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M. Greco : A MODEL FOR THE POMERANCHUK TERMS. -

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ABSTRACT. -

A model for all the Pomeranchuk terms is constructed by taking into account some general properties associated to the leading vacuum singularity. It gives a very good description of the $p\bar{p}$, pp total and differential cross sections, and provides an accurate determination of the Pomeranchuk parameters.

The concept of duality of direct channel resonances and Regge poles exchanges has been used successfully to construct a general scheme to classify particle spectra and to analyze scattering reactions(1). In the framework of this scheme the Pomeranchuk singularity is separated and supposed to be dual to non-resonating background at low energy(2). This hypothesis has yielded a large number of interesting results(2, 3). Ambiguities however arise in testing quantitatively this conjecture, mainly due to the uncertainties associated with the concept of resonance. These ambiguities have prevented in particular to get an accurate computation of $\alpha_P(t)$. Generally the presence of Regge cuts is completely neglected in the above considerations, although strong arguments have been given in favour of their existence(4). Recently some attention has been put on this problem, and it has been suggested(5) that duality can be preserved in the presence of Regge cuts. However it seems to us that quantitative considerations are much more complicated it cuts are involved so that an unambiguous determination of the Pomeranchuk singularity parameters, based only on the above hypothesis is prevented.

It is a commonly accepted idea that the Pomeranchuk contributions to an elastic process represent the global effect of all inelastic channels coupled to it, and this effect sould came out from implementation of unitarity. Intuitive examples of unitarization of the Veneziano model show how elastic and two body inelastic s and t cuts can be generated[6].

The purpose of this note is to present a model for all the Pomeranchuk terms which gives a quantitative example of the hypothesis of Harari[2]. It provides moreover a useful way to get informations on the Pomeranchon parameters and to describe those phenomena in which this singularity plays an essential role. Our belief is that strong limits are fixed on the form of the representation of that terms, due to the general properties one wishes to be satisfied. Consequently approximate forms can be found, in absence of a complete dynamical theory.

We consider spinless equal mass particle scattering, with no direct channel resonances and with the t and u channels identical. Let's define $\alpha(x) = \alpha(0) + \alpha' x$ with $\alpha(0) = 1$. Small deviations from 1 will be discussed below. The amplitude

$$(1) \qquad A_{pole}(u,t) = -\beta R(u,t) \frac{\Gamma\left[2-\alpha(u)-R(u,t)\right]\Gamma\left[2-\alpha(t)-R(u,t)\right]}{\Gamma\left[1-R(u,t)\right]\Gamma\left[2-\alpha(u)-\alpha(t)-R(u,t)\right]}$$

where

$$(2) \qquad R(u,t) = \frac{\sqrt{\alpha(0)-\alpha(4M^2-t)}+\sqrt{\alpha(0)-\alpha(4M^2-u)}}{\sqrt{\alpha(u+t)-\alpha(0)}} = \frac{\sqrt{4m^2-t}+\sqrt{4m^2-u}}{\sqrt{u+t}}$$

and β is a constant, corresponds to a simultaneous Pomeron exchanges in the t and u channels. The function R(u, t) ensures the vanishing of the imaginary part of the amplitude below the elastic thresholds in all three channels $s = 4m^2$, $t = 4m^2$ and $u = 4m^2$. In fact one can easily verify that the amplitude is real within the triangle in the real (s, t) Mandelstam plane formed by the normal thresholds, that is $s \leq 4m^2$, $t \leq 4m^2$, $4m^2 - s - t = u \leq \leq 4m^2$. By looking at the complex s plane, for fixed t, two branch points are found at $s = 4m^2$ and $s = -t$. However a bad analytical behaviour at $s = 4m^2$ arises from the arguments of the Gamma functions. For that reason in the following we shall multiply R(u, t) by f(R(u, t)) in the argument of the Γ's, where f(R(u, t)) is a real analytic function of R(u, t) such that $R(u,t) \times f(R(u,t))$ is finite in the limit $s \to 4m^2$. Furthermore it can be shown that there are no singularities coming from the Gamma function on the real s-axis. It results therefore a smooth behaviour for physical values of the energy, consistently with duality. As far as complex poles are concerned, which eventually could arise from the Gamma functions, the computation is quite hard, but we hope that they could be placed on the second Rieman sheet, due to the cut-structure in the arguments of the Γ's. All the above considerations hold identically in u and t channels.

