F. Selleri : QUANTUM THEORY AND HIDDEN VARIABLES. -

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INTRODUCTION. -

These lecture notes were prepared at Frascati during the months of June and July 1969. They are divided into five chapters. In the first two the foundations of Quantum Mechanics are reviewed critically. I made no effort to hide my dislike of the philosophical implications of Quantum Mechanics. I hope that the reader will forgive this personal attitude. In the third chapter the realistic postulate is stated and some problems connected with Quantum Mechanics are discussed by means of the new standpoint. Some problems are solved while some other ones remain ununderstood. In the last two chapters we review some hidden variable theories and discuss the experiments which could distinguish them from Quantum Mechanics.

These lecture notes should provide an introduction to the field of hidden variables. They do not, however, constitute a comprehensive review to that field. The two main arguments which are missing are De Broglie's theory and Brownturn Mechanics. The theory developed starting from the formal analogies between Quantum Mechanics and the Brownian motion.

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I. - QUANTUM MECHANICS AND METAPHYSICS. -

I. 1. - The physicist's point of view on metaphysical problems.

The physicists and scientists in general are used to think that phylosophy and, in particular, metaphysics are a waste of time and should not be considered seriously by anyone interested in the physical world. We should leave these intellectual exercises to the philosophers and not concern ourselves about what they say, only hoping that they will not try to influence us with their prejudices. This widespread point of view is well illustrated by the following quotation:

> "These phylosophers are always with us, struggling in the periphery to try to tell us something, but they never really understand the subtleties and depths of the problem", R. Feynman, Lectures on Physics (California Institute of Technology, 1963), Vol. I, Ch. 16.

The reasons why we have come to such a situation are rather clear. Firstly several phylosophycal assumptions, which were considered a priori correct, were proven wrong by experiments. Of this kind are, for instance, the notions of absolute space, absolute time and of symmetry of space ("parity conservation"). Secondly many of us still believe that metaphysics in totally irrelevant because our theories can adapt themselves to any metaphysical point of view. Let us illustrate this belief with an example. A body (B) is falling freely under the action of gravity. Theory predicts that the space traveled (x) will be proportional to the time (t) squared

$$(1) \qquad\qquad x = \frac{1}{2} g t^2$$

and this prediction can accurately be verified to be correct by experiments. Let us suppose that the apparatus is so constructed that the only way to observe B is to take pictures of it and let us ask the following metaphysical question: Does B exist even when we do not observe it? Of course nobody knows the answer, but three different possibilities can be conceived:

a) Yes, it exists independently of our observations.
b) No, it exists only when we observe it.
c) Nonsense, the question cannot be asked because we cannot answer it experimentally.

4.

Newtonian physics can easily be shown to be consistent with all of these points of view. In the first case $x = (1/2) gt^2$ predicts the trajectory truly followed by the falling body and, in particular, its positions at the times when pictures were taken; in the second and third cases it predicts the coordinate of the observed body and formula (1) can be considered a summary of the predictions for all possible measurements. The conclusion seems to be that physics is more general than metaphysics and that the latter is therefore totally disconnected from the former and therefore irrelevant to the physicist.

These foundations of our prejudices are in reality rather shaky. That the philo sophers have often been wrong does not mean that they are always wrong or useless, and that our theories can always accomodate themselves to any metaphysical point of view is simply not true. Newtonian mechanics can (in the discussed example) but Quantum Mechanics cannot, as it will be shown later.

I. 2. - The aggressive philosophers of science.

The philosophers of science have repeatedly been trying, in the sixties, to do what Feynman seems to dislike most : to influence us with their throught, claiming that our present axioms are wrong and that we should discuss with them and develop a completely new theory of the physical world. But since we systematically refused to pay any attention to them and since it is a well known psicological phenomenon that frustration generates aggression, they now state the following :

"While in the early 1600's the conservative philosopher would refuse to look through the scientist's telescope, as late in the 1960's most physicists still refuse to use the logoscope built in recent years by the philosopher .... For the first time in history, scientists have managed to outdogmatize philosophers", M. Bunge, Quantum Theory and Reality (Springer, 1967), p. 4.

We will keep this accusation of dogmatism in mind in analyzing some implications of Quan tum Mechanics ; We will find that it is probably right.

I. 3. - An important aspect of Quantum Mechanics.

A very important fact which we wish to point out is that Quantum Mechanics is deeply interrelated to metaphysics. In fact it does not allow an arbitrary answer to the me taphysical question : Does an object exist when we do not observe it ? We will show that a positive answer is incompatible with the axioms of Quantum Mechanics.

Let us consider the well known[1] Einstein-Podolsky-Rosen paradox[2] in the version of Bohm and Aharonov[3]. A spin-zero object at rest in the laboratory spontaneou sly disintegrates into two spin 1/2 objects. An example from particle physics is

(2)
$$\pi^0 \longrightarrow e^+ + e^-$$

and we will refer to the latter for concreteness. The electron and the positron move with equal speed in opposite directions and, since angular momentum is conserved in the decay, their spins ($\vec{\sigma}^{(-)}$ and $\vec{\sigma}^{(+)}$ resp.) are oriented in such a way that the total angular momen tum is zero.

Quantum Mechanics predicts that a measurement of the x-component of, say, the spin of e^- ($\sigma_x^{(-)}$) can give with equal probability $+\hbar/2$ and $-\hbar/2$.

However a simultaneous measurement of $\sigma_x^{(+)}$ and $\sigma_x^{(-)}$ must give opposite results because the wave function of the final e^+, e^- system has an eigenvalue of zero for all the operators $\vec{\sigma}^2$, σ_x, σ_y, σ_z.

The latter measurement leaves the total angular momentum of the system unchanged since the measured observable $\sigma_x = \sigma_x^{(+)} + \sigma_x^{(-)}$ commutes with $\vec{\sigma}^2 = (\vec{\sigma}^{(+)} + \vec{\sigma}^{(-)})^2$. Therefore we know that after e^+ and e^- have interacted with counters A_x and B_x respectively (see Fig. 1) their spins still point in opposite directions. Furthermore let
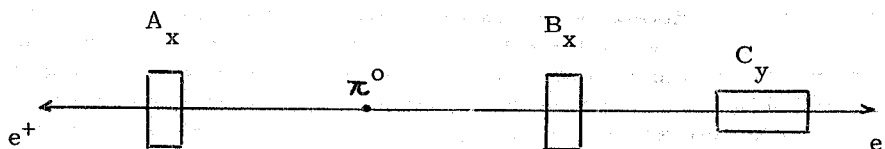
FIG. 1 - Scheme of the EPR experiment. A_x and B_x (C_y) are experimental devices to measure the x-component (y-component) of the spin of the particles crossing them.

us fix our attention on a specific decay process and suppose that the results read on A_x and B_x were

$$\sigma_x^{(+)} = +\hbar/2 \qquad\qquad \sigma_x^{(-)} = -\hbar/2 .$$

According to the apparatus shown in Fig. 1 the positron, from the moment it leaves A_x on, will not interact with anything and, thus, it will not be perturbed in any way.

Therefore $\sigma_x^{(+)} = \hbar/2$ will remain true forever. Let us, next, measure with C_y the observable $\sigma_y^{(-)}$. Since $\sigma_y^{(-)}$ does not commute with $\sigma_x^{(-)}$ in so doing we pertur be the electron and lose all information about $\sigma_x^{(-)}$. Let us suppose that in the specific case considered we obtain:

$$\sigma_y^{(-)} = -\hbar/2$$

then we must conclude that $\sigma_y^{(+)} = \hbar/2$ since the two spins point in opposite directions. But in measuring $\sigma_y^{(-)}$ we have perturbed in no way the e^+ and, therefore, we cannot have lost the information $\sigma_x^{(+)} = \hbar/2$. It follows then that we know simultaneously $\sigma_x^{(+)}$ and $\sigma_y^{(+)}$, in contradiction with the fact that these two observables do not commute. We have thus arri ved at a paradox the so-called ERP-paradox.

Does this mean that Q. M. is wrong? The answer is negative : in fact Q. M. con tains the correct answer to the paradox. In measuring $\sigma_y^{(-)}$ we perturbe the whole system $e^+ + e^-$, as it is clear from the fact that

(3)
$$\left[\sigma_y^{(-)}, \ (\vec{\sigma}^{(+)} + \vec{\sigma}^{(-)})^2 \right] \neq 0 .$$

Therefore we pass from a state with fixed $\vec{\sigma}^2$ (total spin zero) to a state which is a superposition of the possible eigenvalues for $\vec{\sigma}^2$ (total spin zero and one). In measuring $\sigma_y^{(-)}$ we lose the information that the spins point in opposite directions and cannot therefore conclude anymore that $\sigma_y^{(+)} = - \sigma_y^{(-)}$. Therefore the paradox must arise from something else than Q. M. , perhaps from an implicit assumption which we have made in the reasoning and which is not correct.

