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F. Drago : CHARGED-PION PRODUCTION, VENEZIANO REPRESENTATION AND THE PROBLEM OF PION CONSPIRACY

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Charged-Pion Production, Veneziano Representation and the Problem of Pion Conspiracy (*).

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1. – Summary and conclusions.

We study the Veneziano representation for charged-pion photoproduction near the forward direction. The simplest (*i.e.* without satellites) representation that contains the gauge-invariant Born term requires the existence of an $M=1$ pion trajectory. The model predicts correctly some of the peculiar features of the parity doublet which are necessary to fit the data.

Due to the difficulties associated with the $M=1$ assignment, we studied the possibility of constructing solutions with an $M=0$ pion trajectory. In the framework of the model this can be done in an essentially arbitrary way, by adding suitably chosen satellites.

2. – Introduction and notations.

The mechanism of the reaction $\gamma+p\to\pi^++n$ near the forward direction is still unclear. The most economical interpretation of the now well-known narrow forward peak is in terms of a parity-doublet conspiracy of the pion (which means that the pion trajectory belongs to a Toller family with $M=1$). However the $M=1$ assignment for the pion leads to both theoretical and phenomenological difficulties. If on the other hand the pion trajectory has $M=0$ its contribution to the photoproduction cross-section vanishes at $t=0$ and models with poles and cuts are necessary to fit the data [1].

We will discuss here an application of the Veneziano model [2] to the photoproduction of $\pi^\pm$ near the forward direction. Only two amplitudes (for each isospin state) are im-

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(1) For a general discussion see J. D. JACKSON: *Proceedings of the International Conference on Elementary Particles, Lund, 1969* (to be published).

(2) G. VENEZIANO: *Nuovo Cimento*, 57 A, 190 (1968).

portant near the forward direction. We will mainly use t-channel helicity amplitudes free from kinematical singularities: $F_2(s,t)$ and $F_3(s,t)$ are defined in ref. (3) (*). The amplitude $F_2^{(-)}(s,t)$ receives asymptotic contributions from t-channel exchanges with the quantum numbers of the pion and $F_3^{(-)}(s,t)$ from the exchange of trajectories with all the same quantum numbers of the pion, but with opposite natural parity (natural parity = parity × signature).

At $t=0$ the two amplitudes are related by the constraint equation (3)

$$F_2^{(-)}(s, t=0) = -\frac{m_\pi^2}{2M} F_3^{(-)}(s, t=0)\,. \tag{1}$$

3. – The model.

Let us start with the amplitude $F_2^{(-)}(s,t)$. We assume that it receives contributions from the nucleon and Δ trajectories in the s- and u-channel and from the pion trajectory in the t-channel. The simplest Veneziano amplitude that contains the gauge-invariant Born term is

$$\begin{aligned}\frac{F_2^{(-)}(s,t)}{t-m_\pi^2} = -\beta\Bigg\{&\frac{\Gamma[-\alpha_\pi(t)]\,\Gamma[\frac{1}{2}-\alpha_{\mathcal N}(s)]}{\Gamma[\frac{3}{2}-\alpha_\pi(t)-\alpha_{\mathcal N}(s)]} - (s\to u) + \\ &+ p\left[\frac{\Gamma[1-\alpha_\pi(t)]\,\Gamma[\frac{1}{2}-\alpha_\Delta(s)]}{\Gamma[\frac{3}{2}-\alpha_\pi(t)-\alpha_\Delta(s)]} - (s\to u) + \frac{\Gamma[\frac{1}{2}-\alpha_{\mathcal N}(s)]\,\Gamma[\frac{1}{2}-\alpha_\Delta(u)]}{\Gamma[1-\alpha_{\mathcal N}(s)-\alpha_\Delta(u)]} - (s\to u)\right]\Bigg\}\,.\end{aligned} \tag{2}$$

