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E. Etim and P. Picchi: TEST OF VECTOR MESON DOMINANCE
MODEL IN THE COLLIDING BEAM REACTION $e^+e^- \rightarrow P + \gamma$.

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ABSTRACT. -

The commutation rules of gauge field and quark field algebra are used to derive sum rules for the electromagnetic form factors of the decays $(P \equiv \pi^0, \eta, X^0) \rightarrow \gamma + \gamma$ making use of the Bjorken limit theorem.

Assuming that these sum rules can be saturated by vector meson poles and that the resultant form factors correctly describe the processes $e^+ + e^- \rightarrow P + \gamma$, $P \rightarrow e^+ + e^- + \gamma$, $P \rightarrow \gamma + \gamma$ in their appropriate kinematical regions it is found that the prediction of gauge field algebra (field-current identity) but not that of quark field algebra is in conflict with experiment in the reaction $e^+ + e^- \rightarrow \pi^0 + \gamma$; It is also shown that field-current identity fails to predict correctly the width $\Gamma(\pi^0 \rightarrow 2\gamma)$.

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Apart from the Schwinger terms the important difference between gauge field algebra⁽¹⁾ and quark field algebra⁽²⁾ lies in the commutation rule between space components of the currents. If $j_k^a(x)$, $j_l^b(y)$ (SU(3) indices $a, b = 0, 1, \dots, 8$ and Lorentz indices $k, l = 1, 2, 3$) are two currents densities gauge field algebra (GFA) gives

$$(1a) \quad \left[j_k^a(x), j_l^b(y) \right]_{x_0=y_0} = 0$$

while the quark field algebra (QFA) commutation relation is

$$(1b) \quad \left[j_k^a(x), j_l^b(y) \right]_{x_0=y_0} = i \delta_{kl} f_{abc} j_0^c(x) \delta^{(3)}(\vec{x}-\vec{y}) + \\ + i \epsilon_{klm} d_{abc} j_{5m}^c(x) \delta^{(3)}(\vec{x}-\vec{y})$$

The purpose of this paper is to exhibit an important consequence of the difference between eqs. (1a) and (1b) in the processes $e^+e^- \rightarrow P(\pi^0, \eta, X^0) + \gamma$ and $P \rightarrow \gamma + \gamma$ in the particular case $P \equiv \pi^0$. If $F_p(q^2, k^2)$ is the form factor for the decay

$$P \rightarrow \gamma(q, \mu) + \gamma(k, \nu)$$

with amplitude

$$(2) \quad \langle \gamma(q, \mu); \gamma(k, \nu) | P \rangle = i(2\pi)^4 \delta^{(4)}(p-q-k) (2^3 p_0 q_0 k_0)^{-1/2} \times \\ \times \frac{F_p(q^2, k^2)}{m_0} \epsilon_{\mu\nu\lambda\tau} \epsilon_\mu \epsilon_\nu q_\lambda k_\tau$$

where m_0 is the mass of the neutral pion and $F_p(q^2, k^2)$ defined in terms of the hadronic electromagnetic current $j^{\text{em}}(x)$ by

$$(3) \quad \frac{F_p(q^2, k^2)}{m_0} \cdot \epsilon_{\mu\nu\lambda\tau} q_\lambda k_\tau = i \int d^4x e^{iqx} \langle 0 | T(j_\mu^{\text{em}}(x) j_\nu^{\text{em}}(0)) | P \rangle$$

then the total cross-section for the annihilation $e^+e^- \rightarrow P + \gamma$ is expressed as

$$(4) \quad \sigma(e^+e^- \rightarrow P\gamma) = \frac{\alpha}{24 m_0^2} \left(1 - \frac{m_p^2}{q^2}\right) \left|F_p(q^2)\right|^2$$

and the width for the decay $P \rightarrow \gamma(q^2=0) + \gamma(k^2=0)$ as

$$\Gamma(P \rightarrow 2\gamma) = \frac{m_0}{64\pi} \left(\frac{m_p}{m_0}\right)^3 \left|F_p(0)\right|^2$$

According to the Bjorken theorem⁽³⁾ the asymptotic limit of the time ordered product in eq. (3), as $q_0, k_0 \rightarrow \infty$ and $|\vec{q}|, |\vec{k}|$ finite, is determined by the commutator between the currents i. e.