Notice that as far as we limit ourselves to the prediction of experimental results Q. M. gives the right answer. The paradox arises from the part of the reasoning when we say that the positron will not be measured again and threfore it will always have $\sigma_x^{(+)} = \hbar/2$ and from analogous reasonings made always on the unobserved positron after $\sigma_y^{(-)}$ has been determined[4]. The paradox disapears if we take the philosophical point of view (as the Copenhagen school did) that such statements do not make any sense and that sensible questions are only those concerning the results of measurements.

The point of view that there are objectively existing entities called e^+ and e^- possessing physical attributes (spin-components) leads to the paradox and is therefore not compatible with Q. M. Therefore, as we stated before, Q. M. is not consistent with all the possible metaphysical standpoints and limits in fact our metaphysical freedom. If this is so we should then pay more attention to the philosophers of science to make sure that our different metaphysical assumptions are consistent with each other.

In fact the philosophers now claim that they are not.

A further point has to be stressed: the fact that one cannot assume a realistic point of view about the particles partecipating in our experiments is extremely unpleasant to many of us. You become a physicist to discover the "secrets of Nature" and after many years you find that there was no Nature and, thus, nothing to discover. What then is physics? Only a description of our activities, of those manipulations which we call experiments. Of course, I am exaggerating. In fact one can show that Q. M. is only inconsistent with the idea that two systems which have interacted in the past can be assumed to exist separately. But the exaggeration is not very great as an enormous number of interaction has taken place since the birth of the Universe.

I.4. - Schrödinger's cat.

This famous argument, put forward by Schrödinger in 1935[5], shows that the impossibility to think as objectively existing a system which is not observed extends to macroscopic objects like, for instance, a cat.

A box (see Fig. 2) with walls which cannot transmit sound and light contains a cat bound to a wall and a gun. The gun is connected to an apparatus A which makes it shoot (thus killing the cat) whenever a photon hits a sensitive region S.
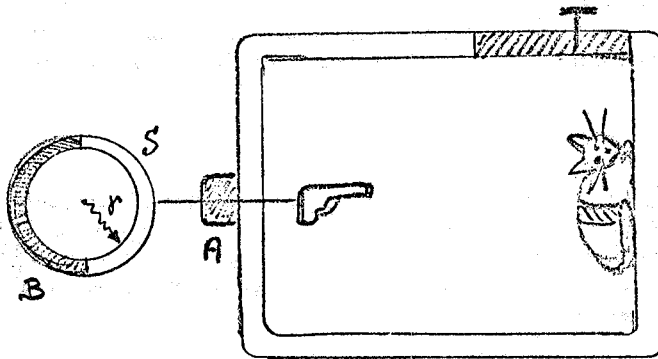

FIG. 2 - Experimental apparatus for the Schrödinger's cat experiment.

An excited atom is contained in spherical cavity whose surface is divided into two parts: the sensitive region S and an absorbing region B. Suppose the atom has a lifetime of five minutes. The time-depent wave function of the whole system can be written

$$|\psi(t)\rangle = e^{-t/2r}|A^*\rangle \, |C.A.\rangle + (1 - e^{-t/2r})|A\rangle \left[\alpha_B |C.A.\rangle + \alpha_S |C.D.\rangle\right]$$

where $r=5$ min is the lifetime of the atom; $|A\rangle$ and $|A^*\rangle$ represent the states for an unexicited and an excited atom, respectively; α_B and α_S are the amplitudes for photon absorption by the regions B and S respectively; $|C.A.\rangle$ and $|C.D.\rangle$ represent the states for a living and a dead cat respectively. For $t \gg r$ and $\alpha_B = \alpha_S = 1/\sqrt{2}$ we can write

$$|\psi(t)\rangle \simeq \frac{1}{\sqrt{2}} |A\rangle \left[|C.A.\rangle + |C.D.\rangle\right] .$$

This is the most complete description of the system which it is possible to give, according to Q. M. Notice that the previous state vector $|\psi(t)\rangle$ contains a part related to a dead cat and a part to a living cat. Thus the commonsense motion that the cat is either alive or dead is not respected here. Furthermore if we open the box and look at the cat we can find, for instance, that it is dead. According to Q. M. it is this act of observation (opening of the box) that leads to the reduction of the wave-packet. In a sense it is the observer who looks at the cat who kills it, because before the observation the cat was at least partly alive. A solution of the paradox is obviously possible if one refuses the metaphysical assumtion that the cat is either living or dead when it is not observed. If one takes the (metaphysical) standpoint that the cat does not exist when it is not observed (or that a question of

existence cannot be asked, which is also a metaphysical standpoint) one runs in no trouble. We conclude that Q. M. limits our metaphysical freedom also at the macroscopic level.

II. - QUANTUM MECHANICS AND DOGMATISM. -

II. 1. - Dogmatic theories.

Every theory is based on a certain number of axioms. We can roughly divide the axioms into two groups : the formal ones, which merely introduce the mathematical formalism on which the theory is based, and the basic one, which lead within the assumed formalism to the physical predictions. We define then as dogmatic a theory whose basic axioms cannot be checked directly with experiments.

The theory of special relativity is not a dogmatic theory. In fact the postulate of constant velocity of light can be checked experimentally by measuring c in different inertial frames. Similarly the principle of relativity can be put to an empirical test by showing that the physical laws are the same in all inertial frames.

Quantum Mechanics is instead a dogmatic theory.

In fact some of the most important axioms - the superposition principle and Schrödinger's equation - must be formulated for a quantity which is not directly observable, the wave function ψ. Of course ψ is used to calculate observable quantities, e. g. the probability density $\rho = |\psi|^2$, and the predictions so obtained have been checked experimentally in an enormous number of cases, and always found to be correct. Obviously any theory worth of any attention must permit at least an indirect verification of the validity of its axioms. Quantum Mechanics is just not directly verifiable, but then, according to our definition, it is a dogmatic theory. This is a further metaphysical feature of Quantum Mechanics which appears particularly unappealing in a theory insisting so much on the fact that the only events which it makes sense to consider are the measurements.

II. 2. - Reduction of the wave-function.

Consider an observable A with eigenstates $|\alpha_i\rangle$ and eigenvalues a_i :

$$(4) \qquad A |\alpha_i\rangle = a_i |\alpha_i\rangle .$$

Let it be given a system on which A can be measured and let $|\psi\rangle$ be its state-vector. Consider the development of $|\psi\rangle$ on the states $|\alpha_i\rangle$

$$(5) \qquad |\psi\rangle = \sum_i c_i |\alpha_i\rangle .$$

If we measure A on $|\psi\rangle$ and obtain a_j as a result, it is an axiom of Q. M. , that the state vector makes a jump from $|\psi\rangle$ to $|\alpha_j\rangle$

$$(6) \qquad |\psi\rangle \xrightarrow[\substack{\text{during the} \\ \text{measurement}}]{} |\alpha_j\rangle .$$

This phenomenon, whose introduction is necessary to ensure the reproducibility of the experimental results, is called the reduction of the wave-function. There has been considerable debate about the very important question whether this postulate is compatible with the time evolution of the wave-function as deduced from Schrödinger's equation.

In fact one could consider the elementary system on which A is measured and the measuring apparatus as a unique physical system for which a global wave function ϕ obeying Schrödinger's equation can be introduced. This ϕ could then be developed on the

states $|\alpha_i\rangle$ describing the microscopic system and, simultaneously, on the states $|\eta_1\rangle$ describing the apparatus

(7)
$$|\rho\rangle = \sum_{il} c_{il} |\alpha_i\rangle |\eta_1\rangle .$$

In 1963 Wigner[6] proved that the wave-vector $|\rho\rangle$ as given in (7) and evolving according to Schrödinger's equation, leade to results contradicting the reduction postulate (6).

The way out of this contradiction is implicit in the formulation of Q. M. of the Copenhagen school. The idea is that the measurement cannot be described by the Schrödin ger equation. The state vector $|\psi\rangle$ only represents the knowledge we have of the elemen tary system. When we read the result of a measurement on an instrument our knowledge changes abruptly and so the state vector must change discontinuously too. In this way the physicist becomes an active part in the process. Whether he becomes or not aware of the experimental result will change the wave function (and thus the properties) of the micro-scopic system :

"........ Ce n'est donc pas une interaction mysterieuse entre l'appareil et l'objet qui produit, pendant la mesure, un nouveau ψ du système. C'est seulement la conscience d'un "Moi" qui peut se separer de la fonction $\psi(x,y,z)$ ancienne et constituer en vertu de son observation une nouvelle objectivite en attribuant dore navant a l'objet une nouvelle fonction $\psi(x) = u_k(x)$", F. London et E. Bauer, La Theorie de l'observation en mecanique quantique (Hermann, Paris, 1939).

At this point we should mention the important paper by Daneri, Loinger and Preperi[7] which seemed to show that the reduction phenomenon is due to the ergodic pro perties of the measuring apparatus. This would have allowed one to avoid the introduction of the experimenter as an actor in the evolution of the microscopic system. It was, howe-ver, shown[8] that there are phenomena, the so called "negative-result measurements", in which the ergodic properties of the apparatus cannot play any role, while at the same time the reduction-phenomenon does take place. This gave rise to an irate answer[9] which the present author does not fully understand. In our opinion probably there was a basic misun der-stanting. Jauch, Wigner and Yanase showed that the DLP-paper[7] did not provide a possible way of avoiding some of most unpleasant features of Q. M. a la Bohr, while Loin-ger[9], seems to accept unconditionally the Bohr formulation.

