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U. Bizzarri and A. Vignati: THE 12 MeV MICROTRON USED AS INJECTOR OF THE FRASCATI ELECTRON-SYNCHROTRON.

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INTRODUCTION. -

The electron accelerator called "Microtron" was proposed in 1944 by V.I. Veksler(1); from that time through about 1963 the development of such a machine was kept to a very low rate, due to the low values of the electron currents obtenible from the original design. In fact in the first prototypes the injected electron current was obtained by field effect extraction from the walls of the resonant cavity. The rather low injected current produced in such a way has been demonstrated to have a peak value corresponding to a wrong phase in order to be accelerated by successive turns, giving rise to an output peak current not exceding the range of 100 μA. A drastic increase of the peak output current of 3 order of magnitudes was obtained as soon as the way was found of injecting electron from an hot cathode source. Such improvement, occurred around 1963 together with minor improvements in the performance of the resonating cavities, allowed the Microtron to jump from the range of the laboratory curiosities into the realm of interesting electron accelerator machines.

In 1964 a program of studing and building a Microtron in the range of 10 MeV energy has been started at Frascati Laboratories in cooperation with the Physics Department of Catania University[12]. After a first model machine of 5.5 MeV energy, it has been possible the suc-cesfull development of 12 MeV energy microtron, and two such machines have been built. The first one has been installed as injector into the Frascati 1,1 GeV Electron-Synchrotron in August 1968; the second one will be put into operation as a medical as well as a physical research facility in a Laboratory of Catania University.

The Microtron working from more than one year as injector into the Frascati Electronsynchrotron produces a very well collimated external beam of electrons, of 60 mA peak current, $2 \div 4\ \mu s$ pulse duration, 20 pulses per second; 12 MeV energy. During that time this machine has shown a very high degree of stability and reliability, the only service required being the periodical change (every about 300 hours of running) of the cathode.

Because of the successful and smooth running of this accelerator, we are ancouraged to give a rather extensive description of the Microtron in the present paper, with reference to the design details of the machine realized in Frascati Laboratories. In fact we believe that the development of the Microtron has reached a stage interesting both for peoples who need a simple, reliable, low cost electron accelerator of energy less than 20 MeV, as for peoples designing higher energy machines based on the same principles.

1. - BASIC THEORY OF THE MICROTRON. -

Electrons running into the microtron are bended in circular orbits of increasing radius by a static and omegeneus magnetic field B. In each turn they cross the gap of a microwave resonating cavity, where they are accelerated by an high electric field (see Fig. 2).

A bunch of electrons accelerated in the first crossing of the cavity, must run the first turn in such a way to find the field into the cavity at second crossing in the right phase to be accelerated again. This is the so called "synchronism condition" which imposes a relationship between the magnetic field B, the accelerating voltage through the cavity ΔV, and the microwave frequency ν_{RF}. Of course a similar condition must be satisfied for all the successive orbits.

Calling T_n the running time of the n-th orbit and eVn the total (including rest) energy of the electrons in this orbit, we have the following relation:

$$T_n = \frac{2 V_n e}{B c^2} \tag{1.1}$$

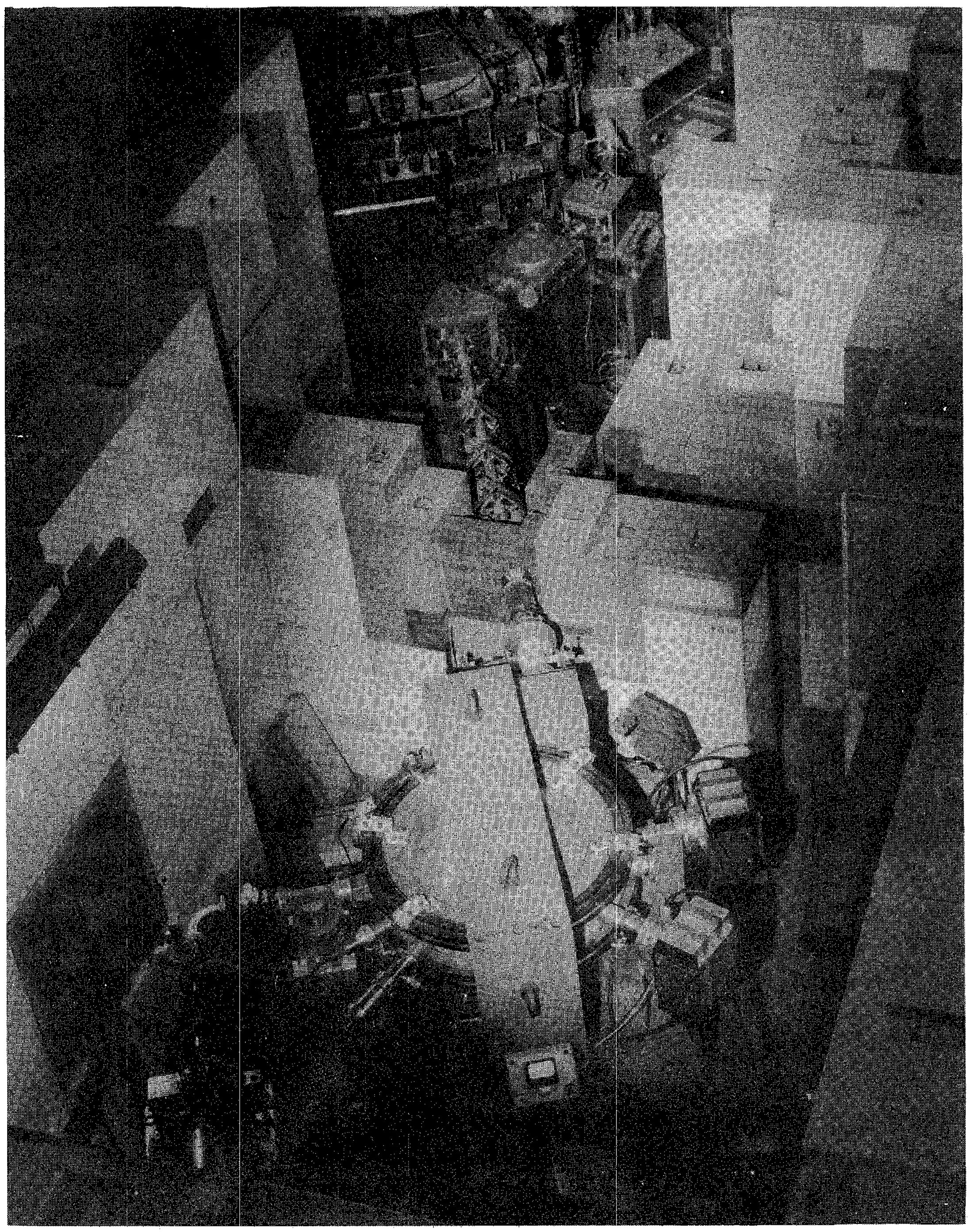


Fig. 1

where: B is the magnetic field (in gauss), and c the light velocity (in cm/s).

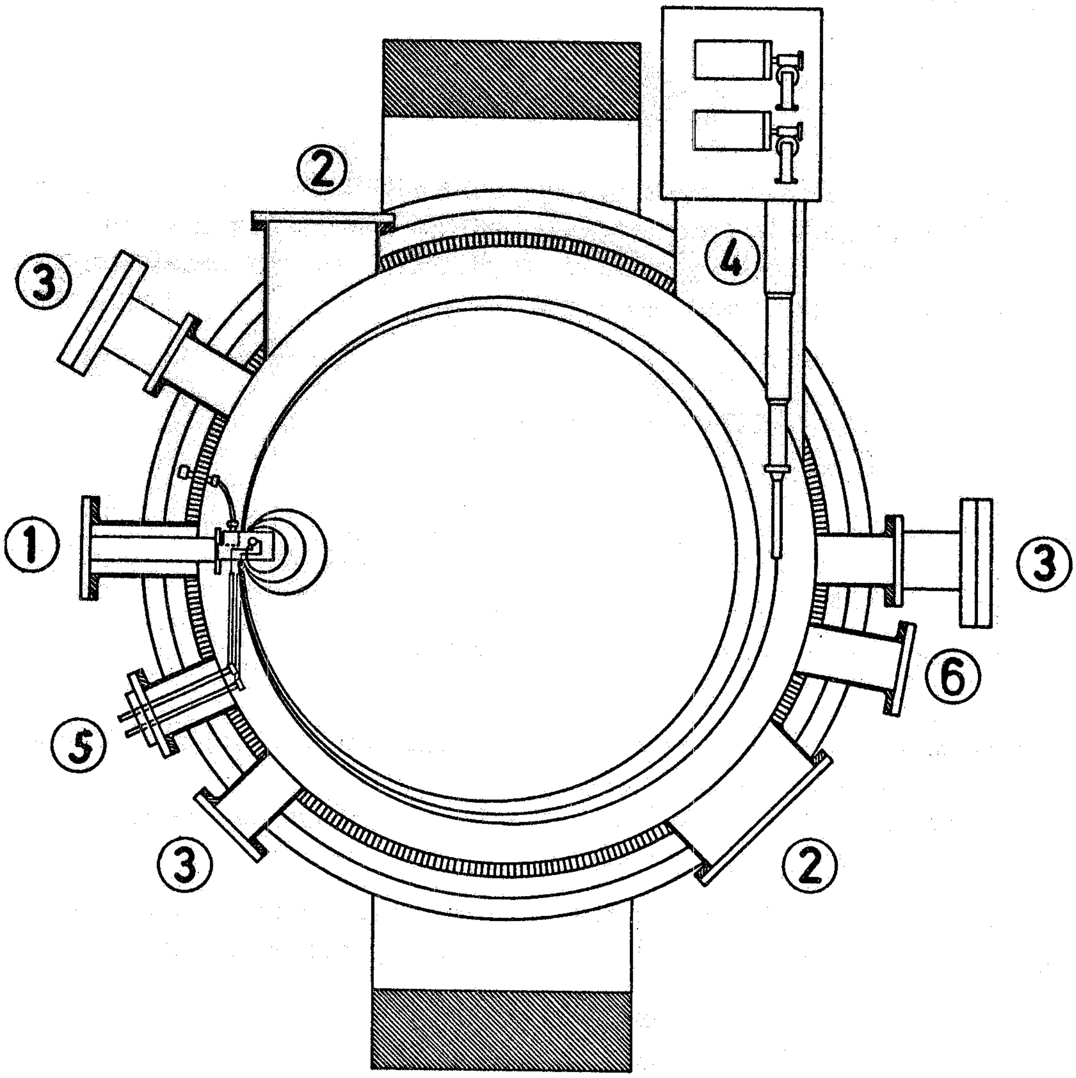


FIG. 2

The synchronism condition imposes that:

(1.2) $$T_1 = a\,T_{RF} \,,$$

(1.3) $$T_{n+1} - T_n = b\,T_{RF} \,,$$

where: $T_{RF} = 1/\nu_{RF}$ is the microwave period and $n = 2, 3, 4, \ldots$ The arbitrary integers a, b characterize the "mode" of working of the microtron.