$$(3') \quad q_0 p_j \epsilon_{ojkl} \frac{F_p(q^2; k^2)}{m_0} \xrightarrow{q_0 \rightarrow \infty} -\frac{1}{q_0} \int d^3\vec{x} e^{-i\vec{q}\cdot\vec{x}} \langle 0 | [j_k^{\text{em}}(0, \vec{x}), j_l^{\text{em}}(0)] | P \rangle$$

The commutator in eq. (3') is easily evaluated once the form of the current $j_\mu^{\text{em}}(\mathbf{x})$ is specified in terms of basic currents of the algebra. Three possibilities are considered:

a) Field-current identity (FCI)⁽⁴⁾: in this case the hadronic electromagnetic current is expressed as a linear combination of the re-normalized fields of the neutral vector mesons

$$(5a) \quad j_\mu^{\text{em}}(\mathbf{x}) = \sum_{V=\rho^0, \omega, \phi} \lambda_V V_\mu(\mathbf{x})$$

$\lambda_V = \frac{g_V^{\text{em}}}{g_V}$ is the γ -V coupling constant and m_V the vector meson mass.

(QGZ)^(5, 6) b) Fractionally charged quark model of Gell-Mann-Zweig

$$(5b) \quad j_\mu^{\text{em}}(\mathbf{x}) = j_\mu^{(3)}(\mathbf{x}) + \frac{1}{\sqrt{3}} j_\mu^{(8)}(\mathbf{x})$$

4.

where j_μ^a ($a=0, 1, \dots, 8$) are the unitary spin currents of the eightfold way.

c) Integrally charged quark model of Han-Nambu (QHN)^(7,8)

$$(5c) \quad j_\mu^{\text{em}}(x) = \sqrt{6} \left[j_\mu^{(3,0)}(x) + \frac{1}{\sqrt{3}} j_\mu^{(8,0)}(x) + \frac{2}{\sqrt{3}} j_\mu^{(0,8)}(x) \right]$$

$j_\mu^{(a,b)}(x)$ ($j_{5\mu}^{(a,b)}(x)$) are generalized $SU(3) \otimes SU(3)$ currents defined in terms of the unitary spin matrices λ_a , charm spin matrices \mathcal{S}_b and the SUB fields $\psi(x)$ ⁽⁹⁾ by

$$j_\mu^{(a,b)}(x) = \bar{\psi}(x) \frac{\lambda_a}{2} \frac{\mathcal{S}_b}{2} \gamma_\mu \psi(x)$$

The linear combination

$$\tilde{j}_{5\mu}^{(a)} = -\sqrt{\frac{2}{3}} j_{5\mu}^{(a,0)} - \frac{4}{\sqrt{3}} j_{5\mu}^{(a,8)}$$

of the axial vector currents will be found useful.

Substituting for $j_\mu^{\text{em}}(x)$ in (3') and making use of the commutation rules

$$(6a) \quad [j_\mu^{\text{em}}(0, \vec{x}), j_\nu^{\text{em}}(0)] = 0$$

$$(6b) \quad [j_\mu^{\text{em}}(0, \vec{x}), j_\nu^{\text{em}}(0)] = -2ie^2 \mathcal{D}^{(3)}(\vec{x}) \varepsilon_{\alpha\mu\nu\lambda} \left[\frac{1}{3} j_{5\lambda}^3 + \frac{1}{3\sqrt{3}} j_{5\lambda}^8 + \frac{2}{9} j_{5\lambda}^0 \right]$$

$$(6c) \quad -2ie^2 \mathcal{D}^{(3)}(\vec{x}) \varepsilon_{\alpha\mu\nu\lambda} \left[\tilde{j}_{5\lambda}^3 + \frac{1}{\sqrt{3}} \tilde{j}_{5\lambda}^8 + \left(\frac{-8}{3} j_{5\lambda}^{(0,0)} + \frac{2\sqrt{2}}{3} j_{5\lambda}^{(0,8)} \right) \right]$$

one finds

$$(3'') \quad q_o p_j \varepsilon_{ojkl} \frac{F_p(q^2, k^2)}{m_o} \xrightarrow{q_o \rightarrow \infty} -S \cdot \frac{2ie^2 \varepsilon_{ojkl}}{q_o} \sum_{P'} N_{P'} \langle 0 | J_5^{(P')} | P \rangle$$

with S a model-dependent parameter equal to zero for GFA and one for QFA and N_p , the appropriate numerical factors multiplying the axial vector currents in eq. (6) having the same SU(3) transformation properties as the corresponding particle P' . The matrix element in eq. (3'') has a traditional form

$$(7) \quad \langle 0 | J_{5j}^{(P')} | P \rangle = i f_{\pi} P_j \delta_{pp'}$$

where f_{π} is the pion weak decay constant.