The conclusions is that psicological phenomena play a role during the measu-rement processes. This would be like saying that thought influences matter. Unfortunately we have seen that in general we cannot talk of matter as something objectively existing.

Therefore the world, as described by Q. M. , is more like the one of Fichtian idealism that the one studies by parapsicology.

II. 3. - Do not try to understand.

Many physicists know how painful it is to teach Quantum Mechanics if one wants to give a physical feeling of what is going on to the students. As we saw there is a very profound reason for this and the "canonically" correct way is to teach in a deductive manner, beginning from a good knowledge of the mathematical formalism and proceeding deductively from axioms. If one, however, insists in teaching in the "wrong" way the following question has to be answered. What is an electron, a wave or a particle ? Then one calmly answers : it is neither but both; you see our rough macroscopic concepts do not apply to the microsco pic objects and one should not try to understand. One can only describe with the beautiful theory of Q. M. Understanding is not possible and never will be. Once a student asked: "If our concepts cannot apply to the microcosm, why instead our differential equations do? ". Somebody, please, has an answer ? The fact that we cannot understand the elementary con

stituents of the world is basic in accepting Q. M. Since all modern physicists must know and use Q. M. it is clear that this attitude must have had a deep influence in the development of modern physics.

In fact in the domain of elementary partcles there are many thinghs which people have not even tried to understand. It has been found that concepts like strangeness, barionic number, isotopic spin, SU(3), leptonic number were helpful to describe the properties of elementary particles. They have therefore been introduced and quickly swept in the limbus of "internal space".

II. 4. - The First Commandment.

"It is not possible to construct a theory with hidden variables, reducing to Quantum Mechanics when the hidden variables are averaged over". This is a free version of the famous von Neumann's theorem[10], published in 1932, which denies the very possibility of a theory more general than Q. M. This theorem lead naturally to the attitude that even the few attempts to build theories different from Q. M. were not worth of any consideration.

On the other hand, however, many people were puzzled by such a formidable conclusion. We quote in the following an excerpt from Bohm and Bub's paper[11]:

",......... if the claims based on von Neumann's theorem are accepted as valid, then it would follow, from the facts confirming the current quantum theory, that a different general structure of concept is impossible. Thus, it is made to appear that the linguistic structure of quantum mechanics prevents even the assertion of the possibility that the basic postulates underlying the theory may be false. In effect, this would mean that certain features of the basic postulates of the current theory are absolute truths that can never be falsified, or shown to be valid only as approximations or limiting cases. This kind of unfalsifiability would be almost as dangerous in any theory as is the claim to unfalsifiability a priori".

Of course any philosopher of science could have told us that von Neumann's theorem cannot be generally true. In fact in 1966, thirtyfour years after the theorem was published, Bell showed[12] to everyone's satisfaction that in the proof of the theorem it was implicitly assumed that the hidden variables satisfied certain properties of linearity. It is enough to postulate "non-linear" hidden variables in order to construct a theory against which von Neumann's theorem cannot apply.

A model-theory of this kind was actually proposed by Bohm and Bub[11]. In general we can now say that the door for hidden variable theories is finally open.

We have introduced in the present paragraph the concept of "hidden variables" without any explanation of it. Up to a point the words "hidden variables" are self-explanatory: they denote physical quantities which have never been observed in the experiments performed up to now. A discussion of these variables is contained in paragraph III. 5. They are also discussed in the Bohm-Bub model (see § IV. 2) and formally introduced in a general way in the proof of Bell's theorem (see § V. 1).

III. - THE NEW PHILOSOPHICAL APPROACH. -

III. 1. - Introduction.

In the two previous sections we have been criticizing Quantum Mechanics in many ways. And yet all these criticisms can be summarized in a simple, apparently inocuous sentence : "Many people do not like Quantum Mechanics". The fact that among these people there are Einstein[13], De Broglie[14] and Schrödinger[15] adds weight to the above point of view, but still nobody can say to have proven that Q. M. is internally inconsistent

or that it leads to disagreement with ewperiments. On the contrary it is perfectly logical and has produced an enormous amount of accurate predictions. Trying to built an alterna tive theory will then seen to many a total waste of time. It is in fact generally agreed that one should not look for new theories until the old one has been shown unable to explain so me empirical facts.

Nevertheless we believe that alternative possibilities should be studied energe tically by theoreticians and that experiments to check the foundations of Quantum Mechanics should be performed in large number and accurately by experimentalists.

This for three fundamental reasons :

a) We finally understood that the theorem of von Neumann is not of general validity. Therefore alternative theories are not excluded anymore by what we know.
b) The philosophical prejudice in favour of a realistic philosophy is strong in the large majority of physicists. This prejudice did not turn against Q. M. simply because very few people knew its real implications. The book of d'Espagnat[4] should hopefully contribute to give a better comprehension of them.
c) Very few and rough experiments have been made to check the most strange consequences of Q. M. We will discuss them in the next sections.

The third point above is a natural consequence of the acritical acceptance of Q. M. by most physicists. Something you learn at the University as a student cannot be basically wrong. Besides a theory you cannot fully understand must be very respectable. Everything happened as if some kind of uncouscious "credo quia absurdum" had conditioned the scientific community. As a consequence investments of money and efforts on experiments checking the most strange aspects of Q. M. were considered as foolish by the prevaling scientific culture. Nobody should feel surprised therefore, if such experiments have been done very rarely.

III. 2. - The realistic postulate.

In the following we will propose a line of thought, which is not new, but which seems to us the only way to built a theory of elementary phenomena in accordance with the realistic phylosophy. We start from the following Realistic postulate : An elementary particle is always associated to a wave objectively existing. This postulate is admittedly rather vague. The only new fact is that the wave is postulated as objectively[16]. This is certainly in contradiction with Q. M. , where even the particle, let alone the wave, cannot be assumed to be objectively existing.

What we claim is that a theory developed starting from the realistic postulate leads to predictions different from those of Q. M. only for experiments which have never been done. We will discuss further this important point later on. Further comments about the postulate are the following. The association of wave Ψ and particle must be such that the probability density for observing the particle be given by $|\Psi|^2$, the familiar result of Q. M. The wave has to be thought of as a real entity in some kind of postulated medium. Thus wave and particle are reminishent of a boat in a lake. Boat and wave are both objectively existing and are found to be associated, in the sense that you cannot find a boat with out a wave; the opposite is, however, possible. As we said before, the realistic postulate is not new. In fact De Broglie and Bohm have been working, among others, on this line of thought.

III. 3. - Is the vacuum empty?

It is aboundantly clear from what we know that the answer is negative. Take Mach's principle for instance : the inertial properties of the bodies are due to the far-away galaxies of our Universe. But we refuse to believe that actions at a distance are possible. Therefore there must be some kind of universal field telling to every electron or proton when it is accelerating with respect to the far galaxies. Another example could be the degenerate neutrino sea whose existence is demanded[17] by all the possible cosmological

models. A third argument against a geometrical, euclidean, empty vacuum comes from CP-nonconservation[18]. This, as was shown by Landau[19], implies that the vacuum is not invariant under reflections. Since it is impossible to think of any physical or geometrical system noninvariant under reflections but invariant under rotations it must neccessarily follow that the vacuum is not isotropic. This must mean that the vacuum contains some angular momentum[20]. This epistemological argument must be based on some sort of physical content: the angular momentum must be carried by a medium or by some zero energy particle, neutrino or spurion, whatever we may wish to introduce, but something. A fourth example of physical vacuum comes from the Bohm-Aharonov effect[21]. An electromagnetic potential $A_p(x)$ such that in a region R the fields $\vec{E}$ and $\vec{H}$ are zero (and terms the energy and the momentum of the field are zero, which is in practice the very definition of vacuum) can affect the wave function of a particle in R by changing its phase. This gives rise to observable effects which have been revealed experimentally[22].

In conclusion it is practically certain that in the vacuum there must be some fields or mediums. Furthermore these little known fields must possess the property of interacting with the particles. We see no reason, then, why there could not be oscillations of these fields associated with the particles.

III. 4. - Metaphysical problems and the realistic postulate.

The criticisms of Q. M. which we discussed in the previous sections are, as we said, of metaphysical nature. They merely illustrate why many people do not like Q. M. or, in other words, why they have prejudices against it. Q. M. remains logically rigorous and empirically highly successfull.

The realistic postulate should be the starting point for building an equally rigorous and not less successfull new theory, that is we hope that this task may in the future be accomplished.