According to our previous definition of the total energy on n-th orbit, we have:

(1.4)
$$V_n = \frac{E_o}{e} + V_i + n\,\Delta V\,,$$

where: E_o/e is the electron rest energy (510 KeV), V_i the injector voltage, ΔV the voltage gain at each crossing of the cavity. By the sostitution of (1.4) in (1,1), and imposing conditions (1.2), (1.3), we obtain the following basic microtron's equations:

(1.5)
$$\Delta V = \left(\frac{E_o}{e} + V_i\right)\frac{b}{a-b}$$

(1.6)
$$B = \frac{2\pi\nu_{RF}}{c^2}\left(\frac{E_o}{e} + V_i\right)\frac{1}{a-b}$$

From these equations it can be observed that:

- $a > b$;
- the maximum value for B - minimum diameter of the magnetic for a given final energy - is obtained with a-b=1;
- with the condition a-b=1, the "mode" a=2, b=1 requires the minimum accelerating voltage ΔV, at cavity gap.

The a=2, b=1 is called the "fundamental mode", and is the one employed in our design. The basic equations in our case are:

(1.7)
$$\Delta V = \frac{E_o}{e} + V_i\,,$$

(1.8)
$$B = \frac{2\pi\nu_{RF}}{c^2}\left(\frac{E_o}{e} + V_i\right).$$

We have assumed an injection voltage:

$$V_i = 70\ \text{KV}.$$

The voltage gain per turn resulting:

$$V = 510 + 70 = 580\ \text{KeV},$$

which is a value rather easily obtainable with microwave tubes (magnetron or klystron) of more than 500 KW power output.

As it will be shown lather the choise of the frequency is imposed by a compromise between magnet dimentions and cathode dimensions.

We assumed ν_{RF} =3.000 Mc/s, from (1.8) it results:

$$B = 1.214 \text{ gauss.}$$

The diameter D of the final orbit is fixed from B and from the number n of orbits, according to the relationship:

$$D = 2 \frac{\left(\frac{E_o}{e} + V_i + n\,\Delta V\right)}{B\,c^2} .$$

In our case with n = 21 orbits, we have:

$$D = 70 \text{ cm};$$

the final kinetic energy results to be of 12,25 MeV.

A choice of a larger value of the frequency ν_{RF} according to eq. (1.8), results in a larger value of B, and smaller magnet diameter, keeping constant the final energy. Unfortunately the dimension of the electron gun, which must be placed between the cavity and the first electron orbit, imposes a minimum value to the diameter of this orbit, which is given in our case by:

$$d_{1st} = 2 \frac{c}{\pi \nu_{RF}} \beta = 5,6 \text{ cm} ,$$

β being the ratio v/c between the first orbit electron velocity and the light velocity. Smaller values of d_{1st} are, of course, possible, but in the design of our microtron we decided to keep the conservative way.

The hypothesis assumed in the preceding basic theory are the following:

1) - constant voltage gain ΔV per turn;
2) - perfectly homogeneous and constant magnetic field;
3) - the transit phase constant;
4) - perfectly circular orbits, crossing tangentially the center of the cavity, and running in a medium plane.

No one of the preceding hypothesis is exactly fullfilled by a real machine, and in order to keep a high intensity beam running, we have to develop a better understanding of the behaviour of accelerated electrons, allowing the optimization of the following parameters of the machine:

- design of the electron-gun and the cavity gap;
- shape of the magnetic field;
- cavity peak voltage;
- shape of the cavity holes for improving the beam focusing.

2. - VOLTAGE GAIN OF ELECTRONS CROSSING THE CAVITY. -

The radiofrequency voltage into the cavity is time dependent following a sinusoidal law. An electron crossing a cavity having a vanishing thin gap, gains a voltage given by:

$$\Delta V = V_o \sin 2\pi \nu_{RF} t_o \tag{2.1}$$

where V_o is the peak voltage, and t_o is the instant of the crossing.

In the case of a cavity having a gap of lenght h, a relativistic electron spends a time $\tau = h/c$ to cross the gap. The voltage gain in this case must be averaged over the time:

$$\Delta V = \frac{V_o}{h} \int_{t_o - \frac{\tau}{2}}^{t_o + \frac{\tau}{2}} v \sin 2\pi \nu_{RF} t \, dt \; ;$$

assuming the velocity of electrons $v = c$, the result is:

$$\Delta V = V_o \frac{\sin 2\pi \nu_{RF} \frac{\tau}{2}}{2\pi \nu_{RF} \frac{\tau}{2}} \sin 2\pi \nu_{RF} t_o \, . \tag{2.2}$$

From the hypothesis that $v = c$, it follows that the voltage gain ΔV is constant in each turn. The hypothesis is well verified for all the turns but the first, where in our case $v \sim 0,8$ c. It is convenient to consider the electrons injected into the second orbit, with a kinetic energy equal to the sum of the injection energy eV_i and the energy gained in the first turn eV_1. In this case the 2nd orbit takes the place of the 1st one, and the microtron works in the mode $a = 3$, $b = 1$. It follows from (1.5) and (1.6) that:

$$\Delta V = (\frac{E_o}{e} + V_i + V_1)\frac{1}{2}$$

$$B = \frac{2\pi\nu_{RF}}{c^2}\Delta V$$

The total energy eV_n after n turns, will be given by:

$$V_{fin} = (n-1)\Delta V + \frac{E_o}{e} + V_i + V_1 = \frac{n+1}{2}(\frac{E_o}{e} + V_i + V_1) = (n+1)\frac{Bc^2}{2\pi\nu_{RF}} .$$

From this equation it results that the final total energy of electrons depends linearly from the magnetic field B. We have found in our machine that B must be changed of $\pm$ 5% for losing all the beam.

3.- STABLE PHASE THEORY.-

According to eq. (2.2) the voltage gain in each turn depends from the time t_o at which the electron crosses the center of the cavity. An electron is called "synchronous" when it crosses the cavity in succes sive turns in such a phase to keep constant the term:

(3.1) $$\sin 2\pi\nu_{RF}\, t_o = \text{const} = \sin \varphi_s$$

in eq. (2.2). The corresponding argument φ_s is called "synchronous phase".

If the "synchronous electron" gains the correct voltage ΔV in order to be accelerated till the last orbit, what happens to the "non synchronous electrons" having a phase $\varphi_k \neq \varphi_s$ in the k-th orbit? In other words how large is the range of the phases accepted by the machine to keep electrons till the n-th orbit. How this range of phases depends from the machine parameter? Many authors(4, 5, 6) have developed theories in the pourpose to answer such questions. We shall not repeat here these theories, but just recall some of the results which have been useful for the desing of our machine.

Let:

$$\tau_k = t_k - \bar{t}_k ,$$

be the time difference between the time t_k a particle running the k-th orbit actually crosses the cavity and the time $\bar{t}_k$ the synchronous particle, running the same orbit, crosses it.

The first difference $\Delta\tau_k = \tau_{k+1} - \tau_k$, is easily shown to be:

(3.2)
$$\Delta\tau_k = T_k - \bar{T}_k ,$$

where T_k is the period of the considered particle and $\bar{T}_k$ the period of the synchronous one running the k-th orbit.

According to eq. (1.1):

$$T_k = \frac{2\pi}{eBc^2} E_k ,$$

E_k being the electrons total energy in k-th orbit. From the synchronism condition (3.1) and from the eq. (1.5) and (1.6) we have

$$\frac{2\pi}{eBc^2} = \frac{b}{eV \sin\varphi_s \cdot \nu_{RF}} .$$

Combining the two last equations, and inserting in (3.2) we obtain:

$$\Delta\tau_k = \frac{b}{eV \sin\varphi_s \cdot \nu_{RF}} (E_k - \bar{E}_k)$$

where $\bar{E}_k$ is the total energy of the synchronous electron. By multiplying both members of last equation by $2\pi\nu_{RF}$, we obtain the following relation between the phases in the two successive orbits k and k+1:

(3.3)
$$\Delta\varphi_k \equiv \varphi_{k+1} - \varphi_k = \frac{2\pi b}{eV \sin\varphi_s} (E_k - \bar{E}_k) ,$$

The energy gain of electrons during the (k+1)-th crossing of the cavity is given by

(3.4)
$$E_{k+1} - E_k \equiv \Delta E_k = eV \sin\varphi_{k+1}$$

By combining eq. (3.3) and eq. (3.4), we get a recurrent formula which allows us to calculate energy and phase of an electron, by assigning the initial values E_1 and φ_1 over the 1st orbit. In order to resolve the preceeding equation we introduce a new variable, the "phase error", defined by:

$$U_k = \varphi_k - \varphi_s ;$$

it follows that:

$$\Delta\varphi_k = \varphi_{k+1} - \varphi_k = (\varphi_{k+1} - \varphi_s) - (\varphi_k - \varphi_s) = U_{k+1} - U_k \equiv \Delta U_k .$$

Now we perform the first difference of eq. (3.3):

$$\Delta(\Delta\varphi_k) = \Delta^2 U_k = \frac{2\pi b}{eV \sin\varphi_s} (\Delta E_k - \Delta\bar{E}_k) ;$$

introducing eq. (3.4) and remembering that:

$$\Delta\bar{E}_k = eV \sin\varphi_s ,$$

we obtain:

