma} F_{\varrho\sigma} \qquad\qquad F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$

Their experiment is designed to separate the C forbidden 3γ decay of $^1S_0$ from the allowed 3γ decay of the $^3S_1$ state by studying the angular distribution of the three photons. Because of Bose statistics, the C non conserving $^1S_0 \to 3\gamma$ rate must vanish for the case of the three photons emerging symmetrically (120°, 120°, 120°) independently of the assumed form of the C non conserving interaction. Therefore the authors measure the ratio of the 3γ rate at the symmetric configuration and at some other set of angles.

We think it is possible to obtain better information about this decay rate observing the interference term between ortho and paraposi- tronium.

II. - (e^+e^-) BOUND STATE AND ANALOGY WITH $|K^0\rangle$, $|\bar{K}^0\rangle$ SYSTEM.

If we put

$$\Phi_+ = f_e(\uparrow)\, f_p(\uparrow) \qquad\qquad \Phi = f_e(\uparrow)\, f_p(\downarrow)$$

$$\Phi_- = f_e(\downarrow)\, f_p(\downarrow) \qquad\qquad \bar{\Phi} = f_e(\downarrow)\, f_p(\uparrow)$$

we can write the triplet and singlet eigenfunction

$$\psi_{+1} = \Phi_+ \qquad \psi_o = \frac{1}{2}(\Phi + \bar{\Phi}) \qquad \psi_{-1} = \Phi_-$$
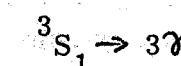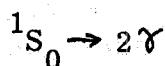
and

$$\psi = \frac{1}{\sqrt{2}}(\Phi - \bar{\Phi})$$

As the C parity of positronium system is $(-1)^{L+S}$, its C conserving de cay into n-photons, must happen according to

$$(-1)^{L+S} = (-1)^n$$

where $(-1)^n$ is the C parity of a n photons system: at the lowest order in α we get

$$^1S_0 \to 2\gamma \qquad\qquad ^3S_1 \to 3\gamma$$

In the following table we list the symmetry properties under C and CP

of the $|K^o\rangle$ and positronium system:

$$CP\,|K^o\rangle = |\bar{K}^o\rangle \qquad\qquad C\,|\Phi\rangle = -|\bar{\Phi}\rangle$$

$$CP\,|\bar{K}^o\rangle = |K^o\rangle \qquad\qquad C\,|\bar{\Phi}\rangle = -|\Phi\rangle$$

$$|K_S^o\rangle = \frac{1}{\sqrt{2}}(|K^o\rangle + |\bar{K}^o\rangle) \qquad\qquad |\Psi\rangle = \frac{1}{\sqrt{2}}(|\Phi\rangle - |\bar{\Phi}\rangle)$$

$$|K_L^o\rangle = \frac{1}{\sqrt{2}}(K^o\rangle - |\bar{K}^o\rangle) \qquad\qquad |\Psi_o\rangle = \frac{1}{\sqrt{2}}(|\Phi\rangle + |\bar{\Phi}\rangle)$$

$$CP\,|K_L^o\rangle = -|K_L^o\rangle \qquad |K_L^o\rangle \to 3\pi \qquad C\,|\Psi\rangle = |\Psi\rangle \qquad |\Psi\rangle \to 2\gamma$$

$$CP\,|K_S^o\rangle = |K_S^o\rangle \qquad |K_S^o\rangle \to 2\pi \qquad C\,|\Psi_o\rangle = -|\Psi_o\rangle \qquad |\Psi_o\rangle \to 3\gamma$$

Now, it is interesting to point out the analogy of the two systems because we can change from the first to the second column by replaing CP with C, $|K^o\rangle$, $|\bar{K}^o\rangle$ with $|\Phi\rangle$, $|\bar{\Phi}\rangle$ and weak with electromagnetic (e. m.) interactions.

III. - QUANTUM MIXTURE IN A MICROWAVE MAGNETIC FIELD. -

The perturbing term due to the microwave magnetic field is

$$(2) \qquad V = \mu_o\,(\sigma_{ez} - \sigma_{pz})\,\frac{H}{2}\,(e^{-i(\omega t + \varphi)} + e^{i(\omega t + \varphi)})$$

where $\mu_o = \dfrac{e\hbar}{2m_eC} = 0,5788 \times 10^{-14}$ MeV Gauss^{-1}

σ_{ez}, σ_{pz} = Pauli spin operators

H = microwave magnetic field amplitude

ω, φ = pulsation and phase of H.