Substituting from (8) into (3'') we finally get

$$(8) \quad F_p(q^2, k^2) \xrightarrow{q_0 \rightarrow \infty} \frac{2e^2 f_{\pi}}{q_0^2} N_p \cdot S$$

According to field-current identity the form factor $F_p(q^2, k^2)$ is dominated by the vector meson poles in the sense of Gell-Mann-Sharp-Wagner⁽¹⁰⁾ and therefore has the form

$$(9a) \quad F_p(q^2, k^2) = \sum_{v, v'} \frac{\lambda_v \lambda_{v'} g_{vv'p}}{(m_v^2 - q^2)(m_{v'}^2 - k^2)}$$

where $g_{vv'p}$ is the dimensionless coupling constant of the vertex $VV'P$. In the quark model it is usual to assume that $F_p(q^2, k^2)$ satisfies a dispersion relation in each of the variables q^2, k^2 ^(6, 8); depending on the subtractedness assumption one gets

$$(9b) \quad F_p(q^2, k^2) = F_p(0, 0) + \sum_v \frac{\lambda_v f_{vp} \alpha}{m_v^2} \left(\frac{q^2}{m_v^2 - q^2} + \frac{k^2}{m_v^2 - k^2} \right) +$$

$$+ \sum_{v, v'} \frac{\lambda_v \lambda_{v'} g_{vv'p} q^2 k^2}{m_v^2 m_{v'}^2 (m_v^2 - q^2)(m_{v'}^2 - k^2)} + \dots$$

$$(9c) \quad F_p(q^2, k^2) = \sum_v \lambda_v f_{vp}^0 \left(\frac{1}{m_v^2 - q^2} + \frac{1}{m_v^2 - k^2} \right) + \sum_{v, v'} \frac{\lambda_v \lambda_{v'} g_{vv'p}}{(m_v^2 - q^2)(m_{v'}^2 - k^2)} + \dots$$

6.

The dots after the double pole terms in the above equations stand for the higher mass contributions and $f_{vp\gamma}$ the coupling constant of the vertex $VP\gamma$. In (9b) use has already been made of the on-mass shell condition

$$\lim_{\substack{q^2 \rightarrow m_v^2 \\ k^2 \rightarrow 0}} (m_v^2 - q^2) F_p(q^2, k^2) = \lambda_v f_{vp\gamma}$$

which when applied to (9a) and (9b) gives rise to the sum rule

$$(10) \quad f_{vp\gamma} = f_{vp\gamma}^0 \cdot S + \sum_{v'} \frac{\lambda_{v'}}{m_{v'}^2} g_{vv'p}$$

Applying now the Bjorken theorem to eq. (9) and make use of (10) we obtain two other sum rules

$$(11) \quad -2e^2 f_{\pi N_p} \cdot S = \sum_v \lambda_v f_{vp}^0$$

$$F_p(0) = 2 \sum_v \frac{\lambda_v}{m_v^2} f_{vp\gamma}^0 + \sum_{v, v'} \frac{\lambda_v \lambda_{v'} g_{vv'p}}{m_v^2 m_{v'}^2}$$

from which it is clear that $f_{vp\gamma}^0$ represents the $VP\gamma$ coupling constant at infinite momentum.

We consider the special case $P = \pi^0$ and $k^2 = 0$ and evaluate the total cross-section in eq. (4) at $q^2 = m_\omega^2$ neglecting for the moment $\rho - \omega$ interference. From eq. (9)

$$F_\pi(q^2 = m_\omega^2) = F_\pi^{(0)} \left[S - i \left(\frac{\lambda_\omega f_{\omega\pi\gamma}}{m_\omega^2 F_\pi(0)} \right) \frac{m_\omega}{\Gamma_\omega} \right]$$