(3.5) $$\Delta^2 U_k = 2\pi b \left(\frac{\sin\varphi_{k+1}}{\sin\varphi_s} - 1\right) .$$

Assuming that $U_{k+1} \ll \varphi_s$, the $\sin\varphi_{k+1}$ could be approximated by:

(3.6) $$\sin\varphi_{k+1} = \sin(\varphi_s + U_{k+1}) \sin\varphi_s + U_{k+1} \cos\varphi_s ,$$

By inserting (3.6) into (3.5), and writing explicitly the second difference $\Delta^2 U_{k+1}$ we obtain the following approximate equation:

(3.7) $$U_{k+2} - 2U_{k+1} (1 + \pi b \operatorname{cotg}\varphi_s) + U_k = 0$$

Solutions of this equation are finite and oscillatory when the characteristic equation has imaginary roots, for values:

$$(1 + \pi b \operatorname{cotg}\varphi_s)^2 < 1.$$

We shall have phase stable solution (for assumed small phase oscillations) only where:

$$-\frac{2}{\pi b} \leq \operatorname{cotg}\varphi_s \leq 0 , \qquad \text{or:}$$

$$90^{\circ} \leq \varphi_s \leq 122^{\circ}\,26' \qquad \text{(supposing } b = 1\text{)}.$$

The optimal synchronous phase is supposed to be at the center of the above defined interval, that is to say:

$$\operatorname{cotg}(\varphi_s)_{opt} = -\frac{1}{\pi b} \; ; \qquad (\varphi_s)_{opt} = 107^{\circ}\,39' .$$

The corresponding optimum RF cavity peak voltage V_o, is given by:

$$V_o \sin \varphi_s = \Delta V \; ; \qquad (V_o)_{opt} = \frac{\Delta V}{\sin(\varphi_s)_{opt}}$$

where ΔV is the resonance voltage gain.

According to the previous calculation, the peak voltage V_o can range from the lower value corresponding to the required voltage gain ΔV, to the higest value allowed by the phase stability corresponding to:

$$\frac{\Delta V}{\operatorname{sen}(122^{o}, 26')} \simeq \Delta V\,(1 + 0, 18) . \tag{3.8}$$

We have solved the phase equations (3.3) and (3.4) for small phase oscillations; allowing large phase oscillations these equations can be numerically solved by the help of a computer. The solutions plotted in a plane of coordinates: φ and δV, where:

φ = phase of the particle in the successive orbits,
$e\,\delta V = E - \bar{E}$ = actual energy - sinchronous particle energy,

give rise to open or closed curves. Points inside a closed curve belong to closed curves conteined into the first one; in other words curves are not intersecting. It follows that it exist a closed limiting curve, all the points inside the curve belonging to closed curves, all points outside belonging to open curves. The closed limiting curve encloses the so called "stable phase region", a region in the φ, $(\delta V/\Delta V)$ plane all the points of which represent phase stable running electrons.

Each value of the peak cavity voltage V_o allowed by the limiting conditions (3.8) gives rise to a phase stable region which can be numerically calculated and plotted (see Fig. 3).

The plot of the phase stable region is a useful tool for evaluating the range of the useful injection phases. In fact an electron which

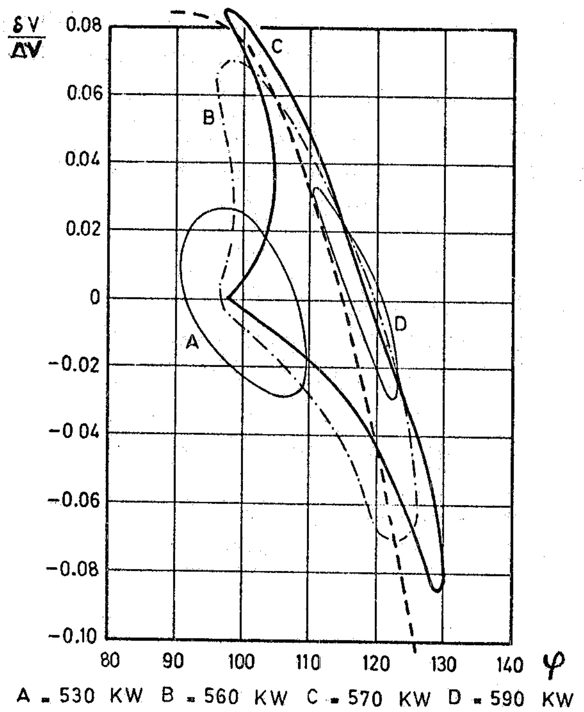


FIG. 3

is injected in to the cavity from the cathode with a phase φ gains a voltage which differs from the voltage gained by the sinchronous electron, by a relative amount given by:

$$(3.9) \qquad \frac{V_o \operatorname{sen} \varphi - V_o \operatorname{sen} \varphi_s}{V_o \operatorname{sen} \varphi_s} = \frac{\delta V}{\Delta V}$$

or simplifying:

$$(3.10) \qquad \frac{\operatorname{sen} \varphi}{\operatorname{sen} \varphi_s} - 1 = \frac{\delta V}{\Delta V}$$

This relationship is represented in to the plane φ, $\delta V / \Delta V$ by a curve which intersects the phase stable region. The points of the curve laing into the phase stable region represent phase stable injected electrons, in other words all electrons accepted for further acceleration from the phase stability point of view. (In Fig. 2 is plotted only to curve corrisponding to V_o= 570 KV).

4.- INJECTION.-

In the introduction of the present paper it has been mentioned the fact that injection has been one of the major problems in the development of the microtron. Cold extracted electrons from the walls of the cavity have been the source of the injected current in first microtrons. The intensity of the cold extracted electron current was thought to be large enough to give rise to an appreciable accelerated current. In recent years many authors have realized that cold electron current, field extracted from the walls of the cavity, is mostly injected with a wrong phase, just at the boundary of the above defined phase stable region. Such a difficulty has been overcome by hot cathode injection, following one of the two proposed methods:

1) electrons are injected by an hot cathode inside the cavity (Kapitza[7,8]);
2) electrons are injected by an hot cathode outside the cavity (Wernholm[6]).

By the method n. 2 a high capture efficiency could be reached (up to 6.5%) and final energy, emittance, geometrical position of the last orbit result quite insensitive to the value of the peak cavity radio frequency voltage. By this method of injection an energy, geometry, and position stable beam of electrons could be extracted from the last orbit of the microtron and easily injected into a larger machine. These are the reasons of our choice of the Wernholm injection method for the microtron built as injector of the Frascati 1.1 GeV.electronsynchrotron.

Kapitza's n. 1 injection method allows continuous change of the final energy of the beam extracted from the machine, through a range of $\pm 20\%$ of the nominal value. Due to the miscellaneous use, both for medical and physical pourposes, foreseen for the microtron to be installed at Catania University, in this machine both injection methods shall be allowed by minor changes into the injector and cavity geometry.

In the following we shall describe the injector of the Frascati machine, according to the Wernholm method. As it is shown in Fig.4, 5, 6.

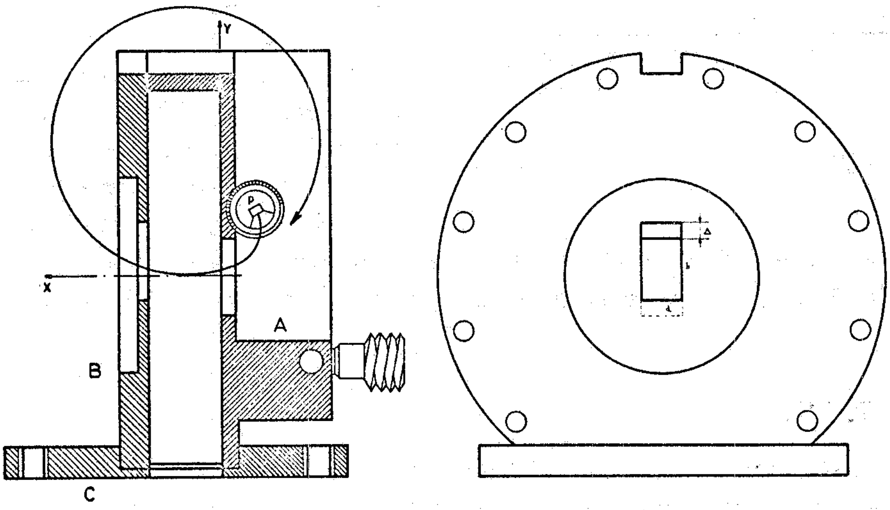


FIG. 4

the electron gun is put outside the cavity into a furrow machined in the wall of the cavity in order to keep clean the way for the first orbit. Electrons emitted by the cathode are accelerated by a pulse of 70 kV applied the cathode support, and move towards the grounded cylindrical envelope of the cathode. In this region they follow an epicycloidal trajectory due to the combined action of perpendicular electric and magnetic fields. Baffled by an hole in the cylindrical envelope of the cathode structure, a beam of electrons runs an half circle of 0.76 cm radius and enter into the first window of the cavity. The fringing electric field from the cavity helps the capture of electrons into the cavity gap.

