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Minimal Solutions to the Conspiracy Problem and Classification of Regge-Pole Families. - II

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Summary. — A non group-theoretical approach to the conspiracy problem, based on analyticity, crossing symmetry and factorization, is presented. The solutions to all the physically interesting cases are given. A classification of Regge-pole families is deduced.

1. - Introduction.

During the last few years the properties of the scattering amplitudes near $t=0$ have been extensively studied in the framework of the Regge-pole model following two different ways: the group-theoretical approach and the analytic approach.

The first one is based on the invariance of the scattering amplitude under the group $O_{3,1}$ or O_4 in the pairwise equal-mass configuration. In fact Toller (1-3) reggeized expansions of amplitudes in terms of the representations of the group $O_{3,1}$; the simpler compact group O_4 was used later by Freedman and Wang (4) and by Domokos (5). The Reggeization of these expansions shows that a Toller pole, that is a pole in the « four-dimensional angular-momentum plane », leads to an infinite family of Regge poles, with definite relations between the trajectories and the residue functions at $t=0$. The families of Regge poles

(1) M. Toller: *Nuovo Cimento*, **37**, 631 (1965); University of Rome reports n. 76 (1965) and n. 84 (1966) (unpublished).
(2) A. Sciarrino and M. Toller: *Journ. Math. Phys.*, **7**, 1670 (1967).
(3) M. Toller: *Nuovo Cimento*, **53** A, 671 (1968); **54** A, 295 (1968).
(4) D. Z. Freedman and J. M. Wang: *Phys. Rev.*, **160**, 1560 (1967).
(5) G. Domokos: *Phys. Lett.*, **24** B, 293 (1967); **19**, 137 (1967).

so generated are characterized, apart from the internal quantum numbers, by a Toller quantum number M, which, for boson trajectories, can take all the integer values.

Since this formalism applies rigorously only at the point $t = 0$ and for equal-mass scattering, more general formalisms have been developed in order to overcome these limitations (6,7). The most general work in this direction has been done by COSENZA, SCIARRINO and TOLLER (8,9) by a generalization of the Lorentz-group formalism and the introduction of quite strong assumptions. This formalism permitted them to construct a large class of families of conspiring Regge trajectories, which satisfy the factorization constraints for an arbitrary number of two-body reactions involving particles with arbitrary masses and spins. The contribution of one of these families of trajectories satisfies all the constraints which can be derived from the analytic properties of the amplitude.

The analytic approach on the other hand is based on the usual assumptions of the S-matrix theory, which, adapted to the Regge-pole theory, permit us to understand the properties of the scattering amplitude near $t = 0$ for any mass configuration. The assumptions made in the analytic approach are the following: *a*) analyticity; *b*) simplicity; *c*) crossing symmetry and; *d*) factorization. Some comment to these properties of the scattering amplitude adapted to the Regge-pole model can be found in ref. (10). The above assumptions require, for every parent trajectory exchanged, the exchange of an infinite family of Regge poles (daughter trajectories), with well-defined quantum numbers with respect to the parent pole (11). The residues and the trajectories of the members of the family are strictly related near $t = 0$ through some analyticity requirements (10). In particular the assumptions *a*)-*d*) permitted these authors to evaluate the contribution of a Regge-pole family to the scattering amplitude at $t = 0$ in the equal-mass configuration and to show in this particular case the complete equivalence between the group-theoretical and the analytic approaches (10,12-14).

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(6) R. DELBOURGO, A. SALAM and J. STRATHDEE: *Phys. Lett.*, **25** B, 230 (1967); *Phys. Rev.*, **164**, 1981 (1967).

(7) G. DOMOKOS and G. L. TINDLE: *Phys. Rev.*, **165**, 1906 (1968).

(8) G. COSENZA, A. SCIARRINO and M. TOLLER: *Phys. Lett.*, **27** B, 398 (1968).

(9) G. COSENZA, A. SCIARRINO and M. TOLLER: University of Rome report n. 158 (1968); CERN preprint TH. 906 (1968).

(10) P. DI VECCHIA and F. DRAGO: *Phys. Rev.*, **178**, 2329 (1969).

(11) D. Z. FREEDMAN and J. M. WANG: *Phys. Rev.*, **160**, 1560 (1967).

(12) P. DI VECCHIA and F. DRAGO: *Phys. Lett.*, **27** B, 387 (1968).

(13) J. B. BRONZAN and C. E. JONES: *Phys. Rev. Lett.*, **21**, 564 (1968).

(14) J. B. BRONZAN: *Daughter sequences in unequal mass vector meson-scalar meson scattering*, Massachussets Institute of Technology preprint, November (1968).

Other relations between the parameters of the various members of the family, like the « mass formulae », have been then evaluated [12,15-16].

In this paper we consider the series of reactions:

$$(1.1)\qquad \begin{cases} S+\mathcal{N}\to J+\mathcal{N}\,, \\ \mathcal{N}+\mathcal{N}\to \mathcal{N}+\mathcal{N}\,, \\ S+S\to J+J\,, \end{cases}$$

where $\mathcal{N}$ is a nucleon and J (S) is a spin-J (S) and mass-m_J (m_S) particle with $m_J \neq m_S \neq$ nucleon mass.

Then we evaluate the « minimal » behaviour of the Regge-pole residue functions satisfying the assumptions *a*)-*d*). In this way it is possible to define a quantum number M and to classify the Regge poles in families with well-defined quantum numbers. This classification is equivalent to the group-theoretical one; moreover the $t=0$ behaviour of the factorized Regge-pole residue functions, satisfying all the kinematical constraints, is the same as that derived in ref. [8,9] by the group-theoretical works.

In Sect. **2** we write down, in terms of the helicity amplitudes, the kinematical constraints at $t=0$ for the reactions of the type (1.1). The details of the calculation are given in Appendix A and B.

In Sect. **3** we write down the factorization requirements near $t=0$ for the Regge-pole residue functions free from any kinematical singularity.

The « minimal » solutions of the factorization requirements and the constraint equations are given in Sect. **4**, where our results are compared with those obtained by CAPELLA, CONTOGOURIS and TRAN THAUH VAN [17], and COSENZA, SCIARRINO and TOLLER [8,9].

Finally in Sect. **5** we show how the introduction of the quantum number M permits us to classify the Regge poles in families with well-defined quantum numbers.

In Appendix C we determine the singularity structure near $t=0$ of the residues of the daughter trajectories in the various helicity amplitudes.

A short account of this work has been published elsewhere [18].

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[15] P. DI VECCHIA and F. DRAGO: *Nuovo Cimento*, **61** A, 421 (1969).
[16] J. B. BRONZAN, C. E. JONES and P. K. KUO: *Phys. Rev.*, **175**, 2200 (1968).
[17] A. CAPELLA, A. P. CONTOGOURIS and J. TRAN THANH VAN: *Phys. Rev.*, **175**, 1892 (1968).
[18] P. DI VECCHIA, F. DRAGO and M. L. PACIELLO: *Nuovo Cimento*, **56** A, 1185 (1968).

2. – Conspiracy relations.