and from (11) the field-current identity sum rule for $F_\pi(0)$ is

$$(12) \quad F_\pi(0) = \frac{\lambda_\rho}{m_\rho^2} f_{\rho\omega\pi} + \frac{\lambda_\omega}{m_\omega^2} f_{\omega\pi\gamma} = \frac{2\lambda_\omega f_{\omega\pi\gamma}}{m_\omega^2}$$

where in (12) we have made use of the equality between the isoscalar and isovector contributions to $F_{\pi}(0)$ and have neglected the ϕ -contribution. Thus in the field current identity model the ratio $|\lambda_{\omega} f_{\omega \pi \gamma} / m_{\omega}^2 F_{\pi}(0)|$ is simply 0,5; in the quark models considered here^(6,8) the value is 0,81. Making use of the experimental value of $|F_{\pi}(0)| = 3,3 \cdot 10^{-3}$ from $\pi^0 \rightarrow 2\gamma$ decay (11) we find

$$\left. \begin{aligned} (13a) \quad \sigma(e^+e^- \rightarrow \omega \rightarrow \pi^0 \gamma) &= (0,708 \pm 0,13) \cdot 10^{-31} \text{ cm}^2 \\ (13b) \quad \sigma(e^+e^- \rightarrow \omega \rightarrow \pi^0 \gamma) &= 1,86 \cdot 10^{-31} \text{ cm}^2 \end{aligned} \right\}$$

to be compared with the value

$$(13c) \quad \sigma(e^+e^- \rightarrow \omega \rightarrow \pi^0 \gamma) = (1,78 \pm 0,47) \cdot 10^{-31} \text{ cm}^2$$

calculated from the ratio

$$\frac{\sigma(e^+e^- \rightarrow \omega \rightarrow \pi^0 \gamma)}{\sigma(e^+e^- \rightarrow \omega \rightarrow \text{ALL})} = \frac{\Gamma(\omega \rightarrow \pi^0 \gamma)}{\Gamma_{\omega}}$$

with $\Gamma(\omega \rightarrow \pi^0 \gamma) = (1,16 \pm 0,2) \text{ MeV}$, $\Gamma_{\omega} = (11,9 \pm 1,5) \text{ MeV}$ and $\sigma(e^+e^- \rightarrow \omega \rightarrow \text{ALL}) = (1,82 \pm 0,34) \cdot 10^{-30} \text{ cm}^2$ (11,12). Thus from equation (13) the gauge field result is in conflict with experiment while the quark model is definitely favoured. In Fig. 1 the total cross-section for the annihilation $e^+ + e^- \rightarrow \pi^0 + \gamma$ is plotted against the CM energy $2E$ taking into account the ρ^0 -meson contribution. The difference between the field-current identity result (full curve) and those of the quark models is indeed striking.

At this juncture it is important to understand why, using equation (12) it has been claimed that the vector meson dominance prediction for the width $\Gamma(\pi^0 \rightarrow 2\gamma)$ was in better agreement with experiment than any other model. In ref. (13) to calculate $F(0)$ from eq. (12) and from the experimental value of $\Gamma(\omega \rightarrow \pi^0 \gamma)$ the following choice was made

$$(14) \quad \begin{aligned} \lambda_{\omega} &= \frac{0,81}{3} \lambda_{\rho} \\ \frac{g_{\rho}^2}{4\pi} &= 2,6 \end{aligned}$$