The almost coaxial structure of the electron gun (see Fig. 7) is built by an external titanium cilynder containing a smaller tantalium

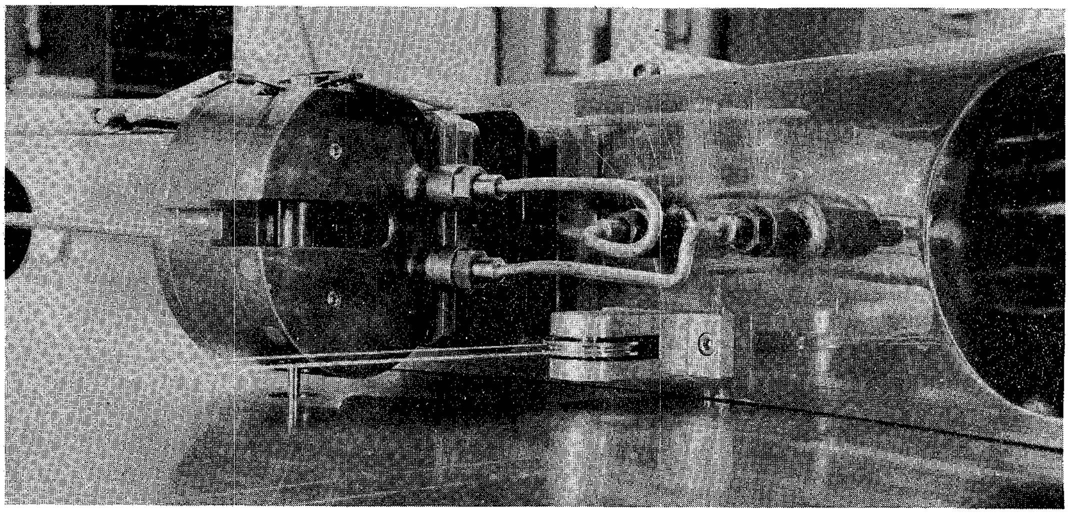


Fig. 5

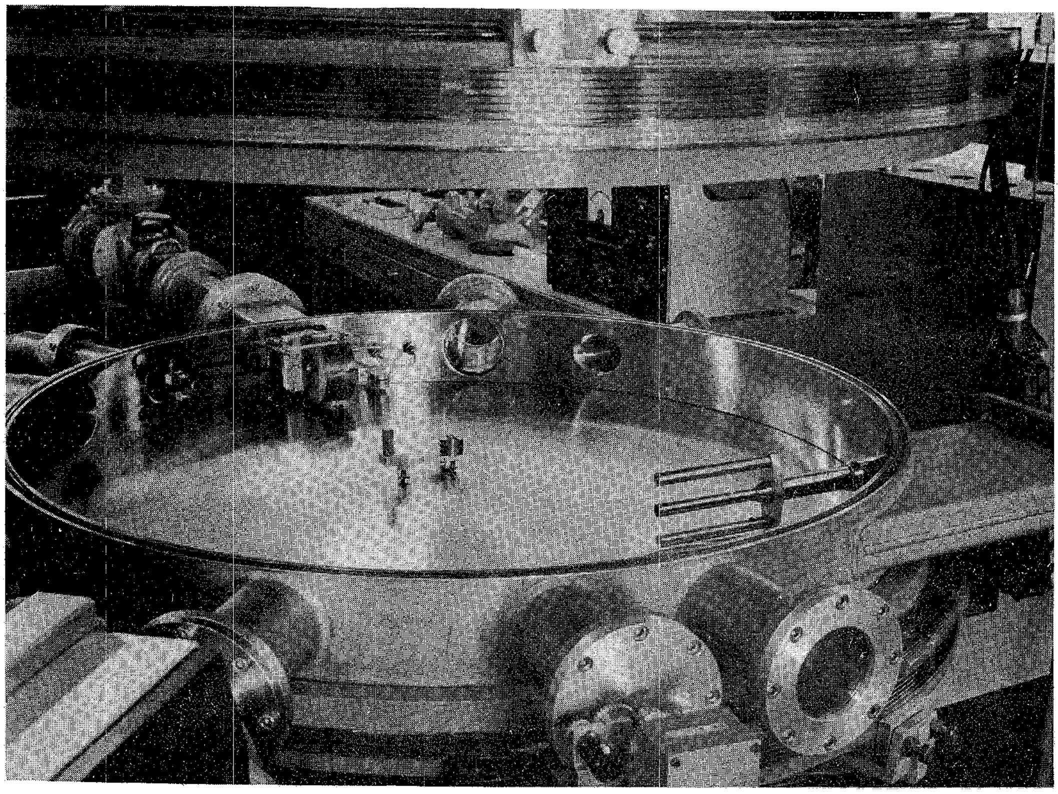


Fig. 6

rod. This latter supports a pill of an high emission density material (LaB_6) which is heated to about 1800°C by an auxiliary electron beam of 50 W power produced by a conventional Wolframium cathode. The cathode structure is supported by an Araldite insulating vacuum tight rod and passes throught an hole in the lower pole face of the microtron's magnet. The position of the cathode with respect to the cavity can by sligtly removely adjusted by mouvement of the supporting structure into the hole.

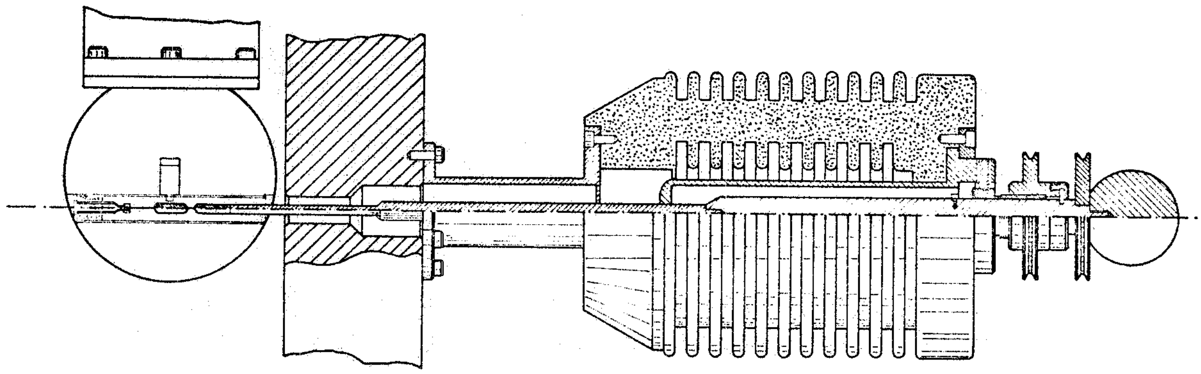


FIG. 7

5. - TECHNOLOGY OF THE LaB_6 EMITTER. -

One of the most delicate problems to be solved in order to built an efficient and long lived hot cathode injector for the microtron is the technology for preparing the LaB_6 pills. We have followed the steps described in the present paragraph.

The termojonic properties of the alkali earths have been studied by Lafferty(9). We refer here in Table I some of the properties of LaB_6.

TABLE I

Melting point	2210°C
Resistivity (1800°C)	ρ = 120 $\mu\Omega$cm
Hall coefficient	R = 7.7 x 10^{-12} V cm/amp gauss
Richardson's law parameters	A = 29 A/cm^2 °K^2 ϕ = 2.66 Volt
Evaporating rate at 1953°K	2.89 g/cm^2 s
Temperature corresponding to a vapour tension of 10^{-5} torr.	T_p = 2090°K

In Table II some properties of materials employed as cathodes are listed.

TABLE II

Material	T_p(%)	Φ (Volt)	T_p/Φ
Ta	2680	4.1	654
W	2860	4.5	658
Th	1910	3.4	563
Ba0 - Sr0	1000	1.5	660
Th over W	1910	2.7	710
LaB_6	2090	2.66	780

Pills of LaB_6 can be obtained by sintering the pressed powder under vacuum at a temperature ranging from 1375°C to 1800°C[9].

Heating the LaB_6 in contact of refractory metals like W, Ta, Mo, etc gives rise to the diffusion of Boron into the metal, and to evapo ration of Lantanum. Long living cathods must be supported by a inactive material. Renium or graphite supports have been used with some difficulties; we preferred an easy to machine Tantalum support, in which the place for the LaB_6 pill is prepared. A thin layer of $MoSi_2$ prevents the diffusion of LaB_6 into the Ta support. The layer of $MoSi_2$ is good conductor, does not react with both materials, has an high melting temperature of 2023°K and seems to be well suited to the pourpose.

The process for preparing this kind of cathodes goes through the following steps:

- a solution of $MoSi_2$ in trichloroethylene is dropped by means of a small brush into the hole drilled into the Tantalium support;
- quick evaporation of the trichloroethylene solvent leaves a layer of $MoSi_2$ powder which is heated under vacuum to a temperature of 2023°K by electron bombardment. The fused $MoSi_2$ covers the inside surface of the Ta hole with a strong thin film;
- powder of LaB_6 is pressed into the hole and is heated under vacuum at 1800°K for about 15 minutes.

Cathodes prepared in this way are ready for use; an emitting surface of about 5 mm^2 gives rise to currents of 5-10 A. The life of such cathodes is well above 1000 working hours.

6.- MAGNETIC FIELD.-

The microtron's magnetic field is supposed to be uniform over the region filled by the beam's orbits. An approximate calculation is described in the present paragraph in order to extimate the amount of disuniformity allowed. The last, and larger orbits are of course the more sensitive to field disuniformity(12, 13). We suppose the field B in the geometrical median plane to be described by the function:

$$B(\theta) = B_o + B_1 \operatorname{sen}\theta + B_2 \cos\theta \qquad (6.1)$$

where θ is the azimuthal angle over the orbit, taking $\theta = 0$ at the center of the resonating cavity gap. In this formula first armonic terms are the only one considered.