We shall use the customary notation $f^t_{cc;ab}$ to denote a helicity amplitude [19] for the t-channel reaction $a+b \to c+d$. Helicity amplitudes free from kinematical singularities in s and t must be used in the derivation of the constraints. The first step is to define amplitudes free from kinematical singularities in s [20]:

$$\bar{f}^t_{cd;ab} = \left(\sqrt{2}\sin\frac{\theta_t}{2}\right)^{-|\lambda-\mu|}\left(\sqrt{2}\cos\frac{\theta_t}{2}\right)^{-|\lambda+\mu|} f^t_{cd;ab}\,, \tag{2.1}$$

where $\lambda = a-b$, $\mu = c-d$ and θ_t is the scattering angle in the centre-of-mass frame of the t-channel.

The works by HARA [21] and WANG [22] show how one can then remove the t kinematical singularities from the parity-conserving helicity amplitudes [20] $f^{t\pm}_{cd;ab}$ free from s kinematical singularities. The Wang result can be written:

$$\bar{f}^{t\pm}_{cd;ab} = K^{\pm}_{cd;ab}(t)\,\tilde{f}^{t\pm}_{cd;ab}\,, \tag{2.2}$$

where $K^{\pm}_{cd;ab}(t)$ is a known factor containing the kinematical singularities at $t=0$.

However the analyticity requirements and the crossing symmetry will provide additional kinematical zeros at $t=0$ in certain linear combinations of the parity-conserving helicity amplitudes. We give now the kinematical constraints at $t=0$ for the various cases in our discussion:

1) EU case (*i.e.* $\mathrm{S}+\mathcal{N}\to \mathrm{J}+\mathcal{N}$). The constraints turn out to be

$$i\tilde{f}^{(+)t}_{cd;\frac{1}{2}-\frac{1}{2}} - \tilde{f}^{(-)t}_{cd;\frac{1}{2}\frac{1}{2}} = O(t) \tag{2.3}$$

for any c and d satisfying the inequality $c \neq d$, and

$$i\tilde{f}^{(-)t}_{cc;\frac{1}{2}-\frac{1}{2}} - \tilde{f}^{(-)t}_{cc;\frac{1}{2}\frac{1}{2}} = O(t) \tag{2.4}$$

for any c (*). The details of the derivation of these constraints are given in

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(19) M. JACOB and G. C. WICK: *Ann. of Phys.*, **7**, 404 (1959).

(20) M. GELL-MANN, M. L. GOLDBERGER, F. E. LOW, E. MARX and F. ZACHARIASEN: *Phys. Rev.*, **133** B, 145 (1964).

(21) Y. HARA: *Phys. Rev.*, **136** B, 507 (1964).

(22) L. L. WANG: *Phys. Rev.*, **142**, 1187 (1966); **153**, 1664 (1967).

(*) The factor $\frac{1}{2}$, which was present in the constraints (2.4) in ref. (18), has been eliminated because of the $\sqrt{2}$ factor in the definition (2.1) of the helicity amplitudes free s kinematical singularities. The constraints EU have been derived independently by J. D. STACK (23).

(23) J. D. STACK: *Phys. Rev.*, **171**, 1666 (1968).

Appendix A. The number of the independent constraints (2.3) is given by $J(2S+1)$, while the number of these (2.4) is given by $(2S+u)/2$ with

$$u\left\{\begin{array}{ll} =0\,, & \text{if } \sigma_J\sigma_S=1\,,\\ =1\,, & \text{if } \sigma_J\sigma_S=-1\,,\end{array}\right. \tag{2.5}$$

where $\sigma_J=\eta_J(-1)^J$ and η_J is the intrinsic parity of the particle J. We supposed above that $J\geqslant S$.

2) UU case (*i.e.* S+S → J+J). The constraints are

$$\tilde{f}^{(+)t}_{cd;ab}+\tilde{f}^{(-)t}_{cd;ab}=O(t^m) \tag{2.6}$$

if $|\lambda-\mu|<|\lambda+\mu|$, and

$$\tilde{f}^{(+)t}_{cd;ab}-\tilde{f}^{(-)t}_{cd;ab}=O(t^m) \tag{2.7}$$

if $|\lambda-\mu|>|\lambda+\mu|$, where $m=\text{minimum}\,(|\lambda|,|\mu|)$. The derivation of these constraints is presented in Appendix B.

3) EE case (*i.e.* $\mathcal{N}+\mathcal{N}\to\mathcal{N}+\mathcal{N}$). In fact in the simplified treatment given here we consider only the nucleon-nucleon scattering in the equal-mass configuration. In this case the constraint is well known ([23]):

$$\tilde{f}^{(-)t}_{\frac12\frac12;\frac12\frac12}-z\tilde{f}^{(+)}_{\frac12-\frac12;\frac12-\frac12}-\tilde{f}^{(-)}_{\frac12-\frac12;\frac12-\frac12}=O(t)\,. \tag{2.8}$$

The derivation can be found in ref. ([24]).

If only Regge poles contribute to the scattering amplitudes, these relations can be satisfied in either of two ways:

1) Each of the two amplitudes involved in an equation has an additional factor of t in the residue: in this case the relation is trivially satisfied (evasion).

2) The amplitudes f involved in the kinematical constraints go to a constant when $t\to0$; this implies a relation between the intercepts $\alpha(0)$ and the residues $\beta(0)$ of the various trajectories which contribute to the f (conspiracy). In the following we will see that the mechanism chosen by the Regge-pole families to satisfy the constraints will enable us to derive a classification of the Regge-pole families.

([24]) M. L. Goldberger, M. T. Grisaru, S. W. Mac Dowell and D. Y. Wong: *Phys. Rev.*, **120**, 2250 (1960).

3. – The Regge residue factorization near $t=0$.

The partial wave expansion for the parity conserving helicity amplitudes, free from S kinematical singularities is given by [20]

$$\bar{f}^{(\pm)t}_{cd;ab} = \sum_J (2J+1)[e^{J+}_{\lambda\mu} F^{J(\pm)}_{cd;ab} + e^{J-}_{\lambda\mu} F^{J(\mp)}_{cd;ab}], \tag{3.1}$$

where the $e^{J\pm}_{\lambda\mu}$ functions are defined in ref. [20].

The contribution of a Regge trajectory $\alpha^{\pm}$ to the previous amplitudes is given by

$$\bar{f}^{(\pm)t}_{cd;ab} \simeq \frac{2\alpha^{\pm}(t)+1}{\sin\alpha^{\pm}(t)} \beta^{\pm}_{cd;ab}(t)\, E^{+\alpha^{\pm}}_{\lambda\mu}(t)\,(-\cos\theta_t) + \\ + \frac{2\alpha^{\mp}(t)+1}{\sin\alpha^{\pm}(t)} \beta^{\mp}_{cd;ab}(t)\, E^{-\alpha^{\mp}}_{\lambda\mu}(\cos\theta_t), \tag{3.2}$$

where the functions $E^{\pm}_{\lambda\mu}$ can be found in ref. [20]. Here and in the following we omit the signature factor which is not essential in our considerations.