The asymptotic behaviour of eq. (1) for t fixed is obtained from ref. (7), by taking into account that:

(3)
$$\lim_{\substack{u \to \pm\infty \\ t \text{ fixed}}} R(u,t) = \pm i$$

and by requiring for the above defined $f(R(u,t))$

(4)
$$\lim_{\substack{u \to \pm\infty \\ t \text{ fixed}}} f(R(u,t)) = 1$$

The results are:

(5a)
$$\lim_{\substack{s \to +\infty \\ t \text{ fixed}}} A_{pole}(u,t) \simeq -\beta(t) \left[\alpha' s\right]^{\alpha(t)} e^{-i\frac{\pi}{2}\alpha(t)}$$

(5b)
$$\lim_{\substack{u \to +\infty \\ t \text{ fixed}}} A_{pole}(u,t) \simeq -\beta^{x}(t) \left[\alpha' u\right]^{\alpha(t)} e^{-i\frac{\pi}{2}\alpha(t)}$$

where $\beta(t)$ is defined as $\beta(t) = \beta(1 - \alpha(t) + i)/i$.
The amplitude is therefore completely imaginary in the forward direction. If possesses however a complex residue function; a fact which underlines, as we would expect, that we are not dealing with a true Regge pole.

The representation (1) can be generalized to take into account the multiple scattering corrections in both t and u channels together with inelastic thresholds cuts.
The result is:

(6)
$$A(u,t) = -\beta \sum_{m,n=1}^{\infty} R_{mn} \frac{\Gamma\left[2 - \alpha_n(u) - R_{mn} f(R_{mn})\right] \Gamma\left[2 - \alpha_m(t) - R_{mn} f(R_{mn})\right]}{\Gamma\left[1 - R_{mn} f(R_{mn})\right] \Gamma\left[2 - \alpha_m(t) - \alpha_n(u) - R_{mn} f(R_{mn})\right]} \times$$
$$\times \frac{1}{mm!} \left\{ - \xi(u,t) \right\}^{m-1} \frac{1}{nn!} \left\{ - \eta(t,u) \right\}^{n-1}$$

where

(7)
$$\alpha_m(x) = m\,(\alpha(0) - 1) + \frac{\alpha' x}{m} + 1 \;,$$

(8) $$R_{mn} \equiv R_{mn}(u,t) = \frac{\sqrt{\alpha(0) - \alpha_m(t-4m^2)} + \sqrt{\alpha(0) - \alpha_n(u-4m^2)}}{\sqrt{mn}\sqrt{\alpha_{m+n-1}(u+t) - \alpha(0)}}$$

(9a) $$\xi(u,t) = \frac{\beta \left| 1 - \alpha_m(t) - R_{mn}\, f(R_{mn}) \right| / \left| R_{11}(u, t=0) \right|}{\lg \left[(1 - \alpha(u))\, R_{11}(u, t=0) \right]}$$

(9b) $$\eta(u,t) = \frac{\beta \left| 1 - \alpha_n(u) - R_{mn}\, f(R_{mn}) \right| / \left| R_{11}(u=0, t) \right|}{\lg \left[(1 - \alpha(t))\, R_{11}(u=0, t) \right]}$$

and the function $f(R_{mn}) \equiv f(R_{mn}(u,t))$ is requested to satisfy properties similar to those of $f(R(u,t))$.