A further simplifying hypothesis is introduced by substituting the sinus and cosinus functions by step functions of the same period, phase and amplitude. The $B_1 \sin\theta$ function is approximated by:

$$+B_1 \quad \text{for} \quad 0^o < \theta < 180^o$$

$$-B_1 \quad \text{for} \quad 180^o < \theta < 360^o$$

The trajectory of electrons of momentum P consequently has the shape of two half cycles (see Fig. 8) having radii:

$$R_1 = \frac{P}{e(B_o - B_1)} \quad \text{end} \quad R_2 = \frac{P}{e(B_o + B_2)} \qquad (6.2)$$

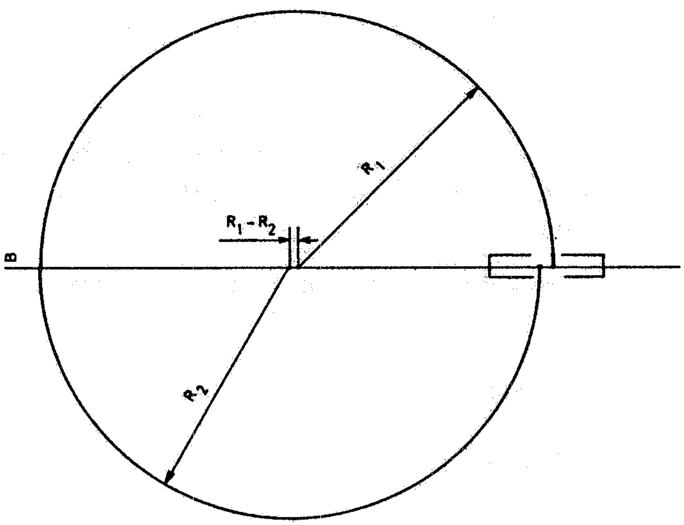


FIG. 8

Such a trajectory can cross again the cavity hole of radius r, if:

$$2(R_1 - R_2) \leq r .$$

From this condition it follows, by simple algebra, that

$$B_1 \lesssim \frac{e r B_o^2}{4 P} \simeq \frac{r B_o}{4 \bar{R}} ,$$

where:

$$\bar{R} = \frac{P}{e B_o} = (n+1) \lambda ;$$

or:

$$\frac{B_1}{B_o} \lesssim \frac{r}{4(n+1)\lambda} ,$$

where: n is the orbit number, λ the wavelenght of the radiofrequency feeding the cavity.

In our case we have:

$$r \cong 0.5 \text{ cm}, \qquad \lambda = 10 \text{ cm}, \qquad n = 20 \text{ (last orbit)},$$

and it results:

$$\frac{B_1}{B_o} = \frac{0.5}{4.21 \times 10} = 0.6 \times 10^{-3} .$$

This figure gives an order of magnitude of the accurancy of magnetic field uniformity needed to keep the beam running the largest orbits, without missing the cavity hole.

A graphical study of the term $B_2 \cos \theta$ in eq. (6.1) shows that a larger value of B_2 is allowed in order to let the beam crossing the cavity. In this case the value of B_2 changes the effective lenght of the orbits, and is mostly limited by the phase conditions.

Assuming again a step function approximation of cos θ, we have:

$$B_2 \cos \theta \simeq \begin{cases} +B_2 & \text{for} \quad 270^o < \theta < 90^o \\ -B_2 & \text{for} \quad 90^o < \theta < 270 \end{cases}$$

From Fig. 9 it is easy to verify that the lenght of the orbit results to be:

$$2\pi\left(\frac{R_1}{4}+\frac{R_2}{2}+\frac{R_1}{4}\right)+2(R_2-R_1)=$$

(6.3)

$$=2\pi\left(\frac{R_1+R_2}{2}\right)+2(R_2-R_1)\simeq 2\pi R_o+(R_2-R_1),$$

where: R_1 and R_2 are defined as in (6.2) changing of course B_1 by B_2.
In the last equation it is supposed:

$$\frac{R_1+R_2}{2}\simeq R_o .$$

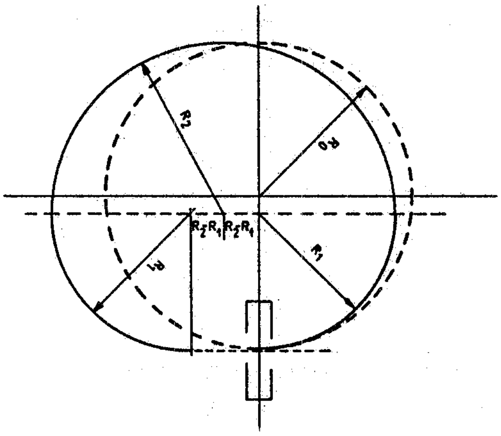


FIG. 9

The term $2(R_2-R_1)$ represents the change in the orbit's lenght due to the field asymmetry; it corresponds to a time lag of electrons circulating with velocity c, given by

$$\Delta T=\frac{2(R_2-R_1)}{C}=\frac{2P}{eC}\left[\frac{(B_o+B_2)-(B_o-B_2)}{B_o^2-B_2^2}\right]\simeq\frac{4P}{eC}\frac{B_2}{B_o^2}$$

The phase shift $\Delta\varphi=\omega\Delta T$ is given by:

(6.4) $$\Delta\varphi=-\frac{4P\omega B_2}{eCB_o^2}=-4(n+1)\frac{B_2}{B_o}.$$

Assuming to allow in the n = 20 orbit a maximum phase shift of $\Delta\varphi=$ = 0.5 rad, we have from (6.3):

$$\frac{B_2}{B_o}\leq 0,6\times 10^{-2}.$$

Finally we have to take into account that the average value of the actual magnetic field $\overline{B}(\theta)$ differs from the nominal value B_o. This error introduces a phase shift $\Delta\varphi'$ which easily shown to be, on the n-th orbit:

$$\Delta\varphi' = 2\pi(n+1)\,\frac{\overline{B}(\theta) - B_o}{B_o} .$$

Assuming n=20, $\Delta\varphi'=15^o$ we have:

$$\frac{\overline{B}(\theta) - B_o}{B_o} \lesssim 0,2\times10^{-2} .$$

Of course we have to consider that both phase shifts: $\Delta\varphi$ and $\Delta\varphi'$, are present at same time, so that it seems conservative to cut by a factor 2 the allowed values of magnetic field dishomogeneity:

$$\frac{B_2}{B_o} \leq 0,3\% \qquad \frac{\overline{B}(\theta) - B_o}{B_o} \lesssim 0,1\% .$$

7. - BEAM EXTRACTION. -

The extraction of electron beam from the microtron seems to be a quite easy operation because of the large orbit separation. In fact the orbits, which are bunched together at cavity gap, are separated by λ/π (3.18 cm in our case) at the diametrically opposite position. Unfortunately the introduction of an iron pipe tangentially to the last orbit, in order to extract electrons by screening the magnetic field, gives rise to a local perturbation of the field which disturbs the orbits in such a way to reduce by a large amount the intensity of the extracted beam.

A method for reducing the perturbation due to the extraction pipe has been proposed by Reich and Löns[14], and has been adopted by us. It has been shown by these authors that the value B of the field in the median plane at a distance x from the perturbing iron pipe depends from the ratio x/a, where a is the magnet gap, according to the function:

$$B\left(\frac{x}{a}\right) = B_o \cdot f\left(\frac{x}{d}, \frac{a}{d}\right), \tag{7.1}$$

where d is the rod diameter and B_o the unperturbed field. A family of curves of the function f(x/d, a/d) is reported in Fig.10. An ispection of these curves shows that for an assigned value of x/d, the value of the function f approaches unity as a/d is decreased. The gap height of the magnet cannot be actually decreased, but Reich and Löns observed that in formula (7.1) the gap height a can be substituted by the distance between the rod and his magnetic image symmetrical with respect to an equipotential plane; a regular ladder of real and image parallel rods, having a distance a', gives rise to the same perturbation of a single rod into a gap of height a'.

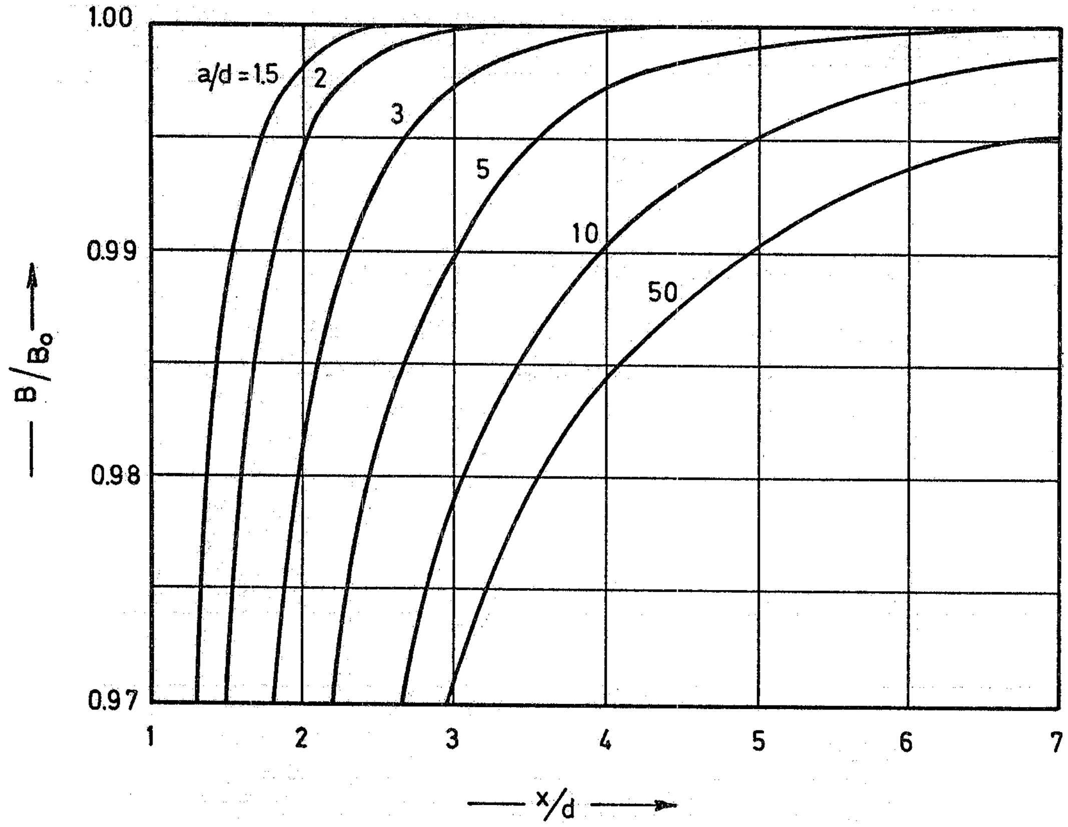


FIG.10

In our extraction system we have introduced, above and below the extraction pipe, two rods of the same diameter d = 16 mm, at a distance of 33.3 mm; this corresponds to a value a' = (1/3) a = (1/3) 10 cm. From Fig.10 it can be calculated the value of the perturbed field expected at the last but one orbit, where we have x/d = 2:

$$\frac{B}{B_o} = 0,96 \quad \text{(Without correcting side rods, for } \frac{a}{d} = 6,25\text{)}$$

$$\frac{B}{B_o} = 0,995 \quad \text{(With two correcting side rods for } \frac{a'}{d} = 2,08\text{)}$$

the perturbation results greatly reduced.