Because of the conservation of the angular momentum, parity, G-parity and isotopic spin in strong interaction physics we can label the single Regge poles by the quantum numbers τ (signature), P (parity), $\sigma = P\tau$ (normality) and $\xi = G(-1)^T$ where T is the isospin of the Regge trajectory and G its G-parity. However the analyticity requires [11], for every parent Regge pole exchanged in unequal-mass reactions the exchange of an infinite family of Regge trajectories with residues conveniently singular near $t=0$ (daughter trajectories). If we label with n the quantum numbers of the n-th daughter trajectory, there exists the following relation between them and those of the parent:

$$\sigma_n = \sigma, \qquad P_n = (-1)^n P, \qquad \tau_n = (-1)^n \tau, \qquad \xi_n = \xi, \tag{3.3}$$

where the quantum numbers without subscript refer to the parent trajectory. Therefore a Regge pole can be labelled by a further quantum number n, which characterizes the behaviour of its residue function near $t=0$ in the unequal-mass scattering. Using the above considerations it is then possible to achieve the classification of Regge trajectories shown in Table I. The classes 3 and 4 do not couple to the nucleon-antinucleon system because of the conservation of angular momentum, parity and G-parity. In the following we will determine the further zeros of the residue functions which are necessary to satisfy the factorization requirements and the constraints and we will get a more com-

TABLE I.

1	$\sigma = +$	$P = \xi$	n even
2	$\sigma = +$	$P = -\xi$	n odd
3	$\sigma = +$	$P = -\xi$	n even
4	$\sigma = +$	$P = \xi$	n odd
5	$\sigma = -$	$P = \xi$	n even
6	$\sigma = -$	$P = -\xi$	n odd
7	$\sigma = -$	$P = -\xi$	n even
8	$\sigma = -$	$P = \xi$	n odd

plete classification of the Regge trajectories in families with a well-defined new quantum number M.

Now we impose the factorization conditions for the residue functions of the Regge poles.

The objects which factorize are the residues of the individual poles in $F^{J\pm}_{cd;ab}$; therefore in our case the residue is

$$\beta^{\pm}_{cd;ab}(t) = K^{\pm}_{cd;ab}(t)(P_{cd}P_{ab})^{\alpha^{\pm}-N} g^{\pm}_{cd;ab}(t)\,\gamma^{\pm}_{cd;ab}(t)\,, \tag{3.4}$$

where $(P_{cd}P_{ab})^{\alpha^{\pm}-N}$ is the threshold factor, $N = \text{minimum}\,(|\lambda|, |\mu|)$, $g_{cd;ab}(t)$ is a factor which takes account of the further singularities present in the residues of the daughter trajectories near $t = 0$ and $\gamma^{\pm}_{cd;ab}(t)$ is the reduced residue free from any kinematical singularity at $t = 0$: it can contain at $t = 0$ only eventual zeros. Hence if reaction 1 is $a + b \to c + d$, reaction 2 is $c + d \to c + d$ and reaction 3 is $a + b \to a + b$ the factorization theorem can be stated as [25]:

$$\begin{aligned}(3.5)\qquad &[K^{\pm}_{1cd;ab}(t)\gamma^{\pm}_{1cd;ab}(t)(P_{cd;}P_{ab})^{\alpha^{\pm}\pm N_1} g_{1cd;ab}(t)]^2 = \\ &= [K^{\pm}_{2cd;cd}(t)\gamma^{\pm}_{2cd;cd}(P_{cd})^{2(\alpha^{\pm}-N_2)} g_{2cd;cd}(t)]\,[K^{\pm}_{3ab;ab}(t)\gamma^{\pm}_{3ab;ab}(P_{ab})^{2(\alpha^{\pm}-N_3)} g_{3ab;ab}(t)]\,.\end{aligned}$$

In Tables II and III we give the behaviour of the factor $K^{\pm}_{cd:ab}(t)$ and $g_{cd;ab}(t)$ for the reactions (1.1), while the behaviour near $t = 0$ of both sides of eq .(3.5) is given in Table IVa) for the reaction $\mathrm{S}+\mathrm{S} \to \mathrm{J}+\mathrm{J}$.

If an equal-mass vertex is present in the reaction considered, then the conservation laws imply the identical vanishing of the residue if the following conditions are not satisfied:

$$(3.6)\qquad \begin{cases} \sigma\xi\tau_n(-1)^{S+1} = 1\,, & \\ S = 1\,, & \text{if } \sigma = +1\,, \end{cases}$$

[25] F. ARBAB and J. D. JACKSON: *Phys. Rev.*, **176**, 1796 (1968).

TABLE II *a*).

	Amplitude $\bar{\mathcal{N}}\mathcal{N} \to \mathrm{JS}$	$\lvert\lambda\rvert$	$\lvert\mu\rvert$	Behaviour near $t=0$ $K_1(t)$
a)	$\bar{f}^{t+}_{C'A';\frac{1}{2}-\frac{1}{2}}$	1	$\neq 0$	$t^{-\frac{1}{2}}$
b)	$\bar{f}^{t-}_{C'A';\frac{1}{2}\frac{1}{2}}$	0	$\neq 0$	$t^{-\frac{1}{2}}$
c)	$\bar{f}^{t-}_{C'A';\frac{1}{2}-\frac{1}{2}}$	1	0	$t^{-\frac{1}{2}}$
d)	$\bar{f}^{t-}_{C'A';\frac{1}{2}\frac{1}{2}}$	0	0	$t^{-\frac{1}{2}}$
e)	$\bar{f}^{t-}_{C'A';\frac{1}{2}-\frac{1}{2}}$	1	$\neq 0$	1
f)	$\bar{f}^{t+}_{C'A';\frac{1}{2}\frac{1}{2}}$	0	$\neq 0$	1
g)	$\bar{f}^{t+}_{C'A';\frac{1}{2}-\frac{1}{2}}$	1	0	1
h)	$\bar{f}^{t+}_{C'A';\frac{1}{2}\frac{1}{2}}$	0	0	1

TABLE II *b*).

Amplitude $\bar{\mathcal{N}}\mathcal{N} \to \bar{\mathcal{N}}\mathcal{N}$	$\lvert\lambda-\mu\rvert$	Behaviour near $t=0$ $K_2(t)$
$\bar{f}^{-t\pm}_{\mu;\lambda}$	even	1
	odd	t

TABLE II *c*).

Amplitude JS → JS	Behaviour near $t=0$ $K_3(t)$
$\bar{f}^{t\pm}_{\lambda;\mu}$	$t^{-\frac{1}{2}N}$ $N = \max(\lvert\lambda+\mu\rvert; \lvert\lambda-\mu\rvert)$

where the quantum numbers in eq. (3.6) refer to the Regge pole exchanged and S is the total spin in the $\mathcal{N}\mathcal{N}$ system. For the residues involving equal-mass vertices, which to not vanish, we give the factorization requirements in Table IV*b*) and IV*c*).

TABLE III.