What we can show right-away, however, is that at least our prejudices of metaphysical nature can be satisfied by the realistic postulate.

a) The phylosopher of science would certainly be satisfied of a theory containing the realistic postulate. In fact this theory would give rise to exactly that shift of phylosophycal attitude which they invoke. In Bunge's words using the realistic postulate would be equivalent to using the "logascope" created by the philosophers of science. We would also feel protected against future accusations of dogmatism.

b) The new theory would not be dogmatic. According to the definition given in Section II. 1 a theory whose basic axioms cannot be checked directly with experiments is dogmatic. We showed that Q. M. is dogmatic. A new theory incorporating the realistic postulate would not be dogmatic however because the superposition principle and Schrödinger's equation would be postulated for an objectively existing ψ and thus necessarily for a directly observable ψ. Therefore these statements could at least in principle be verified with experiments. How in practice ψ can be measured "directly" (that is not merely by observing a probability and equating it to $|\psi|^2$) is an extremely important problem and will be discussed in the next section.

c) Teaching Quantum Mechanics would not give rise to troubles for the physical interpretation of the theory. The question: is an electron a wave or a particle? Would have the answer: it is both. The celebrated two slit experiment which is fundamental for convening to the students the idea that we cannot understand the physical world at the microscopic level, could now be explained very simply (see Fig. 3).

A plane wave accompanies a particle with given momentum. When only slit A is open the wave is diffracted and in those cases where also the particle passes through A we find on the second screen II a distribution of probability proportional to $|\psi_A|^2$. In fact the association of wave and particle has to be made in such a way that the particle is with greater probability in the points where the amplitude of the wave oscillations is larger, just because one wants to have a theory leading to all successfull predictions of Q. M.
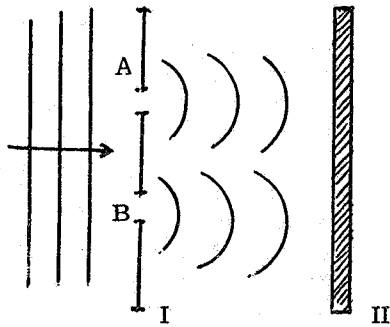
FIG. 3 - Schematic illustration of the two-slit experiment.

If both slits A and B are open (see Fig. 3) the wave is diffracted through both and in the region beyond I interference takes place. If the particle passes through A or B it will be found on the screen II with a probability density $\varsigma = |\Psi_A + \Psi_B|^2$ where Ψ_A (Ψ_B) is the wave diffracted through slit A (B). Closing one slit means absorbing one of the waves (Ψ_A or Ψ_B) and the interference is obviously distroied. One can maintain that the particle has passed either through A or through B in the interference experiment even though this statement cannot be checked experimentally. In other words that the particle passed either through A or through B is a meta physical standpoint which we are free to take now if we wish so. This was not the case in Q. M. where in a sense there is only the particle and Ψ is nothing physical, but represents only the knowledge which we passes of the elementary system.

d) The reduction of the wave packet could probably be attributed to some kind of physical absorption of the wave by the measuring apparatus. This point is, however, delicate and deserves a careful consideration. We simply do not have a complete logical explanation of this phenomenon, at the moment.

e) The ERP paradox must be solved by saying that Q. M. is wrong in its predictions for this paradox. This point is discussed further in the fifth section where it will be shown that Q. M. in this connection has not been checked experimentally in an exhaustive way.

III. 5. - Hidden variables.

The previous considerations lead us naturally to the discussion of the concept of hidden variables. In practice we can say that the physical, objectively existing wave which we have postulated in § III. 2 is a hidden variable. Hidden because it has, until now, never been revealed directly in an experimental manner (how to reveal it directly is discussed in § IV. 1). The wave function is, however, not the only hidden variable which can be introduced. It will help to clarify the matter to recall that there was a time in the history of physics when hidden variables were introduced in a different connection. It was the time when Boltzmann wrote his papers on statistical thermodynamics. He showed that the physical laws obeyed by the observable quantities pressure, volume, temperature, entropy, ... , could be understood in terms of simpler and more appealing properties of "hidden" (for those times) observables like the velocity of the molecules. Of course, Boltzmann was duly ridiculized by his contemporaries. Their arguments were : (i) that thermodynamics was a completely successfull and logical theory and that, therefore, there was no need to look for different theories; (ii) that anyway these molecules or atoms were so small that they could not be observed; (iii) that the atoms were a B. C. idea which had already been abandoned for very good reasons. Looking back from our 1970 standpoint we can now see that the atoms exist, that Boltzmann's thermodynamics is much more satisfactory than the classical one, that the two theories are not completely equivalent because fluctuations from the equilibrium can arise only in the formes and, last but not least, that statistical thermo dynamics is right and classical thermodynamics wrong. Boltzmann's "hidden variables" are now commonly observed. There "hidden variables" of older times are also those which give rise to the brownian motion. It is interesting to notice that for a browmam particle one can introduce a Δx for the position and a Δp for the momentum (mean square deviations) and that they obey an uncertainty relation where essentially the place of h (Planck's constant) is taken by diffusion constant of the medium[23]. Furthermore a $\Psi(\vec{x}, t)$ whose squared modulus gives the probability density for the position of the brownian particle can be introduced. This Ψ can be shown to obey Schrödinger's equation[24]. Furthermore noncommuting operators can be introduced to represent the observables. It will then not be surprising that some people[25] have built theories in which the hidden variables were unobserved

fluctuations of a postulated unobserved medium which acted on the elementary particles. These very appealing theories are, however, not completelysatisfactory(26).

IV. - A NEW CLASS OF EXPERIMENTS. -

The experiments to be discussed in this section have the common characteristic of : (a) being rather easy to perform; (b) disproving Quantum Mechanics if they were ever to give a positive result; (c) looking very strange and unusual at first sight. One should not take the third point as an important issue against them. The fact that they look strange and unusual merely reflects the polarization of thought along the canonical patterns which has existed during the last 40-50 years.

IV. 1. - Experiments on the space part of the wave function.

Hidden variable theories have been discussed in the last section. We anticipated that such theories are still at a preliminary stage of development. We can however stress already at this point that all of them realize an important shift of philosophical attitude : particles and waves become objectively existing entities. Therefore the metaphysical standpoint in the one called realism by d'Espagnat(4). It is also the standpoint invoked by the philosopher of science and these theories must therefore be considered as a brave attempt to put the world back where it was before Bohr.

An important epistemological problem arises at this point : what does it mean in practice that waves and particles exist objectively, or, in other words, which are they the predictions of the new theories which could not be ontained from Q. M. ? This problem is particularly acute for the wave function. In fact a successfull hidden-variable theory must state that, even though the waves exist, all of the energy, the momentum, the angular-momentum, the charge, and so on are strictly associated to the particle. What is it, then, an entity which exists but has not associated to it any observable physical quantity?

Perhaps an answer could be that physical quantities are mainly associated to particles, but that a very small fraction of some of them, so small to have escaped all observations, is associated to the wave. This is, however, an unappealing way-out. A better solution of the epistemological problem can be found if we notice that even without any physical quantity associated to it the wave function could give rise to physically obser vable phenomena(27). In fact we do not only measure energies, momenta, and so on. We also measure probabilities, e. g. the lifetime of an unstable system. The wave function could acquire reality, independently from the particles associated to it, if it could give rise to changes in the transition probabilities of the systems with which it comes in interaction.

An experiment to check the above idea can be the following : a continuous beam of neutrinos traverses a piece of matter in which unstable entities (nuclei, excited atoms or molecules) are contained. The lifetime of these entities is measured in such conditions and compared to the lifetime of the same entities in the absence of any passing beam. If a difference is observed, its only logical explanation is that it is due to the action of the wa ve function, since the neutrinos are extremely weakly interacting particles and only a few of them, at most, can have interacted in the piece of matter with presently available neutrino intensities.

Continuous fluxes of neutrinos are present only near reactors. The wave-lengths are then such as to suggest atomic or molecular (rather than nuclear) unstable systems for performing the experiment.

A necessary condition for the previous experiment being a real test of hidden-variable theories is that the waves accompanying the neutrinos interact with matter. We have no way to know this for sure, of course, but it seems reasonable to assume that the waves associated to different particles are all of the same nature. If this is the case we can conclude that the waves associated to neutrinos will certainly interact with matter. In

fact in the classical two-Alits experiment (performed with electrons or protons) interference phenomena are observed, whose only interpretation can be that two coherent waves originate from the slits themselves, but not from other points of the screen. Therefore the screen acts as an absorber for the incoming wave and this absorption must be attribu ted to some kind of interaction between wave and matter.

Summarizing, if in the proposed experiments a change of decay rate is detec ted it has to be attributed to some effect of the wave function. An extremely small effect of the same kind could be due to the weak interactions of the neutrino (as a result of the process $\nu + e \rightarrow \nu + e$ with bound electrons), but this is far beyond the accuracy of present experimental tecniques.

IV. 2. - The model by Bohm and Bub.

In the present paragraph we discuss the hidden-variable theory by Bohm and Bub[11]. The main features of this theory are the following:

a) It reproduces the statistical predictions of Q. M. if one averages over the hidden-varia bles.
b) It leads authomatically to the reduction of the wave packet, during a measurement, as a consequence of the properties of the hidden-variables.
c) For very short times immediately after a measurement process it leads to predictions different from those of Q. M.