The mechanical arrangement of the extracting pipe and correcting rods[6] is shown in Fig.11; the device can be remotly adjusted by mouvements in the medium plane in order to optimize its position with respect to the beam running the last orbit. By a good positioning of the extracting system an extraction efficiency very near 100% has been obtained.

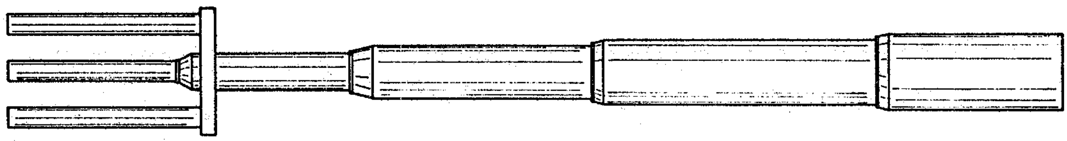


FIG. 11

8. - MAGNET DETAILS AND VACUUM SYSTEM. -

The magnet design is shown in Fig. 2, 12. The two circular flat pole pieces are kept in position at a distance of 10 cm by the stainless steel vacuum chamber. At border of pole pieces a small "tip" corrects the magnetic field from fringing effects.

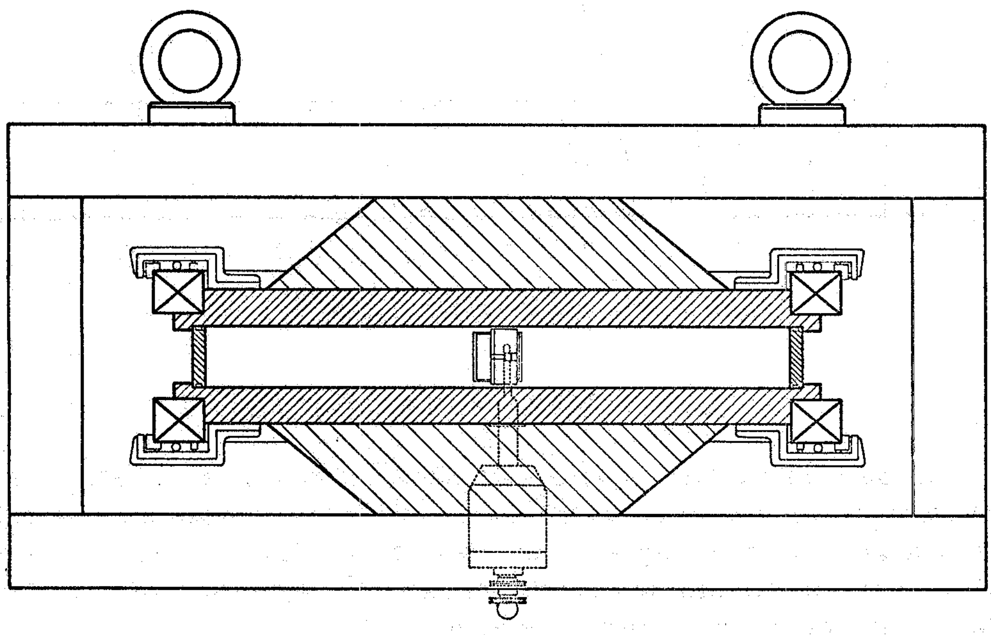


FIG. 12

The shape of the actual magnetic field in the medium plane, measured with a Diecke magnetometer along a diameter, is plotted in Fig. 13. It can be observed that the "tip" overcompensate the fringing effects; because in the useful region the measured disuniformity of the field is less than 0.1% we have left the "tip" unchanged.

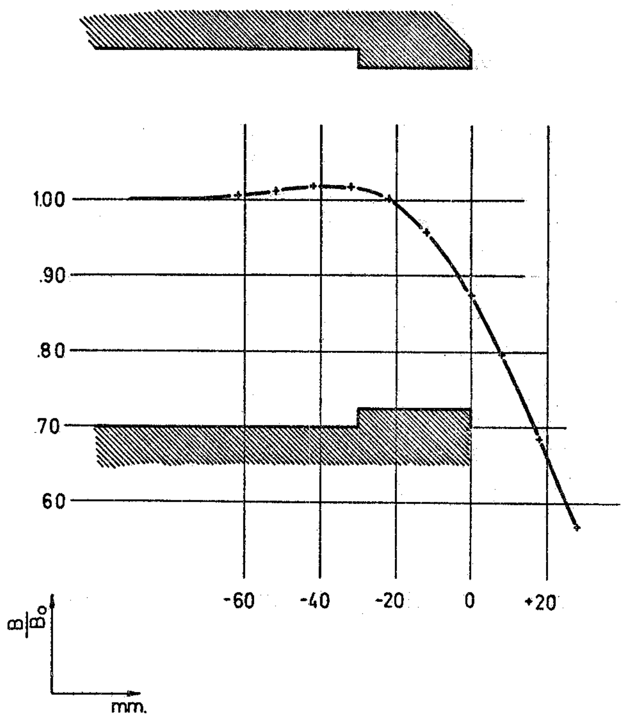


FIG. 13

Magnet excitation is obtained by two 364 turns coils, wounded with copper of rectangular cross section of 5 x 3 mm^2. The resulting resistance of the coils is about 3Ω, and a current of 13 A is needed to reach the value B_o=1, 214 g of the magnetic field.

The vacuum chamber is made by the two iron pole pieces and a stainless steel ring: two 0-rings ensure the vacuum tightness between chamber's parts. The stainless steel ring structure is shown in Figure 2. In the ring 9 circular windows are opened for allowing: 3 vacuum pumping parts, 1 input of the wave guide feeding the power to the cavity, 5 input of power for auxiliary cathode, 4 beam output, 2 optical inspection of the beam, 6 remote control of the arm supporting the Faraday cup measuring beam current.

By 3 jonic titanium pumps, of 100 lit/s capacity a final vacuum of 5×10^{-7} torr is reached into the chamber.

A mechanical pump followed by a diffusion pump and nitrogen trap is runned at the beginning of vacuum pumping operation to bring the pressure below the threshold of jonic pumps working. A vacuum better than 5×10^{-6} torr is needed for beam running; such a vacuum is reached in about 2.5 hours by the above described vacuum system.

9. - CAVITY AND MICROWAVE SYSTEM. -

As it has been described in § 1 a peak voltage of V_o = 580 kV must be mantained at the gap of resonating cavity in order to accelerate and keep in phase the electrons.

Using a cylindrical cavity of radius r = 38, 3 mm and gap h =

= 15 mm we expect a Q_o value of ref. (15):

$$Q_o = \frac{1}{1+\frac{r}{h}} \frac{r}{\delta} = 8.800,$$

$\delta = 1.22 \times 10^{-4}$ mm being the copper skin depth at the frequency: $\nu_{RF} = 3.000$ Mc/s.

The measured Q value results to be:

$$Q = 7500,$$

and the shunt resistance:(15)

$$R_p = 188\, Q \frac{h}{r} = 550\ K\Omega.$$

The power required to mantain a peak voltage of $V_o = 580$ kV at the cavity gap (without the electron beam leading), is:

$$P_o = \frac{1}{2} \frac{V_o^2}{R_p} = 305\ kW,$$

Because of economy reasons we have chosen a 2MW tunable magnetron (TV 1542) as microwave power supply. A modulator supplies to the magnetron power pulses of 45 kV, 90 A, $2 \div 4$ μs length.

Of course higher power and larger duty cycle could by obtained by a klystron microwave power tube, the cost of which is quite larger.

We can evaluate the maximum beam current which can be accelerated by our microwave system; let:

P_m the magnetron power
P_t the power absorbed by the beam
P_o the power fed to the cavity
$\eta = 0.85$ a coefficient taking into account the power lost in the microwave feeding circuit;

we have:

(9.1) $$\eta P_m = P_o + P_t$$

The power P_t absorbed by the beam is the sum of two terms:

$$P_t = n\,P_f + P_e$$

where P_f is the power absorbed by stable phase running electrons per turn and P_e is the power absorbed by lost electrons.

We have:

$$K = \frac{P_e}{P_f} \simeq 20 \div 25 ,$$

because about 5% of injected current is accepted and accelerated by the microtron with the Wernholm injection system.

For n orbits acceleration, it results:

$$P_t = n\,P_f + P_e = V_o\, i_f\,(K+n) , \tag{9.2}$$

where i_f is the beam final current. From (9.1) and (9.2), i_f is given by:

$$i_f = \frac{1}{n V_o}\;\frac{\eta (P_m - P_o)}{1+\frac{K}{n}} . \tag{9.3}$$

Injecting into eq. (9.3) the values of parameters of our microtron:

$$\begin{aligned} \eta &= 0.8 \\ K &= 20 \div 25 \\ n &= 21 \\ nV_o &= 12 \text{ MV} \\ P_o &= 305 \text{ kW} \\ P_m &= 2 \text{ MW} , \end{aligned}$$

we expect a beam current:

$$i_f \simeq 60 \text{ mA} ,$$

a value very near to the measured one.

The microwave resonator is a cylindrical cavity of 38.3 mm radius and 15 mm height obtained by machining a solid block of oxigen free high conductivity copper. The 3 pieces by which the resonator is built, shown as A B and C in Fig. 4, are assembled by screws. Into the bases of the cylinder two rectangular 8 x 16 mm windows are machined for allowing the beam to cross the cavity; as is shown in Fig. 4 the window downstream to the beam is shifted of 3 mm from the axial position with the pourpose of following the first orbit.

The piece of the cavity indicated by A in Fig. 4 presents an hole to give place to the electron gun, and holes are drilled into the bulk material in such a way to form a pipe for cinculating the cooling water.

A rectangular hole cuples the cavity to the waveguide feeding the microwave power. A ceramic window (RCA J 15152) separates the section of the microwave circuit working under vacuum (resonator and the last piece of wave guide) from the remaining section which works at a pressure of 2 Kg/cm^2 of N_2.