Amplitude	Singularity of the residue of the n-th daughter
$(\gamma^{\pm}_{cd;ab})^{\mathrm{UU}}$	$1/t^n$
$(\gamma^{+}_{cd;ab})^{\mathrm{EU}}$	$(1/t)^{n/2}$ if $(-1)^n = 1$ $(1/t)^{(n-1)/2}$ if $(-1)^n = -1$
$(\gamma^{+}_{cd;ab})^{\mathrm{EU}}$ minimum $(\lvert\lambda\rvert, \lvert\mu\rvert) \neq 0$	$(1/t)^{n/2}$ if $(-1)^n = 1$ $(1/t)^{(n-1)/2}$ if $(-1)^n = -1$
$(\gamma^{-}_{cd;ab})^{\mathrm{EU}}$ minimum $(\lvert\lambda\rvert, \lvert\mu\rvert) \neq 0$	$(1/t)^{n/2}$ if $(-1)^n = 1$ $(1/t^{(n+1)/2})$ if $(-1)^n = -1$

TABLE IV a).

Factorization conditions in JS $\to$ JS
$(\gamma^{\pm}_{\lambda;\mu})^2\, t^{\lvert\lvert\mu\rvert-\lvert\lambda\rvert\rvert} = (\gamma^{\pm}_{\mu;\mu})(\gamma^{\pm}_{\lambda;\lambda})$

TABLE IV b).

Factorization conditions in $\bar{\mathcal{N}}\mathcal{N} \to \bar{\mathcal{N}}\mathcal{N}$	$\lvert\lambda-\mu\rvert$
$(\gamma^{\pm}_{\mu;\lambda})^2 = (\gamma^{\pm}_{\mu;\mu})(\gamma^{\pm}_{\lambda;\lambda})$	even
$(\gamma^{\pm}_{\mu;\lambda})^2\, t = (\gamma^{\pm}_{\mu;\mu})(\gamma^{\pm}_{\lambda;\lambda})$	odd

TABLE IV c).

	Factorization conditions in $\bar{\mathcal{N}}\mathcal{N} \to$ JS
a)	$(\gamma^{+}_{cd;\frac12-\frac12})^2\, t^{\lvert\mu\rvert-(1/2)[1+(-1)^n]} = (\gamma^{+}_{cd;cd})(\gamma^{+}_{\frac12-\frac12;\frac12-\frac12})$
b)	$(\gamma^{-}_{cd;\frac12\frac12})^2\, t^{\lvert\mu\rvert-(1/2)[1+(-1)^n]} = (\gamma^{-}_{cd;cd})(\gamma^{-}_{\frac12\frac12;\frac12\frac12})$
c)	$(\gamma^{-}_{cd;\frac12-\frac12})^2\, t^{(1/2)[1-(-1)^n]} = (\gamma^{-}_{cd;cd})(\gamma^{-}_{\frac12-\frac12;\frac12-\frac12})$
d)	$(\gamma^{-}_{cd;\frac12\frac12})^2\, t^{-(1/2)[1+(-1)^n]} = (\gamma^{-}_{cd;cd})(\gamma^{-}_{\frac12\frac12;\frac12\frac12})$
e)	$(\gamma^{-}_{cd;\frac12-\frac12})^2\, t^{-(1/2)[1+(-1)^n]} = (\gamma^{-}_{cd;cd})(\gamma^{-}_{\frac12-\frac12;\frac12-\frac12})$
f)	$(\gamma^{+}_{cd;\frac12\frac12})^2\, t^{\lvert\mu\rvert+(1/2)[1-(-1)^n]} = (\gamma^{+}_{cd;cd})(\gamma^{+}_{\frac12\frac12,\frac12\frac12})$
g)	$(\gamma^{+}_{cd;\frac12-\frac12})^2\, t^{1+(1/2)[1-(-1)^n]} = (\gamma^{+}_{cd;cd})(\gamma^{+}_{\frac12-\frac12;\frac12-\frac12})$
h)	$(\gamma^{+}_{cd;\frac12\frac12})^2\, t^{1/2[1-(-1)^n]} = (\gamma^{+}_{cd;cd})(\gamma^{+}_{\frac12\frac12;\frac12\frac12})$

4. – Minimal solutions of the conspiracy problem.

In this Section, under the assumption that only moving poles are present in the complex J-plane, we give the behaviour of the Regge-pole residues near $t=0$ allowed by the kinematical constraints and the factorization theorem.

In general, from a given solution to all the factorization requirements, one can obtain solutions by increasing the number of t powers of some of the Regge-pole residues in the original solution. If a given solution cannot be obtained from another in this way, we will call it minimal. It is important to note that in general the minimality of a solution is strictly related to the set of reactions in which it is minimal. In fact if we consider two sets of reactions $\{A\}$ and $\{B\}$ a minimal solution in the reactions $\{B\}$ may be not minimal in the set of reactions $\{A\}$.

This means that even if some of the solutions given below are not minimal where looked at from the point of view of a particular reaction, they are minimal in the set of reactions (1.1).

As we said in Sect. **3**, on the basis of general analyticity properties the residues γ can contain only additional zeros at $t=0$. Therefore if the factorization requires that $\gamma \sim t^a$, a is constrained to be a positive integer number greater than or equal to 0.

Taking into account this restriction it is easy to find the minimal solutions of the factorization requirements listed in Table IVa), b), c). For the reactions with unequal masses, of the type $S+S \to J+J$ or $S+S' \to J+J'$, one finds

$$\gamma^{\sigma}_{cd;cd} \sim t^{||\mu|-M|} \tag{4.1}$$

for any class of Regge poles listed in Table I (*).

We can include into the set of reactions (1.1) also processes of the type of $S+S' \to J+J'$ because for them the factorization requirements are identical to those listed in Table IVa). The result (6.1) is coincident with that obtained by LE BELLAC using only UU scattering reactions [26].

When an equal-mass vertex is involved, the selection rules due to parity and G-parity invariance must be taken into account. This implies the identical vanishing of the residue if the conditions (3.16) are not satisfied.

In the equal-mass reaction $\mathcal{N}+\mathcal{N} \to \mathcal{N}+\mathcal{N}$, for the residues which do not

(*) The behaviour given above is different from that reported in ref. [18] because there the singularity structure at $t=0$ of the residues of the daughter trajectories was included in γ.

[26] M. LE BELLAC: *Nuovo Cimento*, **55** A, 318 (1968).

vanish we have

$$\gamma^{\sigma}_{ab;ab} \sim \begin{cases} t^{M}\,, & \text{if } (-1)^{\lambda+n} = \sigma\,, \\ t^{|M-1|}\,, & \text{if } (-1)^{\lambda+n} = -\sigma\,. \end{cases} \tag{4.2}$$

For the families with $\sigma = +$ and $\tau = -\xi$, whose parent and even daughter trajectories do not couple to the $\mathcal{N}\bar{\mathcal{N}}$ system, we have for the odd daughters

$$\gamma^{(+)}_{ab;ab} \sim \begin{cases} t^{M+1}\,, & \text{if } \lambda = 0\,, \\ t^{|M-1|+1}\,, & \text{if } |\lambda| = 1\,. \end{cases} \tag{4.3}$$

Using the factorization conditions given in Table IV*c*) it is then easy to obtain the behaviour of the EU residues (Table V). The minimal solutions (4.1), (4.3), (4.4) and Table V of the factorization equations are consistent with the constraints given in Sect. **2**; such solutions are therefore the minimal solutions of the conspiracy problem.