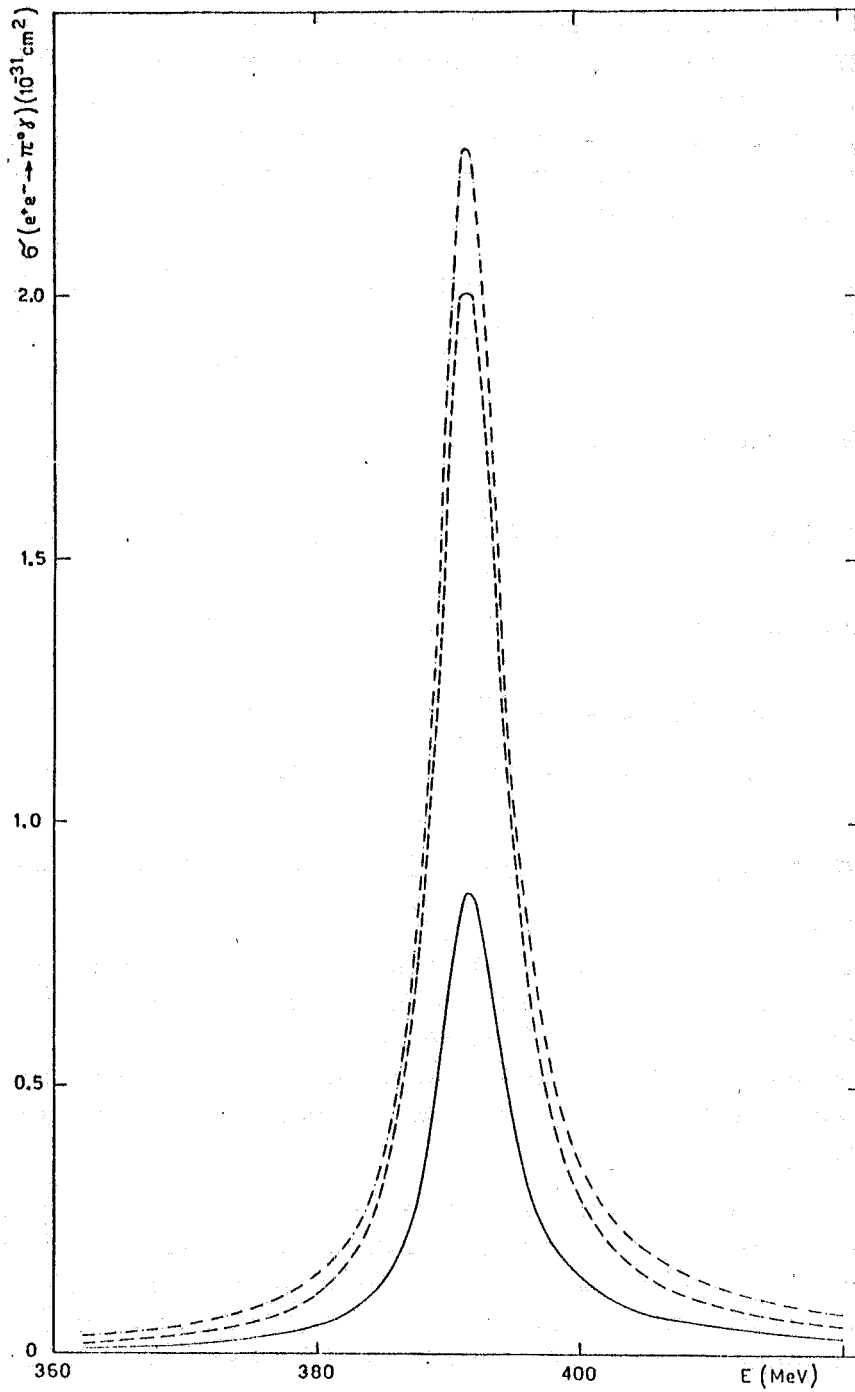


FIG. 1 - Plot of the total cross-section $\sigma(e^+e^- \rightarrow \pi^0 \gamma)$ against $2E$ in the region of the masses of ρ^0 and ω . FCI: full curve; QGZ: dashed curve; QHN: dashed-dotted curve.

Although the choice (14) leads to a reasonable prediction for $|F_{\pi}(0)|^2$ it is in fact self-defeating, for calculating g_{ω} from it gives

$$\frac{g_{\omega}^2}{4\pi} = \alpha \frac{m_{\omega}^4}{\lambda_{\omega}^2} = \frac{g_{\varrho}^2}{4\pi} \left(\frac{3}{0,81}\right)^2 \left(\frac{m_{\omega}}{m_{\varrho}}\right)^4 = 38,2$$

in flagrant disagreement with the experimental value of ⁽¹²⁾

$$\frac{g_{\omega}^2}{4\pi} = 14,8 \pm 2,8$$

Actually the large value of g_{ω} predicted by (14) is not unexpected since (14) is a version of the mixing scheme

$$\frac{m_{\varrho}^2}{g_{\varrho}} = \frac{-\sqrt{3}}{\sin\theta} \frac{m_{\omega}^2}{g_{\omega}} = \frac{\sqrt{3}}{\cos\theta} \frac{m_{\phi}^2}{g_{\phi}}$$

with $\sin\theta = 0,81/\sqrt{3}$ which as well known gives rise to a sum rule

$$\frac{1}{3} m_{\varrho}^3 \Gamma(\varrho \rightarrow e^+e^-) = m_{\omega}^3 \Gamma(\omega \rightarrow e^+e^-) + m_{\phi}^3 \Gamma(\phi \rightarrow e^+e^-)$$

that is badly violated experimentally ⁽¹⁴⁾. We conclude therefore that field-current identity does not make an unambiguous prediction for the decay $\pi^0 \rightarrow 2\gamma$. This conclusion does not change even if one includes the ϕ -contribution in eq. (12).