More details of the microwave circuit design are given in Appendix I, II, III.

In Table III we give the principal parameters of our machine, in Table IV the energy and current values are given of some of the largest microtrone built by various laboratories.

TABLE IV

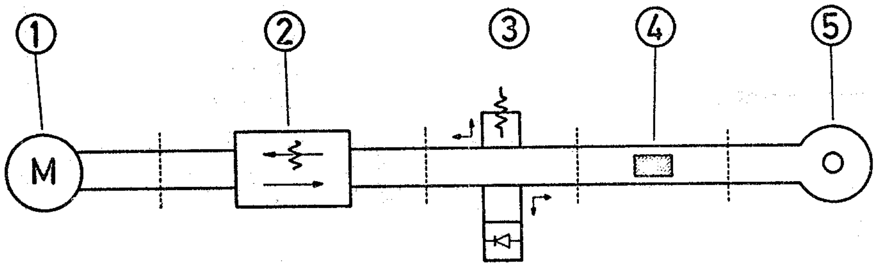
	energy (MeV)	(mA)
Lund	5	100
Stockholm	7	100
Berkeley	7	15
London	6.2	40
Mosca	7	110
Mosca	15	35
Dubna	30	100

TABELLA III

Maximum electron energy	12 MeV
Number of orbits n	21
Peak current	60 mA
Pulse length	2÷4 μs
Number of pulses per second	20-400
Beam emittance:	
horizontal	π 0.8 mrad. cm
vertical	π 0.4 mrad. cm
Momentum spread $\Delta p/p$	$\pm$0.5%
Voltage gain per turn	580 kV
Microwave frequency	3000 Mc/s
Magnetron (TV 1542) peak power	2 MW
Magnetron's power supply:	
peak voltage	45 kV
peak current	90 A
ripple	0.5%
Microwave pulse rise time	0.2 μs
Cavity filling time	0.5 μs
Microwave window	ceramic type RCA-J 15152
Magnetic field	1214 gauss
Pole pieces diameter	1060 mm
Last orbit diameter	700 mm
Magnetic field uniformity inside a diameter of 890 mm	0.1%
Gap height	100 mm
Weight of magnet + coils	2.5 tons
Coils (copper 10 mm^2 rectangular cross section)	2 x 364 turns
Resistance of coils	3 ohm
Coils current	13 A
Current stability	0.1%
Wernholm injector current	1.2 A
Injector voltage pulse	70 kV
Mean life of cathodes:	
auxiliary cathode	~300 h
LaB_6 cathode	~1000 h
Time needed to change a cathode	~3 h
Working vacuum	0.5 - 5 x 10^{-6} torr
Vacuum system 3 titanium pumps	100 lit/s each
1 mechanical pump	30 m^3/h
1 oil diffusion N_a trap	500 lit/s
Beam remote measuring devices and controls:	
a) Faraday cup at successive orbits	
b) Induction output electrode	
c) Position of extracting device	
d) Position of cathode.	

APPENDIX I - Microwave circuit. -

The microwave circuit consists of the magnetron coupled to the resonator by a ferrite insulator and a microwave window (see Fig. 14).



1 Magnetron (TV 1542 or M 5015)
2 Ferrite insulator (2 x ISH 27)
3 Directional coupler (-60 db)
4 Microwave window
5 Cavity

FIG. 14

The ferrite insulator is used to limit the resonator power reflection due to inavoidable mismatching.

The magnetron used in our microtron (TV 1542) requires a standing wave ratio S_o not larger than 1,2.

One can easily calculates S_o, obtaining:

$$S_o = \frac{1 + \Gamma\sqrt{\alpha_1 \alpha_2}}{1 - \Gamma\sqrt{\alpha_1 \alpha_2}}$$

where Γ is the voltage reflection coefficient of the resonator, and α_1, α_2 are the forward and backward power trasmission coefficient of the ferrite insulator. Using two 3 MW Rayteon ISH 27 type insulators in series having $\alpha_1 = 0,95$ $\alpha_2 = 0,1$ one obtains $S_o = 1,2$ in the case $\Gamma = 1$ (detuned cavity).

APPENDIX II - Coupling of the resonator to wave guide. -

The coupling of the resonator must be realized in order to obtain:

a) The maximum power transfer in the accelerated beam,
b) The possibility of changing the current beam in large limits without increasing the S_o value over (1.8), which is the limit value of insulator.

At this point it is convenient to introduce the coupling factor β of the resonator (ref. 15 pag. 235):

$$\beta = \frac{Q_o}{Q_e} = \frac{P_e}{P_o+P_t} ,$$

where: P_e, P_o and P_t are respectively the power dissipated in the input admittance, in the cavity walls and into the beam.

$$Q_o = \frac{2\pi\nu \text{ energy stored in cavity}}{\text{power dissipated in cavity (walls + beam)}} = \frac{2\pi\nu \text{ En. Stor. Cav.}}{P_o + P_t} ,$$

$$Q_e = \frac{2\pi\nu \text{ energy stored in cavity}}{\text{power dissipated in input admittance}} = \frac{2\pi\nu \text{ En. Stor. Cav.}}{P_e} ,$$

P_e depends from the hole shape between cavity and waveguide.

In the hypothesis that the hole is adjusted in order to have $\beta = 1$ (waveguide matched) whith a fixed P_t^x (P_t^x is the power dissipated on I^x beam current, that maches the cavity), one has:

$$P_e = P_o + P_t^x ,$$

and

(1)
$$\beta = \frac{P_o + P_t^x}{P_o + P_t} .$$

This formula is very usefull to calculate β for any value of P_t and in particular the value of $\beta = \beta_\infty$ when $P_t = 0$. The β_∞ is the only parameter that can be measured by microwave test equipment:

(2)
$$\beta_\infty = 1 + \frac{P_t^x}{P_o} = 1 + \frac{I_t^x}{I_o} .$$

The parameters that can realize the conditions a), b) are the power P_m required from the magnetron, the VSWR S which depend by the beam current and by β_∞ (we remember that β_∞ depends only from the coupling hole of the cavity).

The equation 1) can be transformed into:

(3)
$$\beta = \frac{\beta_\infty}{1 + \frac{P_t}{P_o}} = \frac{\beta_\infty}{1 + \frac{I_t}{I_o}} ,$$

(I_o is the equivalent current in the cavity walls). Because (ref. 15 page 235)

$$S = \beta \qquad \text{for} \qquad \beta > 1$$
$$S = 1/\beta \qquad \text{for} \qquad \beta < 1$$

one has:

$$S = \frac{\beta_\infty}{1 + \frac{I_t}{I_o}} \quad , \quad \text{for} \quad I < I^x \ ,$$

(4)
$$S = \frac{1 + \frac{I_t}{I_o}}{\beta_\infty} \qquad \text{for} \qquad I > I^x$$

The equation (3, 4) are plotted in Fig. 15.

The power required from the magnetron P_m is:

(5)
$$P_m = (P_o + P_t + P_r)/\eta \ ,$$

where: P_r is the reflected power from the cavity and η is a factor taking into account the power losses in the microwave circuit. Now we have:

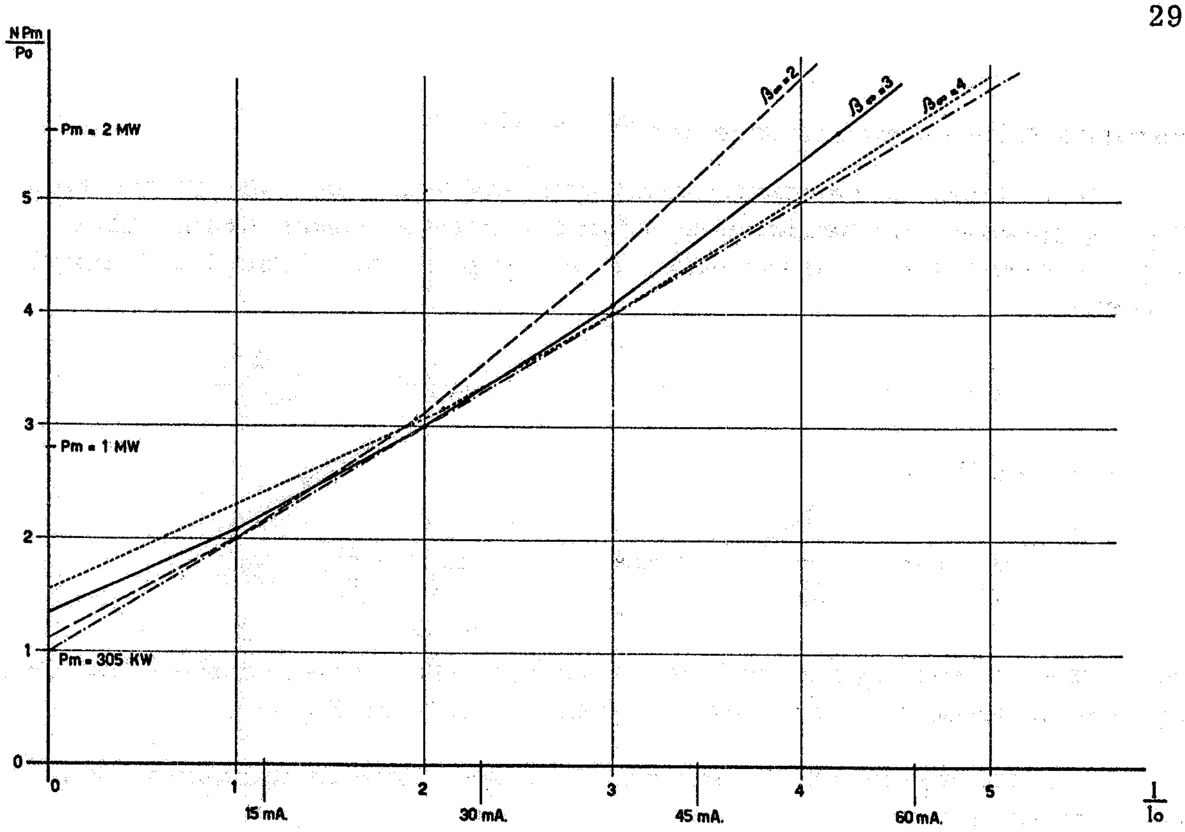
(6)
$$\Gamma^2 = \frac{P_r}{\eta P_m} = \left(\frac{\beta - 1}{\beta + 1}\right)^2 = \left(\frac{1 - (I_t/I_o)}{\frac{\beta_\infty + 1}{\beta_\infty - 1} + \frac{I_t}{I_o}}\right)^2$$

Substituting (6) in (5) and taking into account of (2), (3) we have

(7)
$$\eta \frac{P_m}{P_o} = \frac{1 + (\beta_\infty - 1)(I/I_o)}{- \left(\frac{1 - I_t/I_o}{\beta_\infty + 1)/(\beta_\infty - 1) + I_t/I_o}\right)^2}$$

The equation (7) is plotted in Fig. 16.