TABLE V.

	Behaviour of the EU residues near $t = 0$
a)	$\gamma^{+}_{cd;\frac{1}{2}-\frac{1}{2}} \sim t^{(1/2)\{\lvert\lvert\mu\rvert - M\rvert + 1 + \lvert M-1\rvert - \lvert\mu\rvert\}}$
b)	$\gamma^{-}_{cd;\frac{1}{2}\frac{1}{2}} \sim t^{(1/2)\{\lvert\lvert\mu\rvert - M\rvert - \lvert\mu\rvert + \lvert M-(1/2)[1+(-1)^n]\rvert + (1/2)[1+(-1)^n]\}}$
c)	$\gamma^{-}_{cd;\frac{1}{2}-\frac{1}{2}} \sim t^{(1/2)\{M + \lvert M-(1/2)[1-(-1)^n]\rvert - (1/2)[1-(-1)^n]\}}$
d)	$\gamma^{-}_{cd;\frac{1}{2}\frac{1}{2}} \sim t^{(1/2)\{M + \lvert M-(1/2)[1+(-1)^n]\rvert + (1/2)[1+(-1)^n]\}}$
e)	$\gamma^{-}_{cd;\frac{1}{2}-\frac{1}{2}} \sim t^{(1/2)\{\lvert\lvert\mu\rvert - M\rvert - \lvert\mu\rvert + \lvert M-(1/2)[1-(-1)^n]\rvert + (1/2)[1-(-1)^n]\}}$
f)	$\gamma^{+}_{cd;\frac{1}{2}\frac{1}{2}} \sim t^{(1/2)\{\lvert\lvert\mu\rvert - M\rvert + M - \lvert\mu\rvert\}}$
g)	$\gamma^{+}_{cd;\frac{1}{2}-\frac{1}{2}} \sim t^{(1/2)\{M + \lvert M-1\rvert - 1\}}$
h)	$\gamma^{+}_{cd;\frac{1}{2}\frac{1}{2}} \sim t^{M}$

M is a number that we introduced at this point in order to label the minimal solutions. It can assume all the integer values between zero and infinity. Every family of Regge poles is characterized by a value of M.

An interesting feature of our results is that, for all the values of the masses, a family with a given M contributesa symptotically only to the forward s-channel helicity amplitudes with helicity flip equal to $\pm M$. This property gives to M a clear physical meaning and permits to identify M with the Toller quantum number introduced in the group-theoretical approach.

The previous results for the minimal solutions of the conspiracy problem are coincident with those found by COSENZA, SCIARRINO and TOLLER in the general group-theoretical approach.

However, besides the solutions given above, which are consistent with the group-theoretical one, we find two more solutions for the families with $\sigma=+1$, $\tau=-\xi$:

$$(4.4)\qquad \begin{cases} a) \begin{cases} \gamma^{(+)(\mathcal{N}\bar{\mathcal{N}}\to\mathcal{N}\bar{\mathcal{N}})}_{ab;ab} \sim \begin{cases} t\,, & \text{if } \lambda=0\,,\\ \text{constant}\,, & \text{if } \lambda\neq 0\,,\end{cases}\\ \gamma^{(+)(JS\to J'S')}_{cd;cd} \sim \begin{cases} t^{|\mu|}\,, & \text{if } \mu\neq 0\,,\\ t^2\,, & \text{if } \mu=0\,,\end{cases}\end{cases}\\ b) \begin{cases} \gamma^{(+)(\mathcal{N}\bar{\mathcal{N}}\to\mathcal{N}\bar{\mathcal{N}})}_{ab;ab} \sim \begin{cases} \text{constant}\,, & \text{if } \lambda=0\,,\\ t\,, & \text{if } \lambda\neq 0\,,\end{cases}\\ \gamma^{(+)(JS\to J'S')}_{cd;cd} \sim \begin{cases} t^{|\mu|+1}\,, & \text{if } \mu\neq 0\,,\\ t\,, & \text{if } \mu=0\,.\end{cases}\end{cases}\end{cases}$$

We did not find any way to eliminate these solutions: their meaning and their relations to the group-theoretical results are unclear.

The problem of the minimal solutions of the conspiracy problem in the set of reactions (1.1) has been also treated by CAPELLA, CONTOGOURIS and TRAN THANH VAN [17] using the analytic approach.

They obtained

$$(4.5)\qquad \gamma^{\sigma}_{ab;ab} \sim t^{\frac{1}{2}[1-\sigma(-1)^{M+n+\lambda}]}$$

for the EE mass configuration, while for the UU mass scattering their results are coincident with those given by (4.1). The apparent disagreement between the expressions (4.2), (4.3) and (4.4) is due to the fact that their result is only valid for $M=0,1$ because they did not impose the minimality in all the channels (1.1) at the same time. In so doing they obtained the solutions for $M=0,1$, but lost other solutions which are minimal in the set of reactions (1.1). Finally the behaviour of the EE residues for the family with $\sigma=+$ and $\tau=-\xi$ cannot be obtained from (4.4) because the previous authors did not consider these families.

5. – Classification of Regge-pole families.

The introduction of the new quantum number M permits us to classify the Regge poles in families according to the values of M, σ, ξ.

Class I: $M=0,\ \sigma=+1,\ \tau=\xi$.

A parent trajectory requires the existence of a family of trajectories with « angular momentum »:

$$\alpha_n(0)=\alpha(0)-n\,,$$

where $\alpha(0)$ is the intercept of the parent trajectory. For n even we have

$$\tau_n=P_n=\xi=\tau$$

and for n odd

$$\tau_n=P_n=-\xi=-\tau\,.$$

Only the trajectories with n even can couple to the $\mathcal{N}\bar{\mathcal{N}}$ system. This is the class I of FREEDMAN and WANG [4]. Poles of this class never conspire, as can be seen from the behaviour of the residues listed above.

Class I*a*): $M=0,\ \sigma=+1,\ \tau=-\xi$.

Poles with n even have

$$\tau_n=P_n=-\xi=\tau$$

and with n odd

$$\tau_n=P_n=\xi=-\tau\,.$$

The parent and the even daughters of this class do not couple to the $\mathcal{N}\bar{\mathcal{N}}$ system: this explains why this class is not contained in the Freedman and Wang classification.

Also these poles never conspire.

Class II: $M=0,\ \sigma=-1,\ \tau=-\xi$.

The poles of this class satisfy the constraints (2.4) and (2.8) by a daughter-like conspiracy; that explains why such constraints are called « class II » constraints. All the others are satisfied by evasion. For n even we have

$$\tau_n=-P_n=-\xi=\quad\tau$$

and for n odd

$$\tau_n=-P_n=\quad\xi=-\tau\,.$$

The parent trajectory contributes to the amplitude $\tilde{f}^{(-)t}_{cd;\frac{1}{2}-\frac{1}{2}}$ and conspires with the first daughter that contributes to the amplitude $\tilde{f}^{(-)t}_{cd;\frac{1}{2},\frac{1}{2}}$. The same mechanism of conspiracy applies between the amplitudes $\tilde{f}^{(-)t}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}$ and $\tilde{f}^{(-)t}_{\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2}}$ in the nucleon-nucleon scattering.