General dispersion theory leads to the experimentally confirmed view that the influence of the alternating field will be very small off resonance. We see that in (2) only the term $e^{-i(\omega t + \varphi)}$ can give rise to a resonance phenomenon. The second term produces only dispersion effects which average out during the time of observation[3] and therefore can be dropped.

The matrix element of V is

$$\langle S, S_z|\,V\,|S;S_z'\rangle = \mu_o\,H\,\langle S,S_z|\,\sigma_{ez} - \sigma_{pz}\,|S;S_z'\rangle\,e^{-i(\omega t + \varphi)} =$$

$$= 2\mu_o\,H\,e^{-i(\omega t + \varphi)}\,\delta_{S_z\,S_z'}\,(1 - \delta_{SS'})$$

4.

The only nonvanishing element

$$\langle 10 \mid V \mid 00 \rangle = D\, e^{-i(\omega t)} \qquad D = 2\mu_o H e^{-i\varphi}$$

cause a transition from a singlet state to the triplet state ($S_z = 0$). So the superposition pronciple allow to write the general state of the system in the form

$$(3) \qquad \mid u \rangle = b_o(t)\, e^{-g_1 t/2} \mid 10 \rangle + b(t)\, e^{-g\, t/2} \mid 00 \rangle$$

where $b_o(t)$, $b(t)$ are the amplitude and $g = 8 \times 10^9$ sec^{-1} and $g_1 = 7,2 \times 10^6$ sec^{-1} are the total transition probability of the $\mid 10 \rangle$ and $\mid 00 \rangle$ state respectively.

In the interaction rapresentation the Schrödinger equation for $\mid u \rangle$ is

$$(4) \qquad i\hbar\, \frac{d}{dt} \mid u \rangle = H_{int} \mid u \rangle$$

where

$$H_{int} = e^{-\frac{i}{\hbar} H_o t}\, (-\frac{i}{2} K + V)\, e^{\frac{i}{\hbar} H_o t}$$

H_o is the free Hamiltonian

K is the term responsible of the decay.

Multiplying from left with $\mid 10 \rangle$ and $\mid 00 \rangle$ and remembering the unperturbed Schrodinger equation, we get the two coupled differential equations

$$b_o = -i b\, D\, e^{-pt}$$
$$b = -i b_o\, D^*\, e^{pt} \qquad p = \frac{g - g_1}{2} - i v$$

If a_1 and a_2 are the two roots of the equation

$$\ddot{b} - p\dot{b} + \mid D \mid^2 b = 0$$

and $v = \omega_o - \omega$ is the difference between the proper pulsation of the system and that of the microwave field, we get

$$\mid u \rangle = C_1 \left[i a_1\, e^{ivt}\, D^{-1} \mid 10 \rangle + \mid 00 \rangle \right] e^{-(g/2 - a_1)t} +$$

$$+ C_2 \left[i a_2\, e^{ivt}\, D^{-1} \mid 10 \rangle + \mid 00 \rangle \right] e^{-(g/2 - a_2)t}.$$

with C_1 and C_2 arbitrary constants.

The vector $|u\rangle$ is the sum of two aggregates $|u_1\rangle$ and $|u_2\rangle$ with definite life-times

$$G_1 = g - 2\,\mathrm{Re}\,(a_1) \qquad G_2 = g - 2\,\mathrm{Re}\,(a_2)$$

We choose the proper normalization for $|u_1\rangle$ and $|u_2\rangle$ by imposing that $|u_1\rangle$ tends to $|00\rangle$ and $|u_2\rangle$ to $|10\rangle$ when V tends to 0.