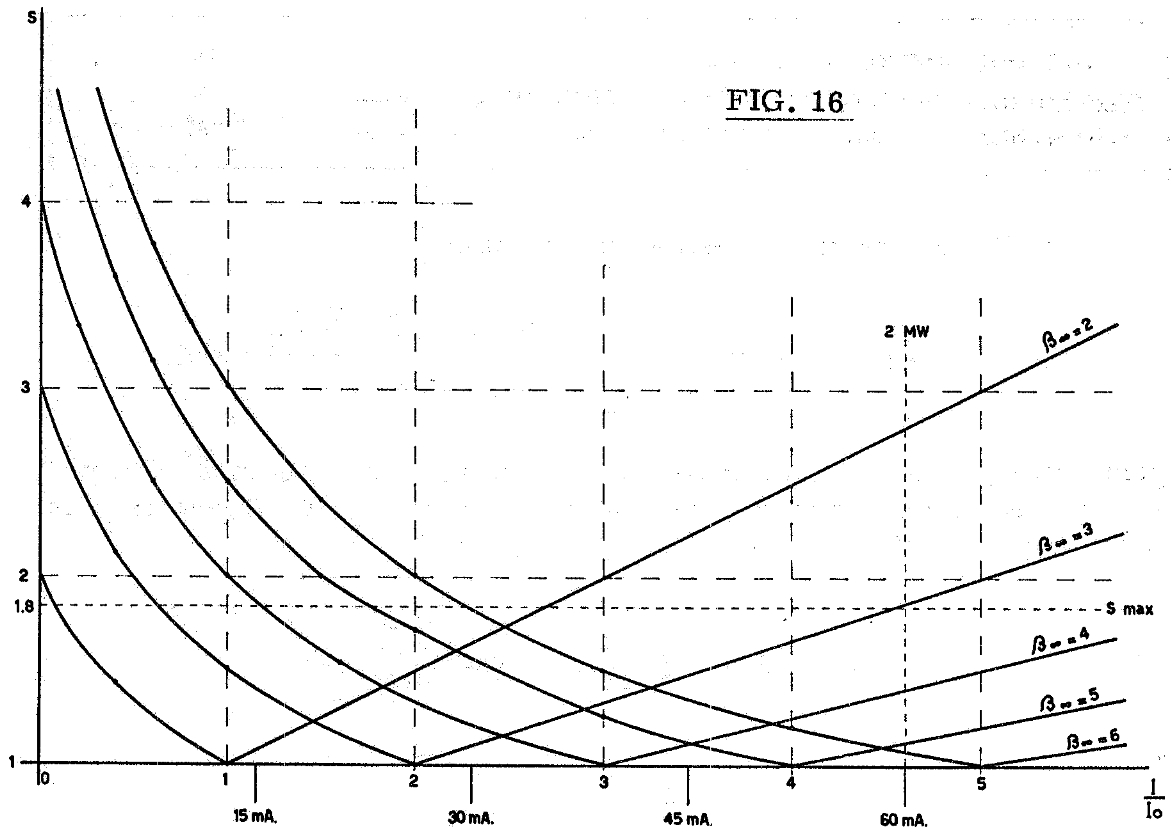
The Fig. 15 and 16 show that the value $\beta_\infty = 3$ realizes the possibility to change che beam current from 15 mA to 55 mA with $S \leq 1.8$ and a power transfer into the beam near the maximum obtainable with perfect matching at the maximum current ($\beta = 5$ $I = 60$ mA).



N Pm / Po
Pm = 2 MW
Pm = 1 MW
Pm = 305 KW
β∞ = 2
β∞ = 3
β∞ = 4
15 mA.
30 mA.
45 mA.
60 mA.
I / Io

FIG. 15

FIG. 16



S
1.8
2 MW
S max
β∞ = 2
β∞ = 3
β∞ = 4
β∞ = 5
β∞ = 6
15 mA.
30 mA.
45 mA.
60 mA.
I / Io

APPENDIX III - Vertical focusing[10, 11, 12]. -

The radio frequency field fringing from the hole of the resonator produces vertical focusing effects on the electron beam. This effect is dependent from the shape of the gap. In fact from the Maxwell equations:

$$\text{div } E = 0 \qquad \text{and} \qquad \text{curl } H = \varepsilon_0 \frac{\partial E}{\partial t} ,$$

one obtains (ref. 2)

$$E_z = - \alpha z \frac{\partial E_x}{\partial x} \qquad \text{and} \qquad B_y = - \frac{z}{C^2} \frac{\partial E_x}{\partial t} ,$$

where xyz are ortogonal reference axes, with x directed along the cavity axis and z along the microtron magnetic field (see Fig. 4).

The parameter α depends from the shape of the hole as it is shown in Table V.

TABLE V

Circular shape of the gap	$\alpha = 1/2$
Rectangular shape of the gap parallel to y axis	$\alpha = 1$
Rectangular shape of the gap parallel to z axis	$\alpha = 0$

From the motion equation one has:

$$(8) \qquad \Delta P_z = \Delta P = - \alpha e \int_{t'}^{t''} \left(\frac{\partial E_x}{\partial x} - \frac{\beta}{C} \frac{\partial E_x}{\partial t} \right) z \, dt$$

Where ΔP_z is the change in momentum along z axis gained by a particle in crossing the fringing region of one hole. In the interesting case $\beta \simeq 1$, equation (8) becomes (ref. 2), pag. 280)

$$(9) \qquad \Delta P = \Delta (m_o \gamma \frac{dz}{dt}) = - \alpha \, 1 \, \frac{dE_x}{dx} z \, dt$$

Where:

$$\Delta t = t'' - t'$$

The equation (9) can be transformed into:

$$\Delta P = -\frac{\alpha z e l \Delta E_x}{C l} = -\frac{\alpha \mathcal{E}_o}{C l} z$$

$$\mathcal{E}_o = h \Delta E_x$$

where $\mathcal{E}_o = h\Delta E_x$ is the energy gained per turn by a particle. Neglecting the length of the fringing field region with respect the gap length, the effects of the gaps entrance hole on the electrons, and of the exit hole are respectively given by the matrices:

$$\begin{vmatrix} 1 & 0 \\ -\frac{\alpha \mathcal{E}_o}{h C} & 1 \end{vmatrix} ; \begin{vmatrix} 1 & \frac{eh}{\gamma_n \mathcal{E}_o} \\ 0 & 1 \end{vmatrix} ; \begin{vmatrix} 1 & 0 \\ \frac{\alpha \mathcal{E}_o}{h C} & 1 \end{vmatrix} .$$

The product of the tree matrices describes the behaviour of the electron in crossing the resonator. One obtains:

$$(10) \qquad \begin{vmatrix} 1 - \alpha/\gamma_n & hC/\gamma_n \mathcal{E}_o \\ -\alpha^2 \mathcal{E}_o/\gamma_n C l & 1 + \alpha/\gamma_n \end{vmatrix}$$

The remaining path of the electron, in the static magnetic field, is descri̲ ved by the matrix:

$$(11) \qquad \begin{vmatrix} 1 & \frac{C}{\mathcal{E}_o}\left(1 - \frac{h}{\gamma_n + 1}\right) \\ 0 & 1 \end{vmatrix}$$

The matrix corresponding to a whole turn is the product of the (10), (11) and is given by:

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

where:

$$a_{11} = 1 - \frac{\alpha}{\gamma_n} - \frac{\alpha^2}{\gamma_n} \frac{1}{h}$$

$$a_{21} = -\alpha^2 \frac{\varepsilon_o}{\gamma_n C h}$$

$$a_{22} = 1 + \frac{\alpha}{\gamma_n}$$

a_{12} may be derived by the equation:

$$\det A = 1$$

One can demonstrate that the coordinate z makes oscillations:

$$z = Z_n \cos(\phi + n\mu) ,$$

where:

$$\cos\mu = 1/2\,(a_{11} + a_{12}) \simeq \alpha \sqrt{\frac{\lambda}{\gamma_n h}} ,$$

$$n = \gamma_n + 1 \quad \text{if the number of revolutions,}$$

$$Z_n = \left(\frac{a_{12}}{\text{sen}\mu}\right)^{1/2} \simeq (a_{12})^{1/2} \Big/ \left(\frac{\lambda}{\gamma_n h}\right)^{1/4} \alpha^{1/2} .$$

In the relativistic region a_{12} is nearly constant and:

$$\sin\mu \simeq \mu = \alpha \sqrt{\lambda / \gamma_n h}$$

and one obtains:

$$\frac{Z_m}{Z_n} = \left(\frac{\gamma_m}{\gamma_n}\right)^{1/4} = \left(\frac{m+1}{n+1}\right)^{1/4} .$$

For m = 21 turns (that is the number of turns in our microtron) and n = 4 (electron nearly relativistic) one obtains:

$$\frac{Z_{21}}{Z_4} = \left(\frac{Z_2}{5}\right)^{1/4} \simeq 1,4 .$$

This result means that the minimum loss of beam from the 4^{th} to 21^{th} orbit is to be expected of 30%, in the hypothesis that the beam is uniformly distributed in vertical.

ACKNOWLEDGEMENTS. -

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