Laboratori Nazionali di Frascati

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POSITIVE-PION PHOTOPRODUCTION WITH COHERENT BREMSSTRAHLUNG - II: ANALYSIS OF THE RESULTS.

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Positive-Pion Photoproduction with Coherent Bremsstrahlung.
II. - Analysis of the Results.

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Summary. – The results of a phenomenological analysis of the data of π^+ photoproduction with coherent bremsstrahlung are reported. By means of a polynomial analysis of the cross-section some multipole amplitudes have been evaluated and critically discussed. A comparison between π^+ and π^0 photoproduction has been carried out. The validity of the procedure followed is discussed.

Introduction.

In the previous part of this work (1) we reported the experimental procedure and the results of the measurements of the asymmetry ratio $A(\theta, E_\gamma)$. In the same paper we reported also a comparison of our data with the predictions of some recent theoretical calculations (see Fig. 11 of ref. (1)). The general trend of the theories is reproduced by the experimental data, but quantitative discrepancies, somewhere large, are present.

In this second part we will analyse such results by means of a phenomenological approach. The aim is to see if a very simplified such model as is discussed in ref. (2) can describe the data of π^+ photoproduction with polarized

(1) M. GRILLI, P. SPILLANTINI, F. SOSO, M. NIGRO, E. SCHIAVUTA and V. VALENTE: *Nuovo Cimento*, **54** A, 877 (1968).
(2) M. NIGRO and E. SCHIAVUTA: *Nuovo Cimento*, **50** A, 358 (1967).

γ-rays. Our data, together with those concerning the cross-section of π^+ photoproduction by unpolarized gamma rays $\langle\sigma(\theta, E_\gamma)\rangle$, can be expressed in terms of the two independent functions:

$$\sigma_\perp(\theta, E) = \big(1 + A(\theta, E_\gamma)\big)\langle\sigma(\theta, E_\gamma)\rangle ,$$

$$\sigma_\perp - \sigma_\parallel = 2A(\theta, E_\gamma)\langle\sigma(\theta, E_\gamma)\rangle .$$

The choice of these two quantities is physically significant, as shown in ref. (2) and will be clear in the following discussion.

The method of the physical analysis and the relative formulae are described in Sect. **1**; the results of such analysis are discussed in Sect. **2**.

A part of the analysis, together with some general conclusions, has been previously reported in ref. (3-6).

1. – Methods of analysis.

Let us write the total photoproduction amplitude, according to CGLN (7)

$$M^\pi = \sum_1^4 \mathscr{F}^K I_K$$

and

$$M^\pi = M^0 + M^{\text{pole}} ,$$

where (8)

$$M^{\text{pole}} = \frac{K(W)ef}{\beta(x_0 - x)} I_3 - \frac{q}{K}\frac{K(W)ef}{\beta(x_0 - x)} I_4$$

is the t-pole amplitude.

Defining the quantity $\sigma_\perp(\theta, E_\gamma)$ as the production cross-section of a single π^+, at a c.m. angle θ, by a photon of energy E_γ with the electric vector perpendicular to the production plane, and taking in account states with $J^P = \frac{1}{2}^\pm$, $\frac{3}{2}^\pm$, and

(3) P. Gorenstein, M. Grilli, P. Spillantini, M. Nigro, E. Schiavuta, F. Soso and V. Valente: *Phys. Lett.*, **19**, 157 (1965).

(4) P. Gorenstein, M. Grilli, F. Soso, P. Spillantini, M. Nigro, E. Schiavuta and V. Valente: *Phys. Lett.*, **23**, 394 (1966).

(5) *Proc. Intern. Conf. Low and Intermediate Energy Electromagnetic Interactions*, vol. **2** (Dubna, 1967), p. 152.

(6) M. Grilli, M. Nigro and E. Schiavuta: *Nuovo Cimento*, **49** A, 326 (1967).

(7) G. F. Chew, M. L. Goldberg, F. E. Low and Y. Nambu: *Phys. Rev.*, **106**, 1345 (1957).

