COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 70/7
4 Marzo 1970

S. Ferrara and A. F. Grillo : VECTOR DOMINANCE MODEL AND INFINITE MULTIPLETS. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-70/7

Nota interna : n. 466
4 Marzo 1970

S. Ferrara and A. F. Grillo : VECTOR DOMINANCE MODEL AND INFINITE MULTIPLETS. -

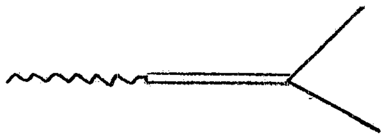
I. - INTRODUCTION.

Recently many models for the electromagnetic form factors of hadrons have been suggested[1]. These models are essentially derived from the strong amplitude proposed by Veneziano. Although the connection with this amplitude is not clear, also for the derivation of the π -form factor using Field Algebra + Veneziano model, these form factors fit in an extremely good way the experimental data, at least in the space-like region, so it seems interesting to look for a dynamical model for their explanation. The purpose of this note is a possible interpretation of these so-called Veneziano-like form factors in the framework of the field theory of composite systems.

We take here the point of view of Fronsdal[2], namely to write down a "phenomenological" propagator (which contains an infinite sequence of poles) and then to compute vertex functions as matrix elements of a vector operator between numbers of an infinite multiplet of some non-compact graph. In this scheme the natural generalization of the Kroll-Lee-Zumino normalization condition[3], by means of an extension of the vector dominance hypothesis, is obtained. The asymptotic behaviour of the π and N form factors predicted in Veneziano-like models are exactly reproduced if the external particles are assigned to the O(4. 2) classification[4]. In this preliminary work we treat the meson field coupled to the photon as a superposition of conventional vector fields[5]. The main problem to be solved, for a self consistent approach, is to treat the intermediate states on the same footing of the external ones i. e. how to couple the electromagnetic field to such an object.

II. - PROPAGATORS.

The matrix element of the electromagnetic current operator is described by the following Born term :



where the wavy line is photon propagator, the solid line is the propagator of the intermediate states and the strong vertex is assumed pointlike. For definiteness we shall consider the isovector part of the current so we are led to consider particles with the rho quantum numbers. From general requirements it is possible to restrict the analytic structure of the propagator $\Delta(p^2)$. This function must be meromorphic with a set of simple poles m_n^2. In order to allow a particle interpretation of these poles all residues must be positive so the propagator must show a structure of alternating poles and zeros.

The expansion of this propagator in a sequence of simple poles relates the bahaviour of the mass spectrum and of the residues. In fact, in order to write

(1) $$\Delta(p^2) = \sum_n \frac{C_n}{p^2 - m_n^2}$$

the ratio C_n / m_n^2 must vanish more rapidly than $1/n$ (Mittag-Leffler theorem).

If we assume a linear mass spectrum and if we require the following asymptotic power behaviour

$$\Delta(p^2) \underset{p^2 \to -\infty}{\sim} (p^2)^{-\beta}$$

then we have

$$\Delta(p^2) = \int_0^1 x^{1-\alpha(p^2)} (1-x)^{-\beta} \phi(x)\, dx$$

where $\phi(x)$ is an analytic function, different from zero, in the interval (0, 1).- $\alpha(p^2)$ is the ρ-trajectory which appear in the π-π Veneziano scattering amplitude. The mass spectrum is already given by:

$$m_n^2 = \frac{n - b}{a} .$$

the simplest choice is $\phi(x)$ =const and we obtain the propagator

$$\Delta(p^2) = \gamma \frac{\Gamma(1-\alpha(p^2))}{\Gamma(1+\beta-\alpha(p^2))} .$$

In order to have positive residues and decreasing propagator for $p^2 \rightarrow -\infty$ the following limitation must hold $0 < \beta < 1$.