We do not discuss here the problem of the baryon parity doublets. We only note that for the parity doublet of the nucleon there is no trouble, since it does not contribute to this amplitude; moreover since no parity doublet of the Δ has been observed, we have put its residue equal to zero in (2). The parameters β and p are obtained by comparing the residues at $\alpha_\pi(t=m_\pi^2)=0$ and $\alpha_{\mathcal N}(s=M^2)=\frac{1}{2}$ in (2) with those obtained from pion and nucleon poles in the Born approximation. We get

$$\beta = eg\alpha'^2 m_\pi^2 \tag{3}$$

and

$$p = -\frac{1}{2\alpha' m_\pi^2}\,, \tag{4}$$

where $e^2/4\pi = 1/137$, $g^2/4\pi = 14.7$ and α' is the common slope of the linear Regge trajectories involved.

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(3) J. Ball, W. Frazer and M. Jacob: *Phys. Rev. Lett.*, 20, 518 (1968).

(*) These amplitudes are related to the CGLN invariant amplitudes (4) in the following way:

$$\frac{F_2^{(-)}(s,t)}{t-m_\pi^2} = A_1^{(-)}(s,t) + tA_2^{(-)}(s,t)\,,$$

$$F_3^{(-)}(s,t) = 2MA_1^{(-)}(s,t) - tA_4^{(-)}(s,t)\,.$$

(4) G. F. Chew, M. L. Goldberger, F. E. Low and Y. Nambu: *Phys. Rev.*, 106, 1345 (1957).

The representation (2) has also been discussed in a recent letter (5) by AHMAD, FAYYAZUDDIN and RIAZUDDIN. However the interpretation that they gave to (2) is completely different from the one presented below.

We will now show that the leading singularity in the t-channel angular-momentum plane is a Regge pole corresponding to a pion with $M = 1$. Moreover the model naturally embodies some of the peculiar properties of the residue functions that are needed to fit the data.

From eq. (2) we get, for large s and small t,

$$(5) \qquad F_2^{(-)}(s,t) = -\frac{\beta}{2\alpha' m_\pi^2}(t+m_\pi^2)\frac{\pi\alpha_\pi(t)}{\Gamma(\alpha_\pi(t)+1)}\frac{1+\exp[-i\pi\alpha_\pi(t)]}{\sin\pi\alpha_\pi(t)}(\alpha' s)^{\alpha_\pi(t)-1}.$$

This amplitude is Regge-behaved and nonzero at $t = 0$. Due to eq. (1) this means that another trajectory $\alpha_c(t)$, with the same quantum numbers of the pion, but with opposite parity, such that $\alpha_c(0) = \alpha_\pi(0)$, must be present in the model: due to the universality of the slope α' the two trajectories will actually be degenerate in the present model.

We stress once again that eq. (1) is not at all trivial, even if, expressed in terms of invariant amplitudes, it reads $A_1(s, t=0) = A_1(s, t=0)$. This only means, in the framework of pure Regge-pole models, that *if* $A_1(s, t=0)$ is Regge-behaved and nonzero at $t = 0$, *then* we have a parity doublet exchange in the t-channel.

We can now easily construct a representation for $F_3^{(-)}(s,t)$ (*) that satisfies eq. (1):

$$(6) \qquad F_3^{(-)}(s,t) = -\gamma\left\{\frac{\Gamma[1-\alpha_c(t)]\Gamma[\frac{1}{2}-\alpha_{\mathcal{N}}(s)]}{\Gamma[\frac{3}{2}-\alpha_c(t)-\alpha_{\mathcal{N}}(s)]} - (s\to u) + \right.$$
$$\left. + q\left[\frac{\Gamma[1-\alpha_c(t)]\Gamma[\frac{1}{2}-\alpha_\Delta(s)]}{\Gamma[\frac{3}{2}-\alpha_c(t)-\alpha_\Delta(s)]} - (s\to u) + \frac{\Gamma[\frac{1}{2}-\alpha_{\mathcal{N}}(s)]\Gamma[\frac{1}{2}-\alpha_\Delta(u)]}{\Gamma[1-\alpha_{\mathcal{N}}(s)-\alpha_\Delta(u)]} - (s\to u)\right]\right\} +$$
$$+ \text{nonleading terms for large } s \text{ and small } t$$

with $\gamma = 2\,Meg\,\alpha'$ and $q = -\frac{1}{2}$.