Class II*a*): $M = 0$, $\sigma = -1$; $\tau = \xi$.

Poles with n even have

$$\tau_n = -P_n = \xi = \tau$$

and with n odd

$$\tau_n = -P_n = -\xi = -\tau .$$

The parent trajectory of this class, which is decoupled from the $\mathcal{N}\bar{\mathcal{N}}$ system at $t = 0$, satisfies all the constraints by evasion. Also this class is therefore absent in the Freedman and Wang classification. Conspiracy between daughters is allowed in the EU configuration.

Class III: $M = 1$, $\tau = \xi$.

In this class we find the well-known parity-doubling phenomenon. The parity doublet structure not only allows satisfaction by conspiracy of the constraints (2.3), (2.6), (2.7), (2.8) but it is also imposed by the general analyticity requirements considered in detail in Appendix C and in ref. [27]. Therefore in some particular conditions the class III conspiracy is a necessity, not just a possibility, deriving from the analyticity and crossing symmetry requirements.

One has the following quantum numbers:

$$\sigma = +1 \begin{cases} n \text{ even}, & \tau_n = P_n = \xi = \tau , \\ n \text{ odd}, & \tau_n = P_n = -\xi = -\tau . \end{cases}$$

Only trajectories with n even can couple to $\mathcal{N}\bar{\mathcal{N}}$ system.

$$\sigma = -1 \begin{cases} n \text{ even}, & \tau_n = -P_n = \xi = \tau , \\ n \text{ odd}, & \tau_n = -P_n = -\xi = -\tau . \end{cases}$$

Class III*a*): $M = 1$, $\tau = -\xi$.

The poles of this class have the following quantum numbers:

$$\sigma = + \begin{cases} n \text{ even}, & \tau_n = P_n = -\xi = \tau , \\ n \text{ odd}, & \tau_n = P_n = \xi = -\tau . \end{cases}$$

Only trajectories with n odd can couple to the $\mathcal{N}\bar{\mathcal{N}}$ system.

$$\sigma = - \begin{cases} n \text{ even}, & \tau_n = -P_n = -\xi = \tau , \\ n \text{ odd}, & \tau_n = -P_n = \xi = -\tau . \end{cases}$$

([27]) M. A. JACOBS and M. H. VAUGHN: *Phys. Rev.*, **172**, 1677 (1968).

The parent trajectories satisfy the constraints by evasion. Conspiracy between daughter trajectories of opposite value of σ is however allowed.

Poles with $M >$ are decoupled, at $t = 0$, from the $\mathcal{N}\overline{\mathcal{N}}$ system, in agreement with the group-theoretical approach.

Such a classification is coincident with that found in the general group-theoretical approach by COSENZA, SCIARRINO and TOLLER and results a generalization of the Freedman and Wang classification. In fact the classes Ia), IIa), IIIa) are absent in their classification because the parent Regge trajectories of these class are decoupled at $t = 0$ in $\mathcal{N}\overline{\mathcal{N}}$ scattering.

* * *

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APPENDIX A

The method used for the derivation of the kinematical constraints in the channel $S+N \to J+N$ is due to COHEN-TANNOUDJI, MOREL and NAVELET [28].

Let us consider the reaction

$$S(a) + \mathcal{N}(b) \to J(c) + \mathcal{N}(d) \tag{A.1}$$

where the expressions between brackets refer to the helicity of the corresponding particle. The t-channel of the reaction (A.1) is

$$\overline{\mathcal{N}}(D') + \mathcal{N}(b') \to J(c') + \overline{S}(A') . \tag{A.2}$$

The helicity amplitudes of the processes (A.1) and (A.2) are related, through the crossing matrix, by

$$\bar{f}^s_{cd;ab} = \left(\sqrt{2}\sin\frac{\theta_s}{2}\right)^{-|\lambda-\mu|} \left(\sqrt{2}\cos\frac{\theta_s}{2}\right)^{-|\lambda+\mu|} \sum_{A'b'c'D'} d^S_{A'a}(\chi_a)\, d^{\frac{1}{2}}_{b'b}(\chi_b) \cdot \\ \cdot d^J_{c'c}(\chi_c)\, d^{\frac{1}{2}}_{D'd}(\chi_d) \left(\sqrt{2}\sin\frac{\theta_t}{2}\right)^{|\lambda'-\mu'|} \left(\sqrt{2}\cos\frac{\theta_t}{2}\right)^{|\lambda'+\mu'|} \bar{f}^t_{c'A';D'b'} , \tag{A.3}$$

where $\cos\chi_a$, $\cos\chi_c$, $\cos(\theta_s/2)$, $\sin(\theta_s/2)$, $\cos(\theta_t/2)$ and $\sin(\theta_t/2)$ are regular functions near $t = 0$, while $\cos\chi_b$ and $\cos\chi_d$ are singular [22].

Drawing out the singular behaviour at $t = 0$, the rotation matrices related

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[28] G. COHEN-TANNOUDJI, A. MOREL and H. NAVELET: *Ann. of Phys.*, **46**. 239 (1968).

to the two nucleons can be written near $t=0$ in the form

$$d^{\frac{1}{2}}(\cos\chi_{b(d)}) \sim \frac{B(D)}{(-t)^{\frac{1}{2}}}\begin{pmatrix} i & -1 \\ 1 & i \end{pmatrix}, \tag{A.4}$$

where

$$B(D) = \left[\frac{M(m_J^2 - m_S^2)}{2[S-(m_{S(J)}-M)^2][S-(m_{S(J)}+M)^2]}\right]^{\frac{1}{2}} \tag{A.5}$$

and M is the nucleon mass.

For sake of simplicity it is convenient to define, for any c' and A', the following system of equations:

$$G_{bd}^{c'A'} = \sum_{b'D'} d^{\frac{1}{2}}_{b'b}(\chi_b)\, d^{\frac{1}{2}}_{D'd}(\chi_d)\left(\sqrt{2}\sin\frac{\theta_t}{2}\right)^{|\lambda'-\mu'|}\left(\sqrt{2}\cos\frac{\theta_t}{2}\right)^{|\lambda'+\mu'|}\tilde{f}^t_{c'A';D'b'}\,. \tag{A.6}$$

Since the rank of the system (A.6) is one it is sufficient to consider only the equation which is obtained for $b=d=\frac{1}{2}$, so that we have near $t=0$:

$$G^{c'A'}_{\frac{1}{2}\frac{1}{2}} = -\frac{BD}{(-t)}\left[\tilde{f}^{t(-)}_{c'A';\frac{1}{2}\frac{1}{2}} - i\tilde{f}^{t(+)}_{c'A';\frac{1}{2}-\frac{1}{2}}\right] \tag{A.7}$$

for $c' \neq A'$ and

$$G^{c'A'}_{\frac{1}{2}\frac{1}{2}} = -\frac{BD}{(-t)}\left[\tilde{f}^{t(-)}_{c'A';\frac{1}{2}\frac{1}{2}} - i\tilde{f}^{t(-)}_{c'A';\frac{1}{2}-\frac{1}{2}}\right] \tag{A.8}$$

for $c' = A'$.