We limit ourselves to the discussion of a dicotomic variable such as helicity of a spin - 1/2 particle or a photon. Let S be such an observable and $|S_1\rangle$, $|S_2\rangle$ its eigenstates. The most general vector for such a system will be

$$(8) \qquad |\psi\rangle = \psi_1 |S_1\rangle + \psi_2 |S_2\rangle$$

with

$$(9) \qquad J_1 + J_2 = 1$$

if we define $J_i \equiv |\psi_i|^2$ (i = 1, 2) for semplicity.

We assume that the description of S is complete only if we introduce a second state vector $|\xi|$ defined by

$$(10) \qquad |\xi\rangle = \xi_1 |S_1\rangle + \xi_2 |S_2\rangle .$$

The difference between $|\psi\rangle$ and $|\xi\rangle$ is the following. $|\psi\rangle$ is the usual quantum-mechanical vector and its components $\psi_1(t)$, $\psi_2(t)$ obey the Schrödinger equation, while $|\xi\rangle$ is a new vector whose (complex) components ξ_1 and ξ_2 (which are the hidden variables of this theory) have a random behaviour. More exactly, in a four dimensional spa ce the representative point of ξ_1, ξ_2 is always found on the hypersphere of unit radius:

$$(11) \qquad |\xi_1|^2 + |\xi_2|^2 = 1 .$$

Furthermore the probability density of the point on the sphere is assumed to be constant.

In this way we have specified competely the behaviour of the two vectors $|\psi\rangle$ and $|\xi\rangle$ in absence of interaction with a measuring device. A process of measurement is assumed to take place during a time interval very short compared with the typical variation time of ξ_1, ξ_2. Therefore ξ_1 and ξ_2 can be considered as constant during measurement. The equation governing the evolution of ψ_1, ψ_2 during a measurement are assumed to be

$$(12) \qquad \frac{d\psi_1}{dt} = \gamma (R_1 - R_2) \psi_1 J_2 \quad ; \qquad \frac{d\psi_2}{dt} = \gamma (R_2 - R_1) \psi_2 J_1 \quad ,$$

where

(13)
$$R_i = \frac{|\Psi_i|^2}{|\xi_i|^2} \qquad (i = 1, 2)$$

and γ is a real positive constant. Notice the shift of attitude of the Bohm-Bub model with respect to Q. M. Instead of a miracolous reduction of the wave packet we have now a precise equation (eq. (12) above) governing the evolution of $|\Psi\rangle$. Notice also that the evolution of $|\Psi\rangle$ depends on the value the hidden variables had immediately before the measurement. In fact they are, as we said, practically constant when (12) holds, and $|\xi_1|, |\xi_2|$ enter in (12) via (13).

To understand the evolution of $|\Psi\rangle$ following from (12) one can multiply the first by Ψ_1^*, the second by Ψ_2^* and obtain

(14)
$$\frac{dJ_1}{dt} = 2\gamma(R_1-R_2)J_1 J_2 \quad ; \qquad \frac{dJ_2}{dt} = 2\gamma(R_2-R_1)J_1 J_2 \ .$$

From these equations it follows $\frac{d}{dt}(J_1+J_2) = 0$, which means that they are consistent with a constant normalization of $|\Psi_1|^2 + |\Psi_2|^2$, as required by (9). It follows furthermore from (14)

(15)
$$\frac{d\log J_1}{dt} = 2\gamma(R_1-R_2)J_2 \quad ; \qquad \frac{d\log J_2}{dt} = 2\gamma(R_2-R_1)J_1$$

where, if $R_1 > R_2$,

(16)
$$\frac{d\log J_1}{dt} > 0 \implies J_1 \text{ increases} ; \qquad \frac{d\log J_2}{dt} < 0 \implies J_2 \text{ decreases} .$$

But J_1 and J_2 are positive and $J_1+J_2 = 1$ remains valid while J_1 increases and J_2 decreases. This means that if the variation is rapid enough (namely, if γ is large enough) at the end of the measurement we will have

$$J_1 = 1 , \qquad J_2 = 0 .$$

But $J_2 = 0$ is equivalent to $\Psi_2 = 0$. Looking back to (8) we see that if $R_1 < R_2$

(17)
$$|\Psi\rangle \longrightarrow e^{i\beta_1}|S_1\rangle$$

because of the measurement.

Similarly one can show that if $R_2 < R_1$

(18)
$$|\Psi\rangle \longrightarrow e^{i\beta_2}|S_2\rangle$$

during the measurement. The case $R_1 = R_2$ will be discussed in a moment. We wish to stress now that the measurement leads to a result, deduced from (12), very analogous to the postulate of reduction of the wave packet, namely to (17) and (18). The choice between (17) and (18) depends on the value of the hidden variables. In fact, using (9) and (11) it is easy to show that

$$R_1 > R_2 \qquad \text{is equivalent to} \qquad |\xi_1| < |\Psi_1|$$

$$R_1 < R_2 \qquad \text{is equivalent to} \qquad |\xi_1| > |\Psi_1|$$

$$R_1 = R_2 \qquad \text{is equivalent} \qquad |\xi_1| = |\Psi_1|$$

One can similarly show that $|\xi_1| < |\Psi_1|$ is equivalent to $|\xi_2| > |\Psi_2|$ and so on. Therefore one can let the result of a measurement depend exclusively on the relative value of

$|\mathcal{F}_1|$ and $|\psi_1|$. Notice that $R_1 = R_2$, equivalent to $|\mathcal{F}_1| = |\psi_1|$ has zero probability and can be neglected. Notice also that if $R_1 > R_2$ holds when the measurement starts it remains valid afterwards, because of the increase of R_1 and of the simultaneous decrease of R_2.

In conclusion, the reduction of the wave packet (eq. (17) and (18)) takes place, and the result of a single act of measurement is predetermined by the relative value of $|\mathcal{F}_1|$ and $|\psi_1|$ immediately before the interaction between the instrument and the microscopic system. If one has $|\mathcal{F}_1| < |\psi_1|$ eq. (17) is deduced and the observable S acquires the value S_1; if $|\mathcal{F}_1| > |\psi_1|$ S acquires the value S_2.

We have so proved the point b) of the beginning of the present paragraph. Let us show next that also a) holds. To do so we have to calculate the probability P_{S_1} that S_1 is obtained as a result of a measurement and to show that it equals $|\psi_1|^2$. Because of the assumption of constant probability density on the sphere $|\mathcal{F}_1|^2 + |\mathcal{F}_2|^2 = 1$ we must calculate simply the surface of that region of such a sphere in which $|\mathcal{F}_1| < |\psi_1|$ holds. Formally

$$(19) \qquad P_{S_1} = N \int d\vec{\mathcal{F}}_1 \, d\vec{\mathcal{F}}_2 \; \delta(\vec{\mathcal{F}}_1^2 + \vec{\mathcal{F}}_2^2 - 1) \; \theta(|\psi_1| - |\mathcal{F}_1|)$$

where N is a normalizing factor. Writing

$$\mathcal{F}_1 = \mathcal{S}_1 e^{i\theta_1} \; ; \qquad d\mathcal{F}_1 = \mathcal{S}_1 d\mathcal{S}_1 d\theta_1$$

and similarly for $\mathcal{F}_2$, one has:

$$P_{S_1} = N \int \mathcal{S}_1 \mathcal{S}_2 \, d\mathcal{S}_1 \, d\mathcal{S}_2 \, d\theta_1 \, d\theta_2 \, \delta(\mathcal{S}_1^2 + \mathcal{S}_2^2 - 1)\theta(|\psi_1| - \mathcal{S}_1) =$$

$$(20) \qquad = N \int_0^\infty \mathcal{S}_1 d\mathcal{S}_1 \theta(|\psi_1| - \mathcal{S}_1) \int_0^\infty \mathcal{S}_2 d\mathcal{S}_2 \delta(\mathcal{S}_1^2 + \mathcal{S}_2^2 - 1) \int_0^{2\pi} d\theta_1 d\theta_2 =$$

$$= N(2\pi)^2 \frac{1}{2} \int_0^\infty \mathcal{S}_1 d\mathcal{S}_1 \theta(|\psi_1| - \mathcal{S}_1) = N \frac{(2\pi)^2}{2} \int_0^{|\psi_1|} \mathcal{S}_1 d\mathcal{S}_1 = N \pi^2 |\psi_1|^2$$

Obviously if $|\psi_1| = 1$ the condition $|\mathcal{F}_1| < |\psi_1|$ holds everywhere (except in one point, while has zero probability and can be neglected). In this case we must have $P_{S_1} = 1$, which shows, comparing with (20), that $N = \pi^{-2}$. We obtain then:

$$(21) \qquad P_{S_1} = |\psi_1|^2$$

which is exactly the quantum mechanical result. There remains to be proven the point c), which will be discussed in the next paragraph.

IV.3. - Papaliolios' experiment.