Let us discuss some properties of eq. (6). From eqs. (8) and (9) we find in each channel new branch points corresponding to inelastic thresholds at $u = 4m^2 + n(n-1)(1-\alpha(0))/\alpha'$, $t = \frac{4m^2 + m(m-1)(1-\alpha(0))}{\alpha'}$ and $s = 4m^2 + l(l-1)(1-\alpha(0))/\alpha'$ with $l = m+n-1$, in the limit $\alpha(0) \neq 1$. Similarly in the same limit, from eqs. (9a) and (9b) other branch points appear at $u = (1-\alpha(0))/\alpha'$ and $t = (1-\alpha(0))/\alpha'$, which have to be connected to the annihilations thresholds $t, u = 4m^2$. It follows therefore a very simple connection between the smallness of the pion mass and the deviation from 1 of $\alpha(0)$. For what concerns the singularities in the complex energy plane, identical considerations hold as made above with respect to $A_{pole}(u,t)$, which has to be identified in eq. (6) as the first term ($m=n=1$) of the whole series.

In the limit of very small momentum transfers (see for example at $t=0$) and high energies, the representation (6) is strictly connected to the eikonal approximation(8), due to the fact that multiple scattering corrections in u channel are qui negligible (at $t=0$ they vanish identically). Equation (6) must indeed be considered a crossing symmetric generalization of the simple eikonal approximation, with the zero-th order term possessing a momentum transfer dependence in the residue function.

The large angle limit of eq. (6) can be obtained analytically under some simplifying hypothesis(9). The result, which is consistent with the fixed angle Cerulus-Martin bound(10), is

(10) $$\lim_{\substack{u \to +\infty \\ t \to -\infty}} A(u,t) \simeq C(u,t)\, e^{-c_1(u,t)\sqrt{-t} - c_2(u,t)\sqrt{u}}$$

where $c_1(u,t)$ and $c_2(u,t)$ are very slowly varying functions of the arguments and $C(u,t)$ contains an additional oscillating factor. Very similar results have been obtained on the basic of general thermodynamical assimptions[11].

We shall present now quantitative tests of our model concerning pp and $p\bar{p}$ elastic scattering. Of course, instead of five amplitudes, we are dealing only with the helicity non-flip amplitude, having completely neglected any spin dependence. Let us consider first pp scattering. At non asymptotic energies ordinary trajectories can be exchanged in u and t channels. These have to be degenerate giving no contribution to the forward imaginary amplitude and are represented by a Veneziano formula[12]:

$$A_d(u,t) = \gamma \frac{\Gamma(1-\alpha_d(t))\,\Gamma(1-\alpha_d(u))}{\Gamma(1-\alpha_d(t)-\alpha_d(u))} \qquad (11)$$

where γ is a constant and $\alpha_d(t)$ is the degenerate $\rho - p'$ trajectory.

In the spirit of considering the Pomeranchuk singularity generated by unitarity from ordinary trajectories[6], we shall assume an universal slope $\alpha'_p = \alpha'_\rho = \alpha'$. Furthermore we shall put $\alpha(0) = 1$. The function $f(R_{mn})$ is crudely approximated by $\exp(1 - |R_{mm}|)$. We have in eq. (6) only one free parameter β which is fitted to 3.25 ± 0.25. The results are the following:

i) The differential cross sections $d\sigma/dt$ are shown in Fig. 1. As it can be seen from there, the agreement is very good, in the full range of t and for different values of the laboratory momentum. At high energy ($P_L \sim 20\,GeV$) the corrects structure at $t \approx -1.2\,(GeV/c)^2$ is reproduced. At Serpukhov energies this structure should appear at smaller t values, in a region not yet experimentally explored. The 58.1 GeV measurements at very low t are very well reproduced too. Note that this is achieved by assigning a normal slope to the Pomeranchon, while from the usual Regge pole model one gets $\alpha'_p \approx 0.4$.

ii) In constrast with a pure eikonal approximation our model predicts the right energy dependence of σ^{pp}_{tot}, which decreases slowly to an asymptotic value of about 30 mb. This is shown in Fig. 2. In our model σ^{pp}_{tot} is a slowly varying function of α' and β but is quite sensitive to $\alpha(0)$. Deviations greater than $\sim 2\%$ from 1, are in contrast with experiments.

iii) The ratio $ReA(0)/ImA(0)$ depends at present energies on the contribution of the secondary trajectories which is computed by fitting in eq. (11) γ to 3.5. The result is in agreement with the experimental data. At higher energies $ReA(0)/ImA(0)$ still remains negative.