In the previous expressions we extracted from the parity-conserving helicity amplitudes the same $t^{-\frac{1}{2}}$ singularity near $t=0$ (Table IIa)). If now we put the relations (A.7) and (A.8) into (A.3) we see that we must have

$$\sum_{\substack{c'A'\\ c'\neq A'}} d^J_{c'c}(\chi_c)\, d^S_{A'a}(\chi_a)\left[\tilde{f}^{t(-)}_{c'A';\frac{1}{2}\frac{1}{2}} - i\tilde{f}^{t(-)}_{c'A';\frac{1}{2}-\frac{1}{2}}\right] + \sum_{\substack{c'A'\\ c'=A'}} d^J_{c'c}(\chi_c)\cdot d^S_{A'a}(\chi_a)\left[\tilde{f}^{t(-)}_{c'A';\frac{1}{2}\frac{1}{2}} - i\tilde{f}^{t(-)}_{c'A';\frac{1}{2}-\frac{1}{2}}\right] = O(t) \tag{A.9}$$

in order to not have a kinematical pole at $t=0$, which is forbidden because the left-hand side of the eq. (A.3) contains quantities free from kinematical singularities in t.

Because of the presence of the $d_{\lambda\mu}$ functions the determinant of the system (A.9) is equal to a finite value; therefore the only allowed solution is

$$\tilde{f}^{t(-)}_{c'A';\frac{1}{2}\frac{1}{2}} - i\tilde{f}^{t(+)}_{c'A';\frac{1}{2}-\frac{1}{2}} = O(t) \tag{A.10}$$

for any $c' \neq A'$ and

$$\tilde{f}^{t(-)}_{c'c';\frac{1}{2}\frac{1}{2}} - i\tilde{f}^{t(-)}_{c'c';\frac{1}{2}-\frac{1}{2}} = O(t) \tag{A.11}$$

for any c'.

APPENDIX B

The kinematical constraints between the t-channel, helicity amplitudes of the reactions

$$S(a) + S'(b) \to J(c) + J'(d) \tag{B.1}$$

can be obtained following the method of COHEN-TANNOUDJI *et al.* ([28]). However we will follow here a more direct approach suggested by FRAUTSCHI and JONES ([29]).

The relation between the amplitudes $\bar{f}^t_{\mu;\lambda}$ and $f^{\pm t}_{\mu;\lambda}$ is

$$\begin{cases} \bar{f}^t_{\mu';\lambda'} = \dfrac{1}{2}[\bar{f}^{t+}_{\mu';\lambda'} + \bar{f}^{t-}_{\mu';\lambda'}]\,, \\ \bar{f}^t_{-\mu';\lambda'} = \dfrac{\sigma_J\sigma_S(-)^{\lambda-N}}{2}[\bar{f}^{t+}_{\mu';\lambda'} - \bar{f}^{t-}_{\mu';\lambda'}]\,, \end{cases} \tag{B.2}$$

where $\lambda' = D' - b'$; $\mu' = c' - A'$, $N = \max(|\lambda'|; |\mu'|)$ and σ_J (σ_S) is the natural parity of the particle J (S). The t-kinematical singularities of the amplitudes $\bar{f}^t_{\mu';\lambda'}$ and $\bar{f}^{t\pm}_{\mu';\lambda'}$ are

$$\bar{f}^t_{\mu';\lambda'} \sim t^{-\frac{1}{2}|\lambda'-\mu'|}\,, \qquad \bar{f}^{t\pm}_{\mu';\lambda'} \sim t^{-\frac{1}{2}N}\,. \tag{B.3}$$

The kinematical constraints follow there immediately from the requirement that the two sides of the identities (B.2) have the same behaviour near $t = 0$.

APPENDIX C

The daughter trajectories have been introduced by FREEDMAN and WANG ([11]) in order to eliminate any kind of singularity at $t = 0$ in the full amplitude. They treated in detail the spinless case and evaluated in this case the singularity of the residue of the n-th daughter at $t = 0$ in the unequal-unequal mass scattering:

$$(\gamma)^{UU} \sim \frac{1}{t^n} \tag{C.1}$$

([29]) S. FRAUTSCHI and L. JONES. *Phys. Rev.*, **167**, 1335 (1968).

and in the equal-unequal mass configuration:

$$\text{(C.2)}\qquad \gamma^{\text{EU}} \sim \begin{cases} \left(\dfrac{1}{t}\right)^{n/2}, & \text{if } (-1)^n = 1\,, \\ \left(\dfrac{1}{t}\right)^{(n-1)/2}, & \text{if } (-1)^n = -1\,. \end{cases}$$

Those are the most singular behaviours required for the residue of the daughter trajectories in order to have an amplitude analytic at $t=0$; of course if the residue of the parent Regge pole vanishes at $t=0$ the residue of the daughter trajectories will be less singular than in (C.1) and (C.2).

We will extend the Freedman-Wang approach to the spin case, where the singularity structure of the daughter residues will show some differences with respect to the spin-less case.

We will see furthermore that in some cases the introduction of another Regge-pole family with opposite value of σ is necessary because the daughter trajectories alone are not sufficient to restore the analyticity of the full amplitude.

We start from the eq. (3.2), which can be written

$$\text{(C.3)}\qquad f^{(\pm)t}_{cd;ab} \simeq \frac{2\alpha^\pm + 1}{\sin\alpha^\pm}\, g^\pm_{cd;ab}(t)\gamma^\pm_{cd;ab}(P_{cd}P_{ab})^{\alpha^\pm - N} E^{+\alpha^\pm}_{\lambda\mu}(-\cos\theta_t) + \\ + \frac{2\alpha^\mp + 1}{\sin\alpha^\mp}\, g^\mp_{cd;ab}(t)\frac{K^\mp_{cd;ab}}{K^\pm_{cd;ab}}(P_{cd}P_{ab})^{\alpha^\mp - N}\gamma^\mp_{cd;ab} E^{-\alpha^\mp}_{\lambda\mu}(-\cos\theta_t)\,,$$

if we use the expression (3.4) for the residues and we eliminate the t kinematical singularities. For sake of clearness it is convenient to consider separately the UU and EU mass configurations.

In the UU case (C.3) becomes

$$\text{(C.4)}\qquad \tilde f^{(\pm)t}_{cd;ab} \sim g^\pm_{cd;ab}\gamma^\pm_{cd;ab}(P_{cd}P_{ab})^{\alpha^\pm - N} E^{+\alpha^\pm}_{\lambda\mu}(-\cos\theta_t) + g^\mp_{cd;ab}\cdot \\ \cdot P_{cd}P_{ab}(P_{cd}P_{ab})^{\alpha^\mp - N - 1} E^{-\alpha^\mp}_{\lambda\mu}(-\cos\theta_t)\gamma^\mp_{cd;ab}\,,$$

where we incorporated the factor $(2\alpha+1)/(\sin\pi\alpha)$ in the residue $\gamma_{cd;ab}$ and we used $K^\pm_{cd;ab}(t) = K^\mp_{cd;ab}(t)$.