As we have seen in the previous section the outcome of a measurement of, say, the helicity H of a photon is completely predictable if $|\psi_1|$ and $|\mathcal{F}_1|$ are known immediately before the measurement. In fact the measurement will give H = +1 if $|\psi_1| > |\mathcal{F}_1|$ and H = -1 if $|\psi_1| < |\mathcal{F}_1|$. The hidden variable $\mathcal{F}_1$ oscillates at random within the unit circle in its complex plane ($|\mathcal{F}_1| \leq 1$). Let r be a time so short that $\mathcal{F}_1$ does not change appreciably in any time interval t - t + r. Let r_M be the time duration of the measurement process. In the BB-theory one assumes that $r_M \ll r$, because one considers $|\mathcal{F}_1|$ constant during measurement. Suppose that after the first measurement M_1 (at time t_1) we perform a second measurement M_2 (at time t_2) and suppose that $t_2 - t_1 \gg r$. This means that before M_2 $\mathcal{F}_1$ has the time to relax again to its normal distribution (constant probability within

the circle $|\mathcal{f}_1| \leq 1$. Therefore we are again in a situation where QM and BB-model lead to the same predictions. If, however, $t_2 - t_1 \ll r$ $\mathcal{f}_1$ has not had time to relax and therefore we have not lost all information on $\mathcal{f}_1$. Different predictions from those of QM can then be obtained. This principle has been used by Papaliolios[28] in his experiment on the validity of the BB-model. He could set up an apparatus for which

$$t_2 - t_1 \simeq 7.5 \times 10^{-14} \text{ sec}$$

while BB estimate $r \simeq \hbar/kT \simeq 10^{-13}$ sec at room temperature. Some effect could be expected because $t_2 - t_1 \gg r$ is certainly not valid here. It should be stressed, however, that the BB estimate is very tentative and a smaller value for r cannot be ruled out. Papaliolios found no evidence of discrepancies with QM. It is worth describing his experiment, however, because it is certainly desirable to carry it out again for shorter values of $t_2 - t_1$.

The experimental arrangement is shown in Fig. 4. There are three linear polarizers A, B and C. A common set of xy-axes is assumed. The direction of polarization transmitted in the three cases is indicated by a double arrow and will be refferred
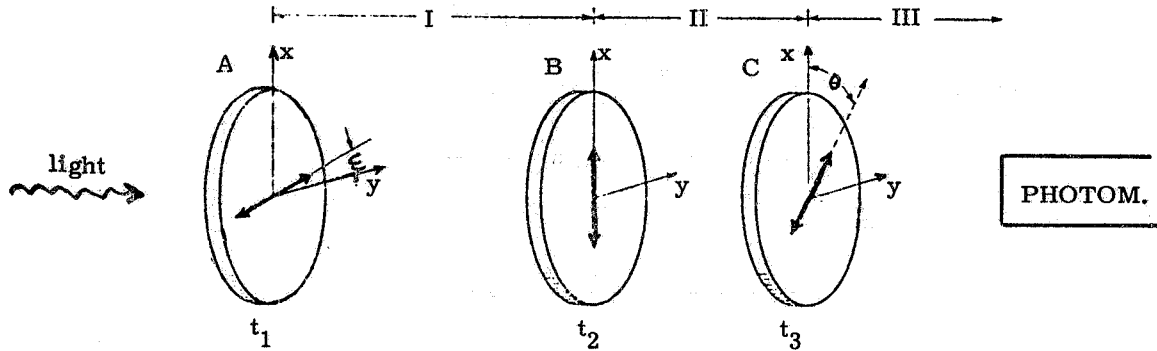

FIG. 4 - Setup of Papaliolios' experiment.

to as $\vec{P}_A$, $\vec{P}_B$ and $\vec{P}_C$ respectively. $\vec{P}_A$ forms an angle $\mathcal{E}$ with respect to the y axis. $\vec{P}_B$ is along x and $\vec{P}_C$ forms an angle θ with respect to the x-axis. A photon crosses the three polarizers at times t_1, t_2, t_3 respectively. We assume that $t_2 - t_1 \gg r$, so that a photon arriving on B behaves quantum mechanically. Therefore A is a device used to let photons with known polarization only arrive on B (this means that Ψ_1 and Ψ_2 are known between A and B).

If a photon crosses B it is also in a known (but different) state of polarization. Furthermore if $t_3 - t_2 \ll r$, we also know something about the hidden variables in region II (space between B and C). Mathematically we have :

$$|\Psi_{(I)}\rangle = \Psi_1^I |x\rangle + \Psi_2^{II}|y\rangle \qquad \Psi_1^I = \sin\mathcal{E}$$

$$= \sin\mathcal{E} |x\rangle + \cos\mathcal{E}|y\rangle \qquad \Psi_2^I = \cos\mathcal{E}$$

where $\mathcal{E} = 10^\circ$ in Papaliolios experiment. A photon crossing B has

$$|\Psi_1^I| > |\mathcal{f}_1| \implies |\mathcal{f}_1| < \sin 10^\circ .$$

The photon arriving on C will have :

$$|\Psi_{(II)}| = \Psi_1^{II}|x\rangle + \Psi_2^{II}|y\rangle \qquad \Psi_1^{II} = 1$$

$$= |x\rangle \qquad \Psi_2^{II} = 0 .$$

Therefore we know ψ_1 and ψ_2 in region II and know furthermore that $|\zeta_1^{II}| <$ $< \sin 10°$. Will this photon cross C? Obviously C constitutes a device to measure a different component of the photon. We must rotate the hidden variables ζ_i and the polarization variables ψ_i by an angle θ to be able to answer the question. If $|x'\rangle$ and $|y'\rangle$ are two new states representing polarization along the axes x', y', rotated by θ with respect to the old axes x, y, one has

$$|x\rangle = \cos\theta\,|x'\rangle + \sin\theta\,|y'\rangle \; ; \qquad |y\rangle = -\sin\theta\,|x'\rangle + \cos\theta\,|y'\rangle$$

where, by writing

$$|\psi_{II}\rangle = \psi_1'\,|x'\rangle + \psi_2'\,|y'\rangle$$

one gets immediately

$$\psi_1' = \cos\theta \; ; \qquad \psi_2' = \sin\theta .$$

Similarly one shall rotate the hidden variables obtaining

$$\zeta_1' = \cos\theta\,\zeta_1 + \sin\theta\,\zeta_2$$

The photon will cross C if $|\psi_1'| > |\zeta_1'|$, namely if

$$|\cos\theta| > |\zeta_1\cos\theta + \zeta_2\sin\theta|$$

which can be written

$$\frac{1 - tg^2\theta}{4\,tg\theta} > \left|\frac{\zeta_1}{\zeta_2}\right|\cos\alpha$$

where α is the relative phase of ζ_1 and ζ_2. But

$$\left|\frac{\zeta_1}{\zeta_2}\right| \leq tg\,\varepsilon \; ; \qquad\qquad \cos\alpha \leq 1 .$$

Therefore the region where the condition

$$\frac{1 - tg^2\theta}{4\,tg\theta} > tg\,\varepsilon$$

is satisfied, corresponds to certain transmission of the γ. But the previous inequality can be written also:

$$\frac{tg(\frac{\pi}{4} - \theta)}{1 - tg^2(\frac{\pi}{4} - \theta)} > tg\,\varepsilon .$$

The above inequality is certainly satisfied if

$$\frac{\pi}{4} - \theta > \varepsilon \qquad\qquad or \qquad \theta < \frac{\pi}{4} - \varepsilon = 45° - 10° = 35° .$$

In conclusion if θ is smaller than $35°$ the photon will be transmitted with certainty. It can analogously be shown that if $\theta > (\pi/4) + \varepsilon = 55°$ the photon will certainly be absorbed. In the region between $35°$ and $45°$ the transmission probability can be shown to be linear (see Fig. 5).
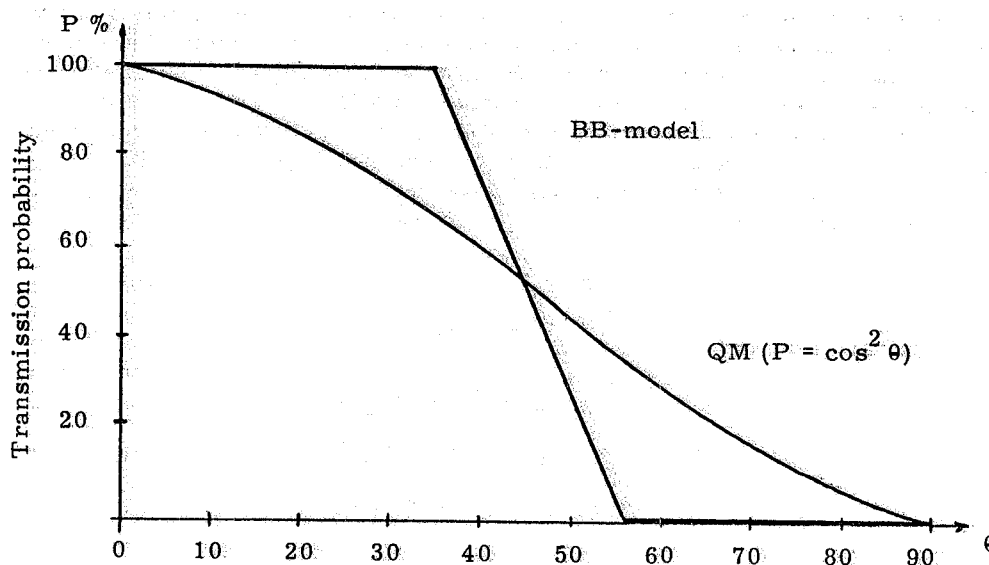
FIG. 5 - Predictions of the BB-theory and of Q. M.