We have assume the composite ρ field $\rho_\mu(p)$ to be conserved and to satisfy the equation

$$(2) \qquad L(p^2)\,\rho_\mu(p) = 0 \qquad\qquad (L^{-1}(p^2) = \Delta(p^2))$$

so that

$$\rho_\mu(p) = \sum_n \psi_\mu^n(p)\,\delta(p^2 - m_n^2) = \sum_n \rho_\mu^n(p)$$

obviously the fields $\rho_\mu^n(p)$ satisfy the K-G equations

$$(p^2 - m_n^2)\,\rho_\mu^n(p) = 0 .$$

The coupling to the strong current is assumed to have the usual form

$$L(p^2)\,\rho_\mu(p) = g_\rho\, J_\mu^s(p)$$

and we can write for each daughter $\rho_\mu^n(p)$

$$(p^2 - m_n^2)\,\rho_\mu^n(p) = g_\rho^n\, J_\mu^s(p)$$

and $g_\rho^n = C_n\, g_\rho$.

If we assume the global interaction with the isovector part of the electromagnetic field to be given by:

$$(3) \qquad J_\mu^{el}(p) = \lambda_\rho\, \rho_\mu(p)$$

then the following relation holds:

$$\langle A | J^{el}(0) | B \rangle = \frac{\Delta(p^2)}{\Delta(0)} \langle A | J^s(0) | B \rangle$$

with the normalization condition

(4) $$\lambda_{\rho}\, g_{\rho} = \Delta^{-1}(0)$$

The assumed analytic properties of the propagator function $\Delta(p^2)$ allows an alternative interpretation of the above scheme which supports the coupling described by eq. (3).

Since the propagator is a meromorphic function of p^2 we can write, from the Weiestrass theorem

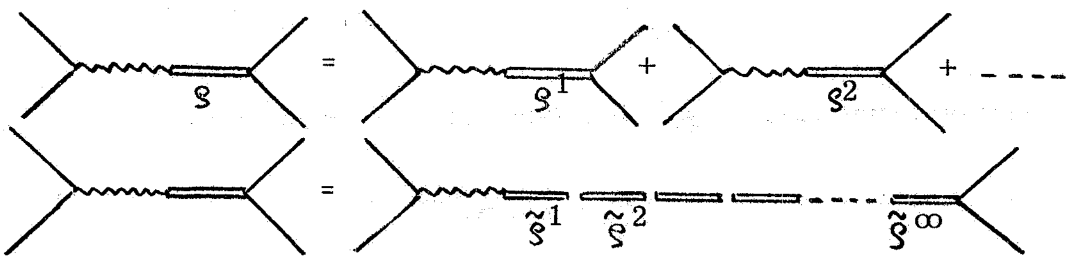
$$\Delta(p^2) = \prod_{i=1}^{\infty} \frac{f_n(p^2)}{p^2 - m_n^2}$$

where the functions $f_n(p^2)$ contain the zeros of $\Delta(p^2)$[6].

The two expressions (1) and (3) for the propagator allow the two possible graphical schemes[7]

a

b



where the $\tilde{\rho}_n(p)$ satisfy the following set of coupled equations:

$$(p^2 - m_1^2)\, \tilde{\rho}_{\mu}^{1}(p) = f^1(p^2)\, \tilde{\rho}_{\mu}^{2}(p)$$

$$(p^2 - m_2^2)\, \tilde{\rho}_{\mu}^{2}(p) = f^2(p^2)\, \tilde{\rho}_{\mu}^{3}(p)$$

. . . .

$$(p^2 - m_n^2)\, \tilde{\rho}_{\mu}^{n}(p) = f^n(p^2)\, \tilde{\rho}_{\mu}^{n+1}(p)$$

. . . .

and the limiting case

$$\rho_{\mu}^{\infty}(p) = \left(\lim_{n\to\infty} \frac{f_n(p^2)}{p^2 - m_n^2}\right) g_{\rho}\, J_{\mu}^{s}(p) \quad ;$$

we then see that

$$\tilde{\rho}_{\mu}^{1}(p) = \rho_{\mu}(p)$$

$$\tilde{\rho}_{\mu}^{\infty}(p) \propto g_{\rho}\, J_{\mu}^{s}(p)$$

So only the $\tilde{\rho}_\mu^1$ couples to the electromagnetic current and only the infinite mass meson couples to the strong current[8].