Defining $k = |a_1| / |D| = |D| / |a_2|$ and $R^{-2} = 1 + k^2$ we can write

$$|u_1\rangle = \left[i\,k\,e^{i(vt + \theta + \varphi)}|10\rangle + |00\rangle \right] R\,e^{-\left(\frac{G_1}{2} - i\,\mathrm{Im}\,a_1\right)t}$$

$$|u_2\rangle = \left[i\,e^{i(vt - \theta + \varphi)}|10\rangle + k\,|00\rangle \right] R\,e^{-\left(\frac{G_2}{2} - i\,\mathrm{Im}\,a_2\right)t}$$

where $\theta = \theta a_1 = -\theta a_2$ since

$$a_1 a_2 = |a_1|\,|a_2|\,e^{i(\theta a_1 + \theta a_2)} = |D|^2$$

It is immediate to verify that

$$G_1 = \frac{g_1\,k^2 + g}{1 + k^2} \qquad\qquad G_2 = \frac{g_1 + g\,k^2}{1 + k^2}$$

If H is of the order of 10 Gauss, as can be done with the present tecniques, $k^2 \ll 1$ and we have again

$$G_1 \cong g \qquad G_2 \cong g_1$$

IV. - INTERFERENCE EFFECTS DUE TO C NONCONSERVATION. -

If we believe that a C violating part in the decay amplitude of the singlet state, exists the three photons decay amplitude is composed of two coherent terms, one from singlet and one from triplet, that can interfere.

Let $\mathcal{H}$ be the total Hamiltonian responsible for the decay of $|u\rangle$ and let us calculate the squared matrix elements

$$|\langle 3\gamma|\mathcal{H}|u_1\rangle|^2 = R^2 \left\{ k^2 |\langle 3\gamma|\mathcal{H}|10\rangle|^2 + |\langle 3\gamma|\mathcal{H}|00\rangle|^2 - \right.$$

$$\left. - 2\,k\,\mathrm{Re}\left[\langle 3\gamma|\mathcal{H}|10\rangle^* \langle 3\gamma|\mathcal{H}|00\rangle\right] \sin(vt + \theta + \varphi) \right\} e^{-G_1 t}$$

$$\left|\langle 3\gamma|\mathcal{H}|u_2\rangle\right|^2 = R^2\left\{\left|\langle 3\gamma|\mathcal{H}|10\rangle\right|^2 + k^2\left|\langle 3\gamma|\mathcal{H}|00\rangle\right|^2 -\right.$$

$$\left.-2\,k\,Re\left[\langle 3\gamma|\mathcal{H}|10\rangle^x\langle 3\gamma|\mathcal{H}|00\rangle\right]\sin(vt-\theta+\varphi)\right\}e^{-G_2 t}$$

From the experimental point of view, we must observe the time distribution of the three γ decays with respect to the positronium formation. If C violating effects are present we have a life-time of about 150 psec. modulated by the interference term.

Care must be taken, however, with the phase φ. In fact it is a random number which can average out the interference term.

In order to compare the limit that can be put to the branching ratio $\mathcal{E}$ with the existing one, we assume the lagrangian (1) and describe the C conserving $^3S_1 \rightarrow 3\gamma$ decay with the usual e. m. interaction.

The nonvanishing matrix elements of (1) are those in which the polarization vector of only one photon is perpendicular to the decay plane. The interference term is a function of two independent variables only and it is maximum when the angle between two photons is such that $\cos\theta_{12} \simeq 0,5$ and q_1 (the momentum of first photon) is $\simeq .3$ MeV.

The oscillating term of $|u_1\rangle$ is superposed to an almost flat contribution coming from $|u_2\rangle$ and from the $m = \pm 1$ states of ortho positronium. Their ratio is $\eta \simeq 10\,h$ if $H \simeq 10$ Gauss.

In the hypothesis that we can appreciate experimentally an oscillating amplitude of the order of 10% ($\eta \simeq 10^{-1}$) we can put $h \lesssim 10^{-2}$ i. e.

$$\mathcal{E} = \frac{rate\ ^1S_o \rightarrow 3\gamma}{rate\ ^1S_o \rightarrow 2\gamma} \simeq 10^{-10}$$

A rough evaluation of $\mathcal{E}$ can be obtained as follows[4]

$$\mathcal{E} \simeq \frac{(phase\ space)_{3\gamma}}{(phase\ space)_{2\gamma}}(qr)^8\alpha$$

where α is the fine structure constant and $(qr)^8$ account for centrifugal barrier effect, q is the mean momentum of decay photons and r is some interaction radius. If we assume r to be $\simeq 1/m_e$, we get $\mathcal{E} \simeq 10^{-8}$.

Concluding, the described method allows the lowering of the present experimental limit on C violation in $^1S_o \rightarrow 3\gamma$ decay of a factor at least 10^4.

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