From all the above data we get, for the Pomeranchuk singularity parameters, $\alpha(0) = 1$ with a 2% uncertainty and $\alpha' = 0.90 \pm 0.05$.

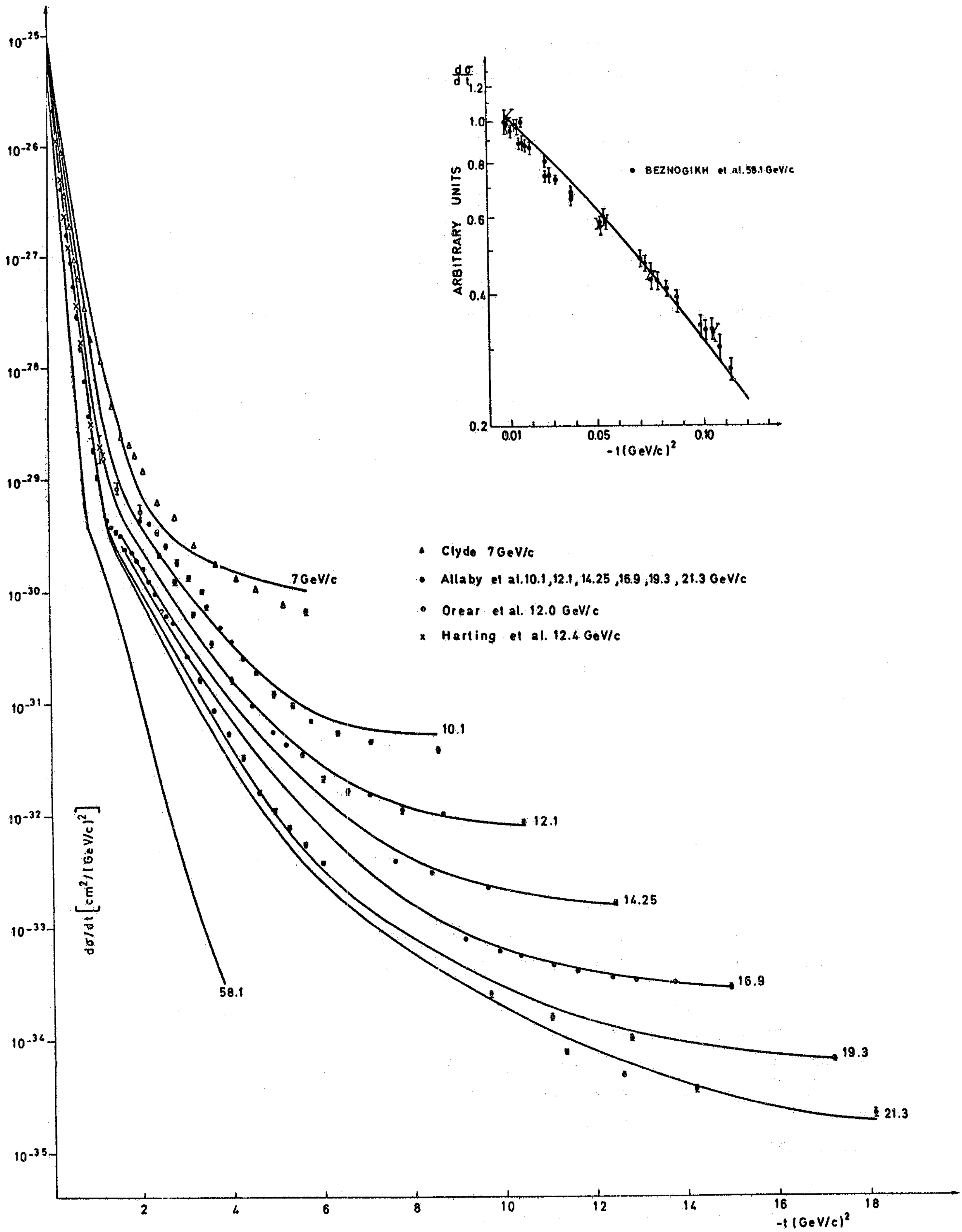

FIG. 1 - pp differential cross sections dσ/dt versus - t. The experimental data are taken from ref. (13). The full lines result from eq. (6).