The connection between the fields $\tilde{\rho}_\mu^n(p)$ and $\rho_\mu^n(p)$ is:

$$\tilde{\rho}_\mu^n(p) = \frac{f_n(p^2)}{C_n} \prod_{i=n+1}^{\infty} \frac{f_i(p^2)}{p^2 - m_i^2} \rho^n(p)$$

and

$$C_n = f_n(m_n^2) \prod_{i \neq n}^{\infty} \frac{f_i(m_n^2)}{m_n^2 - m_i^2} = f_n(m_n^2) \prod_{i=1}^{n-1} \frac{f_i(m_n^2)}{m_n^2 - m_i^2} k_n$$

where k_n defines the "strong coupling constant" of the $\tilde{\rho}_\mu^n(p)$ by means of the relation

$$k_n = f_n(m_n^2) \prod_{i=n+1}^{\infty} \frac{f_i(m_n^2)}{m_n^2 - m_i^2}$$

III. - FORM FACTORS.

The vector vertex functions are defined by means of the basic relations[4]:

$$F_\mu^{i\,AB}(p^2) = \langle A \,|\, J^i(0)\, e^{i \vec{\xi} \cdot \vec{M}} \,|\, B \rangle \qquad (i = \text{electr. or strong})$$

where A, B are rest-frame states, $\vec{\xi}$ is connected to the relative velocity between the two particles by the relation $\vec{V}/c = \mathrm{tg\,h}\,\vec{\xi}$, and $\vec{M}$ are the boost-generators. We see, from (2), that in order to compute the form factors a model for the strong model matrix-element must be assumed. Following Barut we assume that the matrix-element of the strong current is an algebraic vector operator of an appropriate non-compact group. Then $|A\rangle$, $|B\rangle$ are members of a "tower" i. e. an infinite multiplet which transforms under an irreducible representation of the group. For these purposes the group 0(3,1), the Lorentz group, turns out to be too small, so, as pointed out by Barut, the group 0(4,2) has to be considered[9].

Indeed this group is perhaps the "minimal group" which allow to describe a set of parallel Regge trajectories.

We define the π and N electromagnetic form factors by means of the following formula:

$$(5)\qquad G_{\pi}^{el}(p^2) = \frac{\Delta(p^2)}{\Delta(0)}\, G_{\pi}^{s}(p^2) = \frac{\Delta(p^2)}{\Delta(0)}\, \frac{1}{\cosh^2\frac{\xi}{2}}\, F_0^{\pi\pi s}(p^2)$$

where $\cosh^2\frac{\xi}{2} = 1 - \frac{p^2}{4m_{\pi}^2}$

$$(6)\qquad \begin{aligned} G_E^{NNel}(p^2) &= \frac{\Delta(p^2)}{\Delta(0)}\, G_1^{NNs}(p^2) = \frac{\Delta(p^2)}{\Delta(0)} \cosh\frac{\xi}{2}\, F_0^{NNs}(p^2) \\ G_M^{NNel}(p^2) &= \frac{(p^2)}{(0)}\, G_2^{NNs}(p^2) = \frac{\Delta(p^2)}{\Delta(0)} \sinh\frac{\xi}{2}\, F_1^{NNs}(p^2) \end{aligned}$$

where $\cosh^2\frac{\xi}{2} = 1 - \frac{p^2}{4M_N^2}$

In this framework the strong form factors have the same structure as the electromagnetic ones investigated by Barut and collaborators. We observe that this approach clarifies the spin dependence of these form factors so it is possible in principle to compute form factors for higher spin-states. Their asymptotic behaviour is given by:

$$G^{\pi\pi s}(p^2) \underset{p^2\to -\infty}{\sim} |p^2|^{-1}$$

$$G_1^{pps}(p^2) \underset{p^2\to -\infty}{\sim} |p^2|^{-1} ; \qquad G_2^{pps}(p^2) \sim |p^2|^{-2}$$

The electromagnetic form factors behave like:

$$G^{ABel}(p^2) \underset{p^2\to -\infty}{\sim} |p^2|^{-\beta}\, \underset{p^2\to -\infty}{G^{ABs}(p^2)} .$$

It is interesting to observe that the Veneziano-like form-factors as introduced by many authors agree with our scheme.