In the experiment by Papaliolios one expects a result in between the two predictions because neither $t_3 - t_2 \ll r$ (QM-limit), nor $t_3 - t_2 \gg r$ (BB-limit) are satisfied. In reality, as we said, no disagreement with QM was found.

V. - BELL'S THEOREM AND FURTHER EXPERIMENTS. -

V. 1. - Bell's theorem.

No local hidden variable theory can reproduce all of the statistical predictions of Quantum Mechanics. The validity of the previous statement has been proved by Bell[29] a few years ago. The main point is that a hidden variable theory is supposed to be local, meaning in practice that in two regions of space, remote enough from each other, two measuring devices are placed, the results read on the first one cannot depend on the setup of the second one, and viceversa. Suppose now that in two distant regions of space I and II there are two instrument which force the particles crossing them to make a binary decision. The result of this measurement is read to be +1 or -1 depending on the decision taken. For instance a polarizer P whith a photomultiplier behindis an instrument of this kind : if the photon crosses P it will enter into the photomultiplier which will show +1 on a certain scale; if the photon is absorbed by P the photomultiplier will not move from its rest position which is labeled -1. Let A(a) be the result of this measurement, a being an apparatus parameters like, for instance, the rotation angle of the polarizer in the discussed example. Thus $A(a) = \pm 1$. Let similarly B(b) be the result of a measurement performed with the second instrument (b is a parameter of this instrument). Again $B(b) = \pm 1$. These equlities to ± 1 cannot be further specified in Q. M. One can assign a probability to obtain +1 or -1, but one cannot know which choice a given photon will make. Suppose now that there are hidden variables which complete the description. Let us describe these variables with a continuous parameter λ. The results A and B will depend on the hidden variables and the notation

$$(22) \qquad A(a, \lambda) = \pm 1 \; ; \qquad B(b, \lambda) = \pm 1$$

will mean that for some values of λ $A(a, \lambda)$ equals +1 and for some other values it equals -1. Similarly for $B(b, \lambda)$. Therefore the introduction of λ makes the result of a single act of measurement perfectly deterministic in principle. It should be stressed

that different physical assumptions can be made for λ : it could be an internal variable of the particle and could be the same for two particles arriving in regions I and II because of some interaction that the two particles had in the past; it could be a variable representing a local interaction of the particle with a medium, in which case the λ entering in A would not be related to the one entering in B; it could have a double nature representing simultaneously the two previous possibilities. We will prove Bell's theorem starting from the first standpoint, but no difficulty arises if one uses a different one. Let $\varrho(\cdot\lambda)$ be the probability density for the hidden variable λ. Of course

$$(23) \qquad \int_{\Lambda} \varrho(\lambda)\, d\lambda = 1$$

where Λ is the region of variation of λ. We define

$$(24) \qquad \begin{aligned} P_A(a) &\equiv \langle A(a,\lambda)\rangle = \int d\lambda\, \varrho(\lambda)\, A(a,\lambda) \\ P_B(b) &\equiv \langle B(b,\lambda)\rangle = \int d\lambda\, \varrho(\lambda)\, B(b,\lambda) \\ P(a,b) &\equiv \langle A(a,\lambda)B(b,\lambda)\rangle = \int d\lambda\, \varrho(\lambda)A(a,\lambda)B(b,\lambda) \end{aligned}$$

We can write

$$(25) \qquad \begin{aligned} |P(a,b) - P(a,c)| &\leqslant \int d\lambda\, \varrho(\lambda)\, |A(a,\lambda)B(b,\lambda) - A(a,\lambda)B(c,\lambda)| \\ &= \int d\lambda\, \varrho(\lambda)\, (1 - B(b,\lambda)\, B(c,\lambda)) \end{aligned}$$

because of (22). Consider now a value b' of the parameter of the first apparatus (the one placed in region I). Let Λ_+ be the set of all values of λ for which $A(b',\lambda) = +B(b,\lambda)$ is valid. Similarly, let Λ_- be the set for which $A(b',\lambda) = -B(b,\lambda)$. Obviously $\Lambda = \Lambda_+ + \Lambda_-$. We can write

$$(26) \qquad \begin{aligned} \int_{\Lambda} d\lambda\, \varrho(\lambda)B(b,\lambda)B(c,\lambda) &= \int_{\Lambda} d\lambda\, \varrho(\lambda)A(b',\lambda)B(c,\lambda) - 2\int_{\Lambda_-} d\lambda\, \varrho(\lambda)A(b',\lambda)B(c,\lambda) \\ &\geq P(b',c) - 2\int_{\Lambda_-} |A(b',\lambda)B(c,\lambda)|\, \varrho(\lambda)d\lambda = P(b',c) - 2\int_{\Lambda_-} d\lambda\, \varrho(\lambda) \\ &= P(b',c) - \delta \end{aligned}$$

where

$$(27) \qquad \delta = 2\int_{\Lambda_-} d\lambda\, \varrho(\lambda).$$

Notice that we have

$$(28) \qquad \begin{aligned} P(b',b) &= \int_{\Lambda} d\lambda\, \varrho(\lambda)A(b',\lambda)B(b,\lambda) = \int_{\Lambda_+} d\lambda\, \varrho(\lambda) - \int_{\Lambda_-} d\lambda\, \varrho(\lambda) \\ &= \int_{\Lambda} \varrho(\lambda)d\lambda - 2\int_{\Lambda_-} \varrho(\lambda)d\lambda = 1 - \delta. \end{aligned}$$

Inserting (28) in (26) and the result so obtained in (25) we get finally[32] :

$$(29) \qquad |P(a,b) - P(a,c)| \leq 2 - P(b',b) - P(b',c)$$

This inequality, deduced from very general considerations following from the postulated existence of local hidden variables, is extremely important because it is in general not satisfied by Quantum Mechanics. To see this consider again the decay $\pi^0 \rightarrow$

$\rightarrow e^+ + e^-$ discussed in § I. 3. Suppose that in the two regions I and II are placed two Stern-Gerlach apparatus which measure, respectively, the spin components of the elec tron and of the positron along the directions specified by the unit vectors $\vec{\alpha}$ and $\vec{\beta}$. The correlation function $P(\alpha, \beta)$, introduced in (24), can also be calculated from Q. M. Which gives, for a singlet state,

$$(30) \qquad P(\alpha, \beta) = \langle \vec{\sigma} \cdot \vec{\alpha} \; \vec{\sigma} \cdot \vec{\beta} \rangle = -\vec{\alpha} \cdot \vec{\beta} \; .$$

Let us use this result in connection with (29) where we choose $\vec{b}' = -\vec{b}$ and $\vec{a} \perp \vec{b}$. We get so

$$\left| \vec{a} \cdot \vec{c} \right| \le 2 - \vec{b}^2 - \vec{b} \cdot \vec{c}$$

whence

$$\left| \sin \theta \right| \le 1 - \cos \theta$$

if θ is the angle between b and c. The above inequality is not satisfied, in general. For instance for small θ the l. h. l is $O(\theta)$ and the r. h. l is $O(\theta^2)$. We conclude then that no local hidden variable theory can give the same result as Q. M. for a measurement of e^+ and e^- spin correlations in $\pi^\circ \rightarrow e^+ e^-$ decay. Many other applications of the theorem are obviously possible. Some will be discussed in § V. 3.

V. 2. - Older experiments.

In the present paragraph we discuss two experiments, the first by Wu and Shaknov[30] and the second by Kocher and Commins[31], which were designed to check the quantum mechanical predictions on spin correlations.

In the Wu-Shaknov experiment the ground state of positronium (with angular momentum zero) decays into two 0. 5 MeV γ-rays. These propagate in opposite direc- tions and are Compton-scattered by two Al targets. Two photomultipliers reveal the coin cidences for γ-rays scattered at 90° ($\pm 20^\circ$). The number of coincidences was measured for coplanar ($N(\parallel)$) and for perpendicular ($N(\perp)$) arrangements of the photomultipliers. If $R = N(\perp)/N(\parallel)$, the prediction from Q. M. is $R = 2. 00$ while the experimental result ob tained was $R = 2. 04 \pm 0. 08$. The difference from unity of R is understood as a manifesta tion of perpendicular polarization of the two γ-rays, as required from Q. M. This expe- riment supports Q. M. It should be stressed, however, that Bell's theorem cannot be checked here because the γ-rays were not forced to make a binary decision. Therefore it is entirely possible that the same prediction for R could be obtained from a hidden-va riable theory.