As for as $p\bar{p}$ scattering is concerned, eq. (11) can no longer be used to represent the contribution of the secondary trajectories, due to the presence of the imaginary part in $\alpha_d(u)$. Nevertheless in order to give a parametrization to this term, the Regge limit of eq. (11) is used:

$$(12) \qquad A_d(u,t) = \gamma\,\Gamma(1-\alpha_d(t))\left[\alpha(u)\right]^{\alpha_d(t)} e^{-i\pi\alpha_d(t)}$$

which gives a completely imaginary contribution in the forward direction. With the parameters fixed to the above values, we find the following results:

i) the total cross section $\sigma^{p\bar{p}}_{tot}$ is shown in Fig. 2. The predictions of our model are in complete agreement with the experimental data, including the higher energy measurements until 50 GeV by the CERN-Serpukhov collaboration.

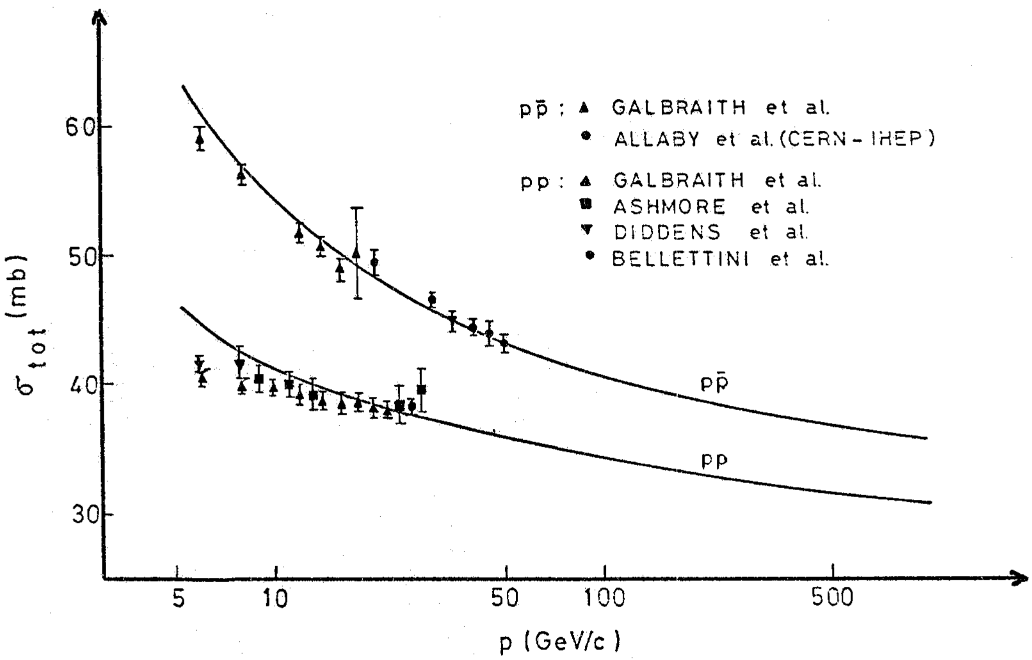

FIG. 2 - pp and $p\bar{p}$ total cross sections versus the incoming particle momentum. p. The experimental data are taken from ref. (14). The full lines result from eq. (6).

ii) The differential cross sections $d\sigma/dt$ are shown in Fig. 3. The gross features of the data are reproduced, in particular the structure at $t \approx -0.8$ and the large angle behaviour. The "dip-bump" structures, which are stronger at low energy, have to be associated with the resonant contributions. They become in fact less pronounced at higher energies (compare for example the recent data at 8 GeV and 16 GeV) where genuine Pomeranchukon effects appear (note the structures at 16 GeV in both pp and $p\bar{p}$).