These form factors are given by the following formula:

$$(7)\qquad G_n^{el}(p^2) = \gamma_n \frac{\Gamma(1-\alpha(p^2))}{\Gamma(\beta+n-\alpha(p^2))}$$

where $n=2$ for the π and $n=3$ for the nucleon and β is a half integer constant (In ref. (1) β is taken 1/2).

Formula (7) can be rewritten as

$$G_n^{el}(p^2) = \frac{\Gamma(1-\alpha(p^2))}{\Gamma(\beta+1-\alpha(p^2))} \, g_n(p^2)$$

where

$$g_\pi(p^2) = \frac{\gamma_\pi}{\beta+1-\alpha(p^2)} \; ; \qquad g_N(p^2) = \frac{\gamma_N}{[\beta+1-\alpha(p^2)][\beta+2-\alpha(p^2)]}$$

Remembering fomulae (5), (6) and (2) the functions $g_n(p^2)$ can be identified with the strong vertices

$$\frac{1}{\Delta(0)} G^{ABs}(p^2) .$$

Furthermore, even if the functions $g_n(p^2)$ are different from the 0(4, 2) vertices their asymptotic behaviour turns out to be the same.

IV. - CONCLUSIONS.

In this paper an attempt has been made in order to give a possible dynamical model for electromagnetic form factors of the Veneziano-type. A generalized rho dominance has been assumed, as in the most recent works (the photon couples to the rho and all its spin one daughters). In this scheme the matrix element of the electromagnetic current has been derived as a product of a "propagator" and of a strong vertex function (generalized Born term).

The main defect of our approach is that the ρ field has been assigned to a conventional field (vector transforming as a reducible representation of the Poincare group), while the external particles lie in a 0(4. 2) representation. In order to have a more consistent scheme also the intermediate objects must belong to an infinite multiplet and a more complicated coupling of these field to the electromagnetic one must be introduced. From this point of view the rho field must be assigned to a meson tower of 0 (4, 2) which can be described by a generalized tensor field. This tensor can be decomposed into irreducible spin-J tensors(2). As each spin-J representation has an infinite multiplicity an infinite sequence of spin-J particles can be accomodated. The spin-one component can define a conserved four vector (our ρ_μ) which is coupled, as in (2), to the electromagnetic current. In this framework also the isoscalar form factor (generalized ω and ϕ dominance) can be taken into account by enlarging the rest-frame symmetry group to SU(3) $\otimes$ 0(4, 2). A more detailed analysis of these points will be given in a future work.

REFERENCES AND FOOTNOTES.

(1) - Y. Oyanagy, Tokyo Univ. Preprint UT 16-69 (1969); R. Jengo and E. Remiddi, CERN Preprint TH-1050 (1969); P. Di Vecchia and F. Drago, Lett. Nuovo Cimento 1, 917 (1969); R. Jengo and E. Remiddi, Lett. Nuovo Cimento 1, 922 (1969); H. Suura, Phys. Rev. Letters 23, 551 (1969); P. H. Frampton, Chicago Preprint EFI 69-90 (1969).

(2) - C. Fronsdal, Proc. 1967 Intern. Conf. on Particles and Fields, Rochester (1967); C. Fronsdal, Phys. Rev. 168, 1845 (1968).

(3) - N. M. Kroll, T. D. Lee and B. Zumino, Phys. Rev. 157, 1376 (1967).

(4) - A. O. Barut, Nuclear Phys. B4, 455 (1968); A. O. Barut, D. Corrigan and H. K. Kleinert, Phys. Rev. Letters 20, 167 (1968).

(5) - A similar approach has been investigated, from a different point of view, by M. Kobayashi, Progr. Theoret. Phys. 42, 1319 (1969).

(6) - In the case of eq. (2) the functions $f_n(p^2)$ are the following:

$$f_1(p^2) = \Upsilon(\beta + 1 - \alpha(p^2))\, e^{c/\beta}$$

$$f_n(p^2) = e^{-\beta/n-1}(n + \beta - \alpha(p^2))$$

where c is the Eulero-Mascheroni constant.

(7) - C. Bernardini, private communication.

(8) - It is a natural assumption to describe such an infinite mass object by means of a point-like interaction.

(9) - A. O. Barut and H. Kleinert, Coral Gable Conferences, Fourth Conference, Univ. of Miami (1967).