(8) W = total energy, $K(W) = \sqrt{2}\,(M/W)\,\beta$, $e = 1/137$, $(2Mf)^2 = g_r^2/4\pi = 14.7$, $\lambda_r = 1.41\cdot 10^{-13}$ cm (Compton wavelength of pion).

with $J > \frac{3}{2}$ only through M^{pole}, we can write the following expressions ([9]):

$$
(1.1)\quad \begin{cases}
\dfrac{K}{q}\,\sigma_\perp(\theta, E_\gamma) = A_\perp(E_\gamma) + B_\perp(E_\gamma)\cos\theta + C_\perp(E_\gamma)\cos^2\theta + D_\perp(E_\gamma)\cos^3\theta\,, \\
A_\perp = |E_{0+}|^2 + 4|M_{1+}|^2 + |M_{1-}|^2 + 4\,\mathrm{Re}\,(M^*_{1+}M_{1-}) + \\
\qquad\qquad + |E_{2-} + 3M_{2-}|^2 + 2\,\mathrm{Re}\,E^*_{0+}(E_{2-} + 3M_{2-})\,, \\
B_\perp = 2\,\mathrm{Re}\,E^*_{0+}(M_{1+} + 3E_{1+} - M_{1-}) + 2\,\mathrm{Re}\,E^*_{2-}(M_{1+} - M_{1-} + 6E_{1+}) + \\
\qquad\qquad + 6\,\mathrm{Re}\,M^*_{2-}(5M_{1+} + M_{1-} + 3E_{1+})\,, \\
C_\perp = 9|E_{1+}|^2 - 3|M_{1+}|^2 + 6\,\mathrm{Re}\,(M^*_{1+}E_{1+}) - 6\,\mathrm{Re}\,(M^*_{1+}M_{1-}) - \\
\qquad\qquad - 6\,\mathrm{Re}\,(E^*_{1+}M_{1-}) - 12\,\mathrm{Re}\,M^*_{2-}(E_{0+} + E_{2-})\,, \\
D_\perp = -36\,\mathrm{Re}\,M^*_{2-}(M_{1+} + E_{1+})\,.
\end{cases}
$$

In (1.1) the multipoles are defined as follows:

$$
(1.2)\qquad M_{l,J} = M^0_{l,J} = M^\pi_{l,J} - M^{\text{pole}}_{l,J}\,,
$$

where $M^{0,\text{pole}}_{l,J}$ are the projections of M^0 and M^{pole} respectively, on the l, J-state ([10]).

Starting from some theoretical predictions for the multipole amplitudes, *e.g.* those of BERENDS *et al.* ([11]), we calculate the cross-section and the asymmetry ratio with this simplified model. The results of this calculations are compared, in Fig. 1*a*)-1*d*), with the predictions of ref. ([11]), in which were included *S*, *P*, *D*, *F* multipoles, the higher waves being introduced in the Born approximation. The two calculations agree everywhere within the theory uncertainties (*).

In the same hypothesis and with the same notations we can define the function ([2,12]):

$$
(1.3)\qquad |\Delta|^M = \frac{-(K/q)\big((\sigma_\perp - \sigma_\parallel)/(1-x^2)\big)\big(\beta(x-x_0)\big)^2 - \delta}{\beta(x_0 - x)}\,, \qquad x = \cos\theta\,,
$$

([9]) For the multipoles we follow the CGLN notations.

([10]) As can be demonstrated (see for example INFN-67/1, Internal Report), the expressions (1.1), with the definitions (1.2), contain the M^{pole}_{lJ} contributions with $J > \frac{3}{2}$.

([11]) F. A. BERENDS, A. DONNACHIE and D. L. WEAVER: *Nucl. Phys.*, B 4, 1 (1968).

(*) Only at small angles and for energies above 350 MeV there are some differences; in these regions however we have no experimental point.