From eq. (6) we obtain for large s and small t

$$(7) \qquad F_3^{(-)}(s,t) = \frac{\gamma}{2}\frac{\pi\alpha_c(t)}{\Gamma(\alpha_c(t)+1)}\frac{1+\exp[-i\pi\alpha_c(t)]}{\sin\pi\alpha_c(t)}(\alpha' s)^{\alpha_c(t)-1}.$$

We will now briefly discuss some very interesting results which are automatically contained in the model:

1) As also remarked by AHMAD, FAYYAZUDDIN and RIAZUDDIN, in the limit $s\to\infty$, $\alpha_\pi(t) = 0$, the model reduces to the electric Born term, which is known to give a reasonable fit to the forward cross-section.

2) The pion residue function is proportional to $(1 + \frac{1}{2}(t - m_\pi^2)/m_\pi^2)$ and therefore varies very rapidly in the interval $-m_\pi^2 \leqslant t \leqslant m_\pi^2$. A rapid variation of this kind is known to be necessary to fit both the magnitude of the differential cross-section

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(5) M. AHMAD, FAYYAZUDDIN and RIAZUDDIN: *Phys. Rev. Lett.*, 23, 504 (1969).

(*) Note that $F_3^{(-)}(s,t)$ being a nonsense-nonsense amplitude at $\alpha(t) = 0$ cannot contain a pole at $\alpha(t) = 0$.

and the data with polarized photons with an $M=1$ pion. This behavior has to be compared with the one obtained from high-energy (3) and continuous-moment sum rule (6) fits: $(1+\lambda(t-m_\pi^2)/m_\pi^2)$, a typical value of λ being $\lambda=0.42$.

In other words the model predicts a very rapid variation of the unnatural-parity contribution and its vanishing at $t=-m_\pi^2$. This zero is extremely important to explain the behavior of the polarized-photon data: the predictions are compared to the data (7) in Fig. 1 and the agreement is very good in the extreme forward direction. Of course for $-t \gg m_\pi^2$ other trajectories like the ρ, A_2 and B, which are not important near $t=0$, will come into the game and will change considerably the predictions shown in the Figure.

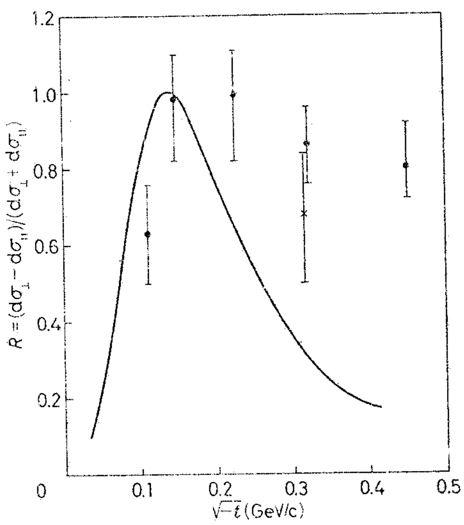

Fig. 1. – The quantity $R=(d\sigma_\perp - d\sigma_\parallel)/(d\sigma_\perp + d\sigma_\parallel)$ is compared with the prediction of the model with a conspiring pion. The data are from ref. (7): • 3.4 GeV, × 5 GeV.

3) From the behavior of the residue of the «conspirator» pole at $\alpha_c(t)=0$ in the nonsense-nonsense amplitude $F_3^{(-)}$ we see that the «conspirator» trajectory chooses nonsense at $\alpha_c(t)=0$. That means that no new 0^+ particle is expected. This result is also in agreement with the study of finite-energy sum rules (8), once the $M=1$ assignment has been assumed.

We have thus shown that the simplest (*i.e.* without satellites) Veneziano representation requires an $M=1$ pion trajectory and correctly predicts some of the peculiar properties required to fit the data.