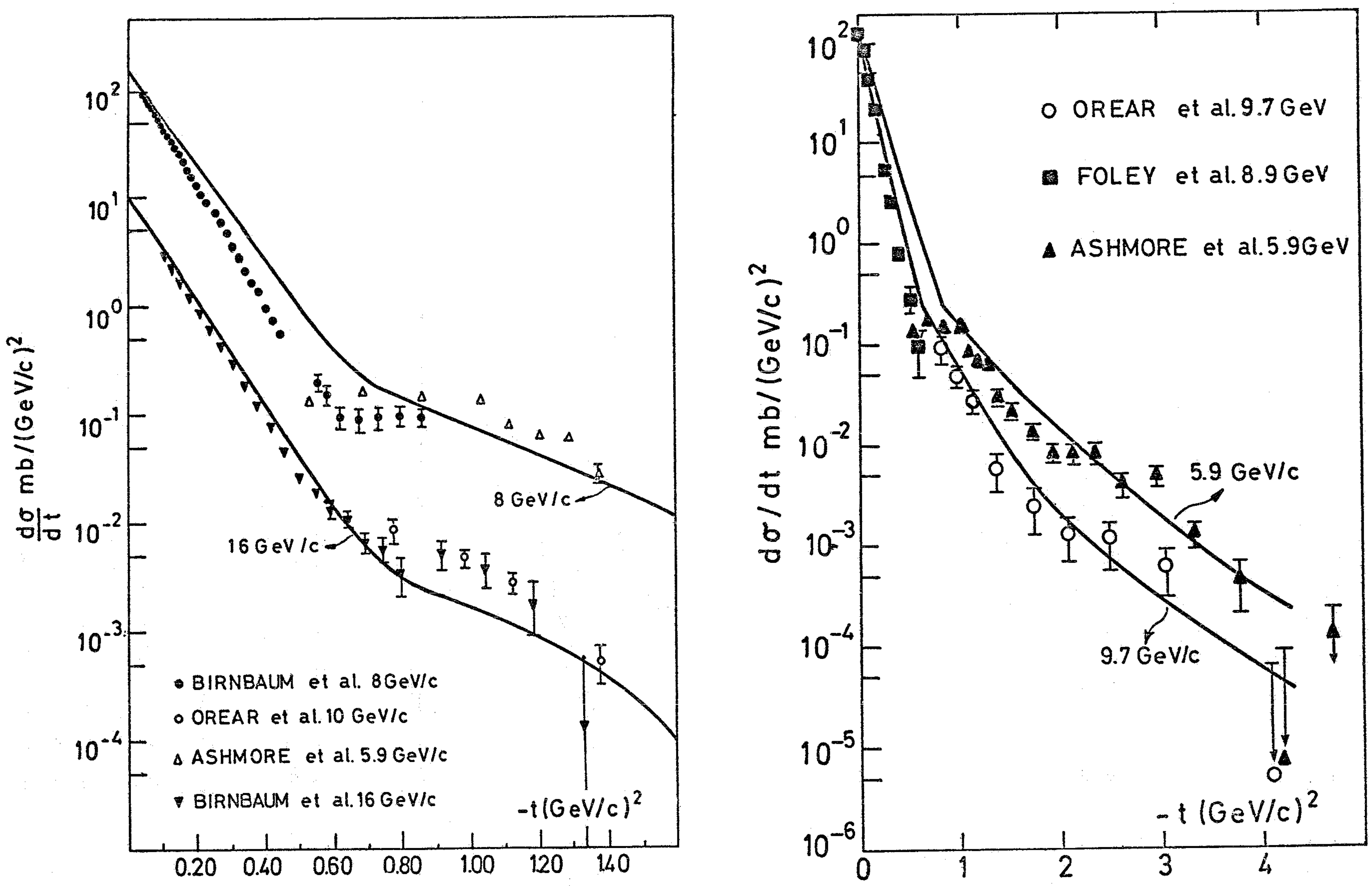

FIG. 3 - $p\bar{p}$ differential cross sections $d\sigma/dt$ versus $-t$. The experimental data are taken from ref. (15). The full lines result from eq. (6).

Our results can be summarized by saying that the representation we have constructed for the Pomeranchuk terms taking into account some general properties of the scattering amplitude, can be used successfully to describe many experimental facts and provides an accurate determination of the Pomeranchuk singularity parameters.

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