([12]) K, q are the momenta of γ and π in the c.m. system, ω the total energy of π and $\beta = q/\omega$, $\beta(x_0 - x) = -(t-1)/2\omega K$, t = momentum transfer,

$$
\delta = \lim_{x\to x_0}\left(-\frac{K}{q}\,\frac{2\langle\sigma\rangle A}{1-x^2}\,\big(\beta(x-x_0)\big)^2\right).
$$

in which the effect of the 1st-order pole in t-channel (very close to the physical region: $x = x_0 = 1/\beta$), is completely removed.

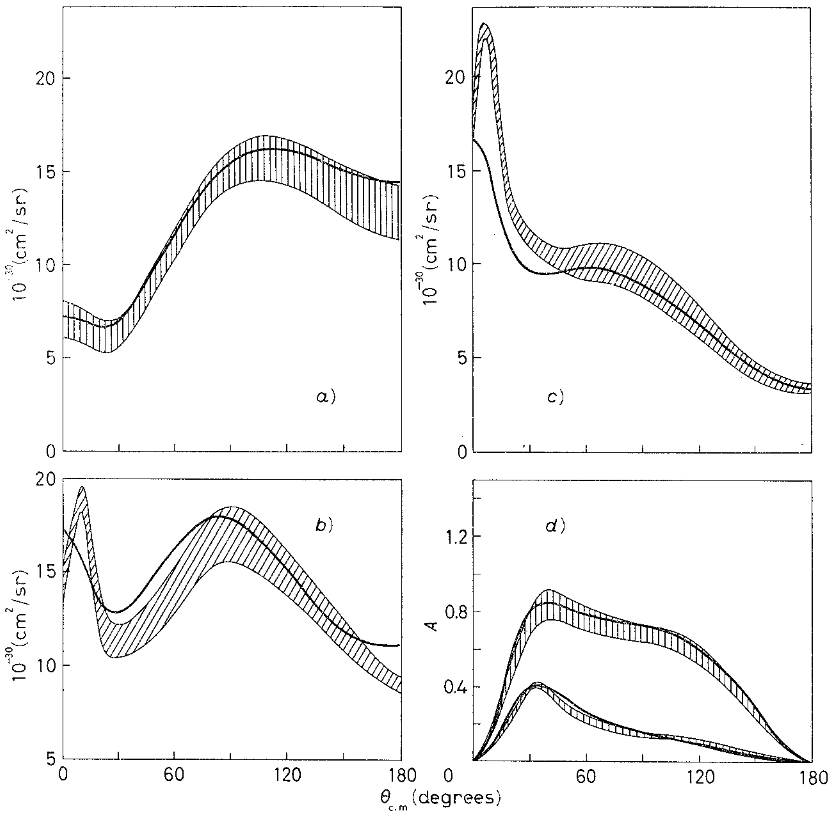

Fig. 1. – Cross-sections and asymmetry ratios calculated with the simplified model (———, see text) compared with the previsions of BERENDS *et al.* [11] (dashed area). *a*) $E_\gamma = 250$ MeV; *b*) $E_\gamma = 400$ MeV; *c*) $E_\gamma = 350$ MeV; *d*) up $E_\gamma = 400$ MeV, down $E_\gamma = 250$ MeV.

The multipole expansion of $|\Delta|^M$ is the following:

$$(1.4)\qquad \begin{cases} |\Delta|^M = \gamma + \alpha\big(\beta(x_0 - x)\big) + \eta\big(\beta(x_0 - x)\big)^2, \\ \gamma = 2\,R_3\left\{\mathrm{Re}\left(3\left(1 - 2\dfrac{\omega}{K}\right)E_{1+} - \dfrac{q}{K}\,E_{0+} + M_{1-} - M_{1+}\right) + \right. \\ \qquad\qquad\qquad\qquad \left. + \dfrac{1}{\beta}\,\mathrm{Re}\left(3(M_{2-} - E_{2-}) + 2\,\dfrac{q}{K}\,E_{2-}\right)\right\}, \end{cases}$$