However it has all the troubles of the $M=1$ models (1) and it is therefore interesting to try to construct an amplitude that contains an $M=0$ pion trajectory. In this case of course absorptive corrections are crucial to fit the data.

It is obviously impossible to construct a Veneziano amplitude without satellites and with an $M=0$ pion trajectory, since we want it to contain the Born term which has a nonzero residue at $t=0$.

For simplicity of writing let us just consider the first term in eq. (2): the discussion of the second term is very similar. We consider the amplitude

$$-\beta\left\{\frac{\Gamma[-\alpha_\pi(t)]\,\Gamma[\frac{1}{2}-\alpha_N(s)]}{\Gamma[\frac{3}{2}-\alpha_\pi(t)-\alpha_N(s)]}+\sum_{r=1}^{N}\gamma_r\,\frac{\Gamma[r-\alpha_\pi(t)]\,\Gamma[\frac{1}{2}+r-\alpha_N(s)]}{\Gamma[\frac{1}{2}+2r-\alpha_\pi(t)-\alpha_N(s)]}-(s\to u)\right\} \tag{8}$$

and we expand it in power series using the method of ref. (9). We can now choose the coefficients γ_r in such a way that at $t=0$ the amplitude (8) behaves as $(\alpha' s)^{\alpha_\pi(0)-N}$;

(6) P. Di Vecchia, F. Drago, C. Ferro-Fontán, R. Odorico and M. L. Paciello: *Phys. Lett.*, 27 B, 296 (1968).

(7) C. Geweniger, P. Heide, U. Kotz, R. Lewis, P. Schmuser, H. Skrom, H. Wahl and K. Wegener: *Phys. Lett.*, 28 B, 155 (1968).

(8) A. Bietti, P. Di Vecchia, A. Drigo and M. Paciello: *Phys. Lett.*, 26 B, 457 (1968).

(9) M. L. Paciello, L. Sertorio and B. Taglienti: *Nuovo Cimento*, 62 A, 713 (1969).

which means that we have removed the first $(N-1)$ Toller poles with $M=1$ present in the original amplitude.

It can be easily seen that to do their job the γ_r must satisfy the system

$$(9)\qquad A_{n-1}(\alpha' M^2, 1+\alpha' M^2-\alpha_\pi(0)) + \sum_{r=1}^{n} \gamma_r \frac{\Gamma[r-\alpha_\pi(0)]}{\Gamma[-\alpha_\pi(0)]}\cdot \cdot A_{n-r}(r+\alpha' M^2, 2r+\alpha' M^2-\alpha_\pi(0)) = 0$$

for $1 \leqslant n \leqslant N$, where the coefficients $A_n(\lambda, \mu)$ are defined and discussed in ref. [5]. The system (9) is equivalent to a recurrence formula that gives γ_s if $\gamma_1, \gamma_2, \ldots, \gamma_{s-1}$ are known. However it should be remarked that the cancellation mechanism introduced in (8) is highly arbitrary and it is presumably possible to find many other satellite contributions that work equally well. Moreover it is not very clear what happens if we let $N\to\infty$: in this case in fact the series in (8) must develop a pole at $\alpha_{\mathcal{N}}=\frac{1}{2}$ which is not present in the single terms and therefore will change the Born term.

By modifing in a similar way the second term in (2) we obtain the asymptotic expansion

$$(10)\qquad F_2^{(-)}(s,t) = -eg\alpha' t \frac{\alpha'(t+m_\pi^2)+2}{2(\alpha' m_\pi^2+1)} \frac{\pi\alpha_\pi(t)}{\Gamma(\alpha_\pi(t)+1)} \frac{1+\exp[-i\pi\alpha_\pi(t)]}{\sin\pi\alpha_\pi(t)} (\alpha' s)^{\alpha_\pi(t)-1},$$

that obviously corresponds to the exchange of an $M=0$ pion trajectory.

In conclusion we have shown that the simplest Veneziano representation for charged-pion photoproduction requires pion conspiracy, with all its defects. Moreover there is an essentially infinite freedom in the construction of evasive solutions.

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