Doing that would give

$$\frac{1}{2} F_{\pi}(0) = \frac{\lambda_{\omega}}{2} \frac{f_{\omega\pi\gamma}}{m_{\omega}} + \frac{\lambda_{\phi}}{2} \frac{f_{\phi\pi\gamma}}{m_{\phi}}$$

from which one gets ⁽¹⁵⁾

$$\frac{f_{\phi\pi\gamma}}{4\pi} = (0,13 \pm 0,1) \alpha$$

and $\Gamma(\phi \rightarrow \pi^0\gamma)$ between 25 and 900 KeV, values which far exceed the

experimental upper limit of $\Gamma(\phi \rightarrow \pi^0 \gamma) < 15 \text{ KeV}^{(16)}$. If this upper limit is not to be exceeded and the ϕ -contribution still retained to be able to explain the discrepancy in eq. (13a) and Fig. 1 one would need $\Gamma(\pi^0 \rightarrow 2\gamma) > 10 \text{ eV}$ which when compared with the experimental value of $\Gamma(\pi^0 \rightarrow 2\gamma) = (7, 4 \pm 1, 5) \text{ eV}$ is as unsatisfactory as, if not more so than, eq. (13a).

More generally the process $e^+e^- \rightarrow P + \gamma$ can be used as a test of the validities of models of elementary particles. This will soon be possible with the colliding beam facility at Frascati⁽¹⁷⁾. All that is needed for such a test are measurements of cross-sections in the vicinity of the known resonances ρ^0, ω, ϕ since these cross-sections are sensitive to the structure of the form factor

$$(15) \quad F_P(q^2) = S \cdot \sum_V \frac{\lambda_V}{m_V^2} f_{V\gamma}^0 + \sum_V \frac{\lambda_V^f}{m_V^2 - q^2} f_{V\gamma}^f$$

containing the model dependent parameters S and $f_{V\gamma}^0, f_{V\gamma}^f$ is fixed by the radiative decays $V \rightarrow P + \gamma$ of the vector mesons. As in the case of $\pi^0 \rightarrow 2\gamma$ decay discussed above these colliding beam tests will also serve to establish which models predict correctly the $P \rightarrow 2\gamma$ width. For instance taking into account η, X^0 mixing, with mixing angle θ_p , one has

$$F_\eta(0) = \sin \theta_p F_{\eta_1}(0) + \cos \theta_p F_{\eta_8}(0)$$

$$F_{X^0}(0) = -\sin \theta_p F_{\eta_8}(0) + \cos \theta_p F_{\eta_1}(0)$$

In a vector dominance model⁽¹⁸⁾ the ratio $|F_{\eta_1}(0)/F_{\eta_8}(0)| = 7, 6$ which is about twice the value given by the quark model of Han-Nambu⁽⁸⁾ $|F_{\eta_1}(0)/F_{\eta_8}(0)| = 3, 58$. Since, even with such a large difference between model predictions, the present experimental errors and the fact that in many cases only upper limits exist do not allow a definitive statement to be made regarding the validities of particle models using the decays $P \rightarrow 2\gamma$, the proposed tests with storage rings will turn out very useful in this respect. One feature of such tests is the possibility of detecting any asymmetry in the form of the resonance curve about the resonant mass due to the presence in eq. (15) of the real infinite momentum background term proportional to S . The asymmetry, measured by the ratio

$$A(x) = \frac{\sigma(m_V + x)}{\sigma(m_V - x)}$$

also depends on the relative sign between $F_p(0)$ and $\lambda_{\nu} f_{\nu p} \gamma / m_{\nu}^2$. For instance from Fig. 1 and negative relative sign we find $A(x = \sqrt{\omega}) = 40\%$ for the quark model of Han-Nambu⁽⁸⁾.

Such an asymmetry can be detected using Adone.

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