$$
(1.4)\qquad \left\{\begin{aligned}
\alpha = {} & 9|E_{1+}|^2 - 3|M_{1+}|^2 + 6\,\mathrm{Re}\,(E_{1+}^* M_{1-} - E_{1+}^* M_{1+} - M_{1+}^* M_{1-}) + \\
& + 12\,\frac{\omega}{K} R_3\,\mathrm{Re}\,E_{1+} + 3|E_{2-}|^2 - 9|M_{2-}|^2 - \\
& - 6\,\mathrm{Re}\,\big(E_{2-}^* M_{2-} + E_{0+}^*(E_{2-} + M_{2-})\big) - \\
& - \frac{36}{\beta}\,\mathrm{Re}\,(E_{2-}^* E_{1+} + M_{2-}^* M_{1+}) + \frac{6}{\beta} R_3\,\mathrm{Re}\,(E_{2-} - M_{2-})\,, \\
\eta = {} & \frac{36}{\beta}\,\mathrm{Re}\,(E_{1+}^* E_{2-} + M_{1+}^* M_{2-})\,, \\
R_3 = {} & \sqrt{2}\left(\frac{M}{W}\right)\beta e f\,.
\end{aligned}\right.
$$

On the basis of the theoretical calculations of the multipole amplitudes [11] we can neglect, in (1.1), (1.4), $\mathrm{Im}\,M_2$, $\mathrm{Im}\,E_2$ with respect to the real parts.

2. – Fits of the experimental data; results.

We calculated $\sigma_\perp$ and $|\Delta|^M$ from the experimental data by means of the definitions

$$
(2.1)\qquad \left\{\begin{aligned}
& \sigma_\perp(\theta, E_\gamma) = \big(1 + A(\theta, E_\gamma)\big)\,\langle\sigma(\theta, E_\gamma)\rangle\,, \\
& |\Delta|^M = \frac{-(K/q)\big((2\langle\sigma\rangle A)/(1-x^2)\big)\big(\beta(x_0 - x)\big)^2 - \delta}{\beta(x_0 - x)}\,,
\end{aligned}\right.
$$

where $\langle\sigma(\theta, E_\gamma)\rangle$ is the photoproduction cross-section by unpolarized γ-rays. We have fitted all the available data for $\langle\sigma\rangle$, as given by BEALE *et al.* [13], with a Moravcsik polynomial expansion in x and, for every θ, we interpolated the results of the fits as continous functions of E_γ. The final results of this bi-dimensional fitting procedure [14] were included in (2.1). For A we used our data reported in ref. [1].

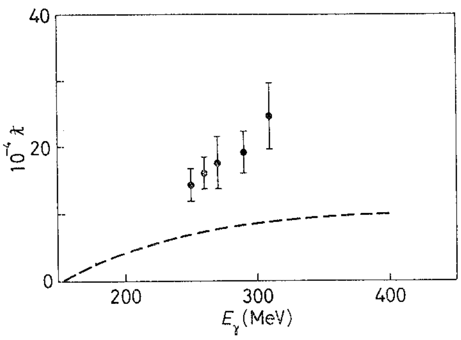

Fig. 2. – Results for Re M_{2-}. ——— BERENDS *et al.* [11].

According to the expansions (1.1), (1.4), we have fitted $\sigma_\perp$ and $|\Delta|^M$ as polynomial functions, at fixed E_γ, of x and $\beta(x_0 - x)$ respectively. The results are summarized in the Tables I, II.

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[13] J. T. BEALE, S. D. EKLUND and R. L. WALKER: CALT 60-108 (Internal Report).
[14] Laboratori Nazionali di Frascati, Internal Report.

In $\sigma_\perp$ were included the values of $\langle\sigma\rangle$ for $\theta = 0°$ and $\theta = 180°$, where $A(\theta, E_\gamma) = 0$.