In the Kocher-Commins experiment the 6^1S_0 excited state of Ca decays to the 4^1P_1 state by emitting a photon with a wavelength of 5513 Å. The 4^1P_1 state decays im- mediately to the ground state (4^1S_0) with a 4227 Å photon. In this way the Ca-atom emits two photons and passes from a J=0 state to another J=0 state. The total angular momen- tum of the two photons must obviously be zero and this fact is reflected in a squared ma- trix-element for the complete transition proportional to ($\vec{\mathcal{E}}_1 \cdot \vec{\mathcal{E}}_2$)^2 where $\vec{\mathcal{E}}_1$ and $\vec{\mathcal{E}}_2$ are the polarization vectors of the two photons. Parallel polarization is obviously favou- red. The experimental apparatus by Kocker and Commins consisted of two photomulti- pliers put in opposite directions with respect to the point where the excited Ca-atom de- cayed. In front of the photomultipliers there were filters (one letting only photons with λ_1 = 5513 Å through the other λ_2 = 4227 Å) and linear polarizers. It was so verified that for perpendicular setting of the polarizer no coincidences above background were registred, while for parallel setting a high coincidence rate was present. The result was therefore in agreement with the predictions of Quantum Mechanics. No indication pro or against Bell's theorem can however be obtained.

V. 3. - Newly proposed experiments.

As we saw in the last paragraph the experiments by Wu and Shaknov[30] and by Kocher and Commins[31] do not check the inequality (29) which expresses Bell's theorem and holds for local hidden variable theories. Therefore it is entirely possible, as far as we know today, that a local hidden variable description of the physical world is valid, instead of Q. M. It is therefore important to find experiments which can discriminate between the two possibilities.

An experiment of this type was in fact proposed by Clauser, Horne, Shimony and Holt[32]. In order to understand their reasoning let us see how the correlation function defined by (24) and entering in (29) is related to observable quantities. Let $w\left[A(a)_+, B(b)_+\right]$ be the probability (in a statistical sense) to obtain $A(a) = +1$ and $B(b) = +1$ in the measurements discussed in par. V. 1. Similarly we define the quantities $w\left[A(a)_+, B(b)_-\right]$, $w\left[A(a)_-, B(b)_+\right]$ and $w\left[A(a)_-, B(b)_-\right]$. If Λ_{++} is the set of values of the hidden variable λ for which $A(a, \lambda) = +1$ and $B(b, \lambda) = +1$ hold simultaneously, one has

$$w\left[A(a)_+, B(b)_+\right] = \int_{\Lambda+1} d\lambda\, \varsigma(\lambda)\ .$$

The previous equation can be written

$$w\left[A(a)_+, B(b)_+\right] = \int_{\Lambda} d\lambda\, \varsigma(\lambda) \left\{ \frac{1 + A(a, \lambda)}{2} \quad \frac{1 + B(b, \lambda)}{2} \right\}$$

because the quantity whithin curly brackets vanishes when ever $A(a, \lambda) = -1$ and/or $B(b, \lambda) = -1$. Generalizing the previous argument we can write

(31) $$w\left[A(a)_\pm, B(b)_\pm\right] = d\lambda\, \varsigma(\lambda) \left\{ \frac{1 \pm A(a, \lambda)}{2} \quad \frac{1 \pm B(b, \lambda)}{2} \right\}$$

It is now a simple matter to show that

(32) $$P(a, b) = w\left[A(a)_+, B(b)_+\right] - w\left[A(a)_+, B(b)_-\right] - w\left[A(a)_-, B(b)_+\right] + w\left[A(a)_-, B(b)_-\right]$$

The previous equation relates $P(a, b)$ to observable probabilities. Let us next define $w\left[A(a)_\pm, 0\right]$ us the probability that $A(a, \lambda) = \pm1$ when the polarizer II is taken away in the KC-experiment. In a similar way we define $w\left[0, B(b)_\pm\right]$. Obviously one has

(33)
$$w\left[A(a)_+, 0\right] = w\left[A(a)_+, B(b)_+\right] + w\left[A(a)_+, B(b)_-\right]$$

$$w\left[0, B(b)_+\right] = w\left[A(a)_+, B(b)_+\right] + w\left[A(a)_-, B(b)_+\right]$$

and similarly for $w\left[A(a)_-, 0\right]$ and $w\left[0, B(b)_-\right]$. Finally one has

(34) $$1 = w\left[A(a)_+, B(b)_+\right] + w\left[A(a)_+, B(b)_-\right] + w\left[A(a)_-, B(b)_+\right] + w\left[A(a)_- B(b)_-\right]$$

The reason for introducing equations (33) and (34) is in the fact that $w\left[A(a)_+, B(b)_+\right]$, $w\left[A(a)_+, 0\right]$, $w\left[0, B(b)_+\right]$ can all be measured as coincidence rates while the other quantities entering in (32) are difficult to measure directly. Using (33) and (34) in order to eliminate $w\left[A(a)_+, B(b)_-\right]$, $w\left[A(a)_-, B(b)_+\right]$ and $w\left[A(a)_-, B(b)_-\right]$ from (32) one gets

(35) $$P(a, b) = 4 w\left[A(a)_+, B(b)_+\right] - 2 w\left[A(a)_+, 0\right] - 2 w\left[0, B(b)_+\right] + 1$$

Supposing that $w\left[A(a)_+, 0\right]$ and $w\left[0, B(b)_+\right]$ do not really depend on a and b (a situation

realized when simultaneously emitted photons are polarized relatively to each other, but not in absolute) one gets, for the l. h. s. and r. h. s. of eq. (29)

(36) $\qquad \left| P(a,b) - P(a,c) \right| = 4 \left| w\left[A(a)_+, B(b)_+ \right] - w\left[A(a)_+, B(c)_+ \right] \right|$

(37) $2 - P(b',b) - P(b',c) = 2 - \left\{ 4w\left[A(b')_+, B(b)_+ \right] - 2w\left[A_+, 0 \right] - 2w\left[0, B_+ \right] + 1 \right\}$

$\qquad\qquad\qquad\qquad - \left\{ 4w\left[A(b')_+, B(c)_+ \right] - 2w\left[A_+, 0 \right] - 2w\left[0, B_+ \right] + 1 \right\}$

From (36) and (37) it follows:

(38) $\quad \left| w\left[A(a)_+, B(b)_+ \right] - w\left[A(a)_+, B(c)_+ \right] \right| \leq - w\left[A(b')_+, B(b)_+ \right] - w\left[A(b')_+, B(c)_+ \right]$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + w\left[A_+, 0 \right] + w\left[0, B_+ \right]$.

This relation, equivalent to (29), has however the advantage of being expressed in terms of quantities directly measurable as coincidence rates. The inequality (38) is of general validity. Clauser, Horne, Shimony and Holt[32] propose to check it in an amproved experiment of the Kocher-Commins type. All the coincidence probabilities appearing in (38) are proportional to the coincidence counting rates. For instance one has

(39) $\qquad\qquad\qquad w\left[A(a)_+, B(b)_+ \right] = \dfrac{R(a,b)}{R_0}$

where $R(a,b)$ is the coincidence rate when the first (second) polarizer is oriented along the direction specified by $a(b)$ (which could be the angles between the polarizer axes and a fixed direction) and R_0 is the coincidence rate when both polarizers are removed. Similar expressions hold for the other probabilities entering in (38). One gets, with obvious notation,

(40) $\qquad \left| R(a,b) - R(a,c) \right| \leq R(a,-) + R(-,a) - R(b',b) - R(b',c)$

The quantum mechanical predictions for the counting rates for the $J=0 \to J=1 \to J=0$ electric dipole cascade of calcium are the following :

1) $R(a,b)$ depends only on the relative orientation of the polarizer axes b-a;

2) $R(a,-)$ and $R(-,a)$ are equal to each other and do not depend on a (the last part of this stateement was also assumed to be true in deriving (38) and (40));

3) For ideal polarizers and point-like photomultipliers one has

(41)

$$R(\phi) = R_0 \dfrac{1}{4}(1 + \cos 2\phi) = \dfrac{R_0}{2} \cos^2 \phi$$

$$R(a,-) = R(-,a) = \dfrac{1}{2} R_0$$

The staments 1) and 2) should be checked experimentally, but one does not expect discrepancies here. The important point is that the quantum mechanical predictions (41) do not satisfy the inequality (40) in general. In fact let us choose

$\left. \begin{array}{l} b - a = 30^\circ \\ c - a = 60^\circ \end{array} \right\} \implies c - b = 30^\circ$

$b - b' = 0^\circ \qquad\qquad \implies c - b' = 30^\circ$

One gets from (41)

$R(a,b) = R(b - a) = R(30^\circ) = 3\, R_0/8$

$R(a,c) = R(c - a) = R(60^\circ) = R_0/8$

$$R(b', b) = R(b - b') = R(0^{\circ}) = 4 \, R_o/8$$

$$R(b', c) = R(c - b') = R(30^{\circ}) = 3 \, R_o/8$$

whence the l. h. s. and r. h. s. of (40) become

$$|R(a, b) - R(a, c)| = 2 \, R_o/8$$

$$R(a, -) + R(-, b) - R(b', b) - R(b', c) = R_o/8$$

Obviously (40) is not satisfied. As the discrepancy is small ($\approx R_o/8$), one must be careful with the true efficiency of the polarizers and with the finite angle subtended by the photomultipliers. The quoted authors[32] showed that also by considering these effects one can check whether (40) is satisfied or not in the physical world.

Many other experiments to check (40) should be possible and are highly desirable in order to study in different field the possibility of a local hidden variable description of the microcosm.

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