We note that the factors

$$(P_{cd}P_{ab})^{\alpha^\pm - N} E^{+\alpha^\pm}_{\lambda\mu}(-\cos\theta_t) \quad\text{and}\quad (P_{cd}P_{ab})^{\alpha^\mp - N - 1} E^{-\alpha^\mp}_{\lambda\mu}(-\cos\theta_t)$$

have the same singularity structure at $t=0$ of $P_{\alpha^\pm - N}(-\cos\theta_t)(P_{cd}P_{ab})^{\alpha^\pm - N}$ involved in the spinless case. The only difference therefore with the spinless case arises from the presence of two terms in (C.4) and from the fact that the second term contains a factor that behaves like $1/t$ near $t=0$. This singular behaviour of $P_{cd}P_{ab}$ complicates in the spin case the analysis of the singularity structure of the residues of the daughter trajectories.

Obviously in the amplitudes with minimum $(|\lambda|, |\mu|) = 0$ the singularity structure of the daughter poles is the same as that in the spinless case because of the identical vanishing of the $E^{-}_{\lambda\mu}$ function. For the other amplitudes because of the presence of the factor $P_{cd}P_{ab}$ in the second term of eq. (C.2) the introduction of daughter trajectories is necessary to eliminate some singularities, but is not in general sufficient to have an amplitude analytic at $t = 0$.

In order to restore the analyticity of the full amplitude we need the contribution of another Regge-pole family with opposite value of σ with respect to the family primarily introduced in the amplitude. One can then study the singularity structure of the daughter trajectories of these two families and it is easy to check that such structure is not different from that found by FREEDMAN and WANG in the spinless case. Finally we can conclude that in the spin case for the UU mass configuration the singularity structure of the residues of the daughter trajectories is the same of that given in (C.1). Furthermore such analysis permitted us to get that the analyticity requires the existence of the parity-doubling phenomenon [27].

Obviously the possibility of the parity doublet can be avoided if $\gamma^{\sigma}_{cd;ab} \sim t$ at least when $t \to 0$ for the pole primarily exchanged, but we are interested at this stage in the most singular behaviour of the residues of the daughter trajectories and we expect that such further zeros will come out from the factorization conditions.

Let us then consider the EU mass configuration. The expression (C.3) in this case becomes

$$
\text{(C.5)} \qquad \tilde{f}^{(\pm)t}_{cd;ab} \sim g^{\pm}_{cd;ab}\gamma^{\pm}_{cd;ab}(P_{cd}P_{ab})^{\alpha^{\pm}-N} E^{+\alpha^{\pm}}_{\lambda\mu}(-\cos\theta_t) + \\
+ \left[\frac{K^{\mp}_{cd;ab}(t)}{K^{\pm}_{cd;ab}} P_{cd}P_{ab}\right] g^{\mp}_{cd;ab}(P_{cd}P_{ab})^{\alpha^{\mp}-N-1} E^{-\alpha^{\mp}}_{\lambda\mu}(-\cos\theta_t)\gamma^{\mp}_{cd;ab}\,.
$$

Also in this mass configuration the only difference with the spinless case arises from the presence of two terms in (C.5) and from the expression

$$
\frac{K^{\mp}_{cd;ab}(t)}{K^{\pm}_{cd;ab}} P_{cd}P_{ab}\,,
$$

which in some amplitudes behaves like $1/t$ for $t \to 0$. Because of the identical vanishing of the E^{-} for λ and μ not both different from zero, the analysis can therefore be restricted to the amplitudes with $\mu \neq 0$ and $\lambda = 1$; for the other amplitudes the singularities near $t = 0$ of the residues of the daughter trajectories is evidently the same than in the spinless case.

If $\lambda = 1$ and $\mu \neq 0$ the expression (C.5) becomes

$$
\text{(C.6)} \qquad \tilde{f}^{(+)t}_{cd;\frac{1}{2}-\frac{1}{2}} \sim g^{+}_{cd;\frac{1}{2}-\frac{1}{2}}\gamma^{+}_{cd;\frac{1}{2}-\frac{1}{2}}(P_{cd}P_{ab})^{\alpha^{\pm}N} E^{+\alpha^{+}}_{1\mu}(-\cos\theta_t) + \\
+ g^{-}_{cd;\frac{1}{2}-\frac{1}{2}}(P_{cd}P_{ab})^{\alpha^{-}-N-2} E^{-\alpha^{-}}_{1\mu}(-\cos\theta_t)\gamma^{-}_{cd;\frac{1}{2}-\frac{1}{2}}\,,
$$

$$
\text{(C.7)} \qquad \tilde{f}^{(-)t}_{cd;\frac{1}{2}-\frac{1}{2}} \sim g^{-}_{cd;\frac{1}{2}-\frac{1}{2}}\gamma^{-}_{cd;\frac{1}{2}-\frac{1}{2}}(P_{cd}P_{ab})^{\alpha^{-}-N} E^{+\alpha^{-}}_{1\mu}(-\cos\theta_t) + \\
+ \frac{f(t)}{t}(P_{cd}P_{ab})^{\alpha^{\pm}-N-1} E^{-\alpha^{+}}_{1\mu}(-\cos\theta_t)\gamma^{+}_{cd;\frac{1}{2}-\frac{1}{2}}\,,
$$

where $f(t)$ is a function regular at $t = 0$, which incorporates the nonsingular part of $(P_{cd}P_{ab})$. Because of the factor $1/t$ in the second term of (C.7), also in this mass configuration the parity doubling phenomenon is in general necessary to restore the analyticity of the amplitude at $t = 0$; the singularities can be cancelled only if in the first term of the right-hand side of eq. (C.7) contribute the odd daughters of a Regge-pole family with $\sigma = -1$, whose residues have the singularity structure

$$\gamma^{-}_{cd;\frac{1}{2}\frac{1}{2}} \sim \left(\frac{1}{t}\right)^{n+1} \qquad \text{with} \qquad (-1)^n = -1 \tag{C.8}$$

which is different from the expression (C.2) valid in the spinless case (*).

In conclusion the analyticity properties require the singularity structure (C.8) for the residues of the odd daughters belonging to a family with $\sigma = -1$, while for all the other amplitudes and Regge-pole families, the singularities are given by (C.2).

(*) We are grateful to Prof. L. BERTOCCHI and Dr. A. SCIARRINO for enlightening discussions about this point.

RIASSUNTO

Si presenta un approccio non gruppistico al problema delle cospirazioni, basato sulle proprietà di analiticità, simmetria di crossing e fattorizzazione. Si determinano le soluzioni per tutti i casi di interesse fisico e si deduce una classificazione per le famiglie di poli di Regge.

Минимальные решения для проблемы конспиративности и классификация семейств полюсов Редже. - II.

Резюме (*). — Предлагается не теоретико-групповой подход к проблеме конспиративности, основанный на аналитичности, кроссинг-симметрии и факторизации. Получаются решения для всех физически интересных случаев. Проводится лклассификация семейств полюсов Редже.

(*) *Переведено редакцией.*