As it can be seen from the results the contribution of the term $D_\perp = 36\,\mathrm{Re}\,M^*_{2-}(M_{1+}E_{1+})$ becomes relevant in the energy range $(250 \div 310)$ MeV, where $\mathrm{Re}\,M_{33}$ is large. Introducing in the last equation of (1.1) the theoretical values for $\mathrm{Re}\,M_{1+}$ and $\mathrm{Re}\,E_{1+}$ we can deduce $\mathrm{Re}\,M_{2-}$ (see Fig. 2). This mul-

TABLE I. – *The coefficients of the polynomial fits of $\sigma_\perp(\theta)$.*

E_γ (MeV)	$(q/K)A_\perp$ (μb/sr)	$(q/K)B_\perp$	$(q/K)C_\perp$	$(q/K)D_\perp$	$P(\chi^2)$	
					$N=2$	$N=3$
210	11.92 ± 0.33	-1.85 ± 0.17	-2.99 ± 0.40		0.27	
220	13.39 ± 0.29	-2.46 ± 0.17	-4.29 ± 0.37		0.31	
230	15.21 ± 0.21	-3.16 ± 0.12	-5.83 ± 0.26		0.71	
240	17.22 ± 0.34	-3.88 ± 0.18	-7.50 ± 0.42		0.22	
250	19.71 ± 0.21	-6.89 ± 0.43	-9.51 ± 0.24	2.58 ± 0.45	0.09	0.90
260	22.00 ± 0.24	-7.96 ± 0.46	-11.40 ± 0.28	3.41 ± 0.50	0.02	0.91
270	24.50 ± 0.43	-8.64 ± 0.81	-13.50 ± 0.51	4.04 ± 0.88	0.00	0.60
290	29.28 ± 0.69	-8.05 ± 1.31	-17.74 ± 0.81	4.35 ± 1.43	0.00	0.25
310	33.22 ± 1.20	-6.99 ± 2.48	-21.36 ± 1.27	5.31 ± 2.59	0.02	0.17
330	31.30 ± 0.79	0.73 ± 0.23	-19.42 ± 0.88		0.45	
350	27.77 ± 0.39	3.47 ± 0.11	-15.90 ± 0.42		0.96	
370	23.37 ± 0.36	5.33 ± 0.11	-11.84 ± 0.40		0.97	
400	17.52 ± 0.27	6.71 ± 0.06	-6.51 ± 0.29		0.97	
430	12.81 ± 0.13	7.64 ± 0.5	-2.08 ± 0.14		0.98	

TABLE II.

E_γ	γ	α	$P(\chi^2)$
210	0.13 ± 1.94	0.17 ± 2.04	0.75
220	1.43 ± 1.29	-1.97 ± 1.35	0.87
230	4.46 ± 1.09	-6.10 ± 1.22	0.86
240	6.29 ± 1.31	-8.95 ± 1.70	0.66
250	8.84 ± 0.93	-13.91 ± 1.34	0.85
260	9.96 ± 1.12	-16.95 ± 1.74	0.73
270	10.60 ± 1.35	-19.94 ± 2.17	0.53
290	9.64 ± 1.69	-23.83 ± 2.73	0.32
310	9.15 ± 2.52	-31.10 ± 4.60	0.23
330	5.52 ± 2.02	-29.59 ± 3.35	0.48
350	-0.31 ± 1.60	-21.75 ± 2.60	0.80
370	-3.19 ± 1.12	-16.32 ± 1.73	0.94
400	-4.78 ± 1.26	-10.26 ± 1.83	0.90
430	-5.97 ± 0.76	-4.48 ± 0.99	0.98

tipole results systematically larger than in Donnachie's [11] calculation. Unfortunately the smallness of the coefficient $D_\perp$ prevents us from evaluating $\mathrm{Re}\, M_{2-}$ outside the reported energy range.

The fits $|\Delta|^M$, because of the absence of experimental points at $\beta(x_0 - x) > 1$ (backward production), are insensitive to a quadratic term, so an independent conclusion about M_{2-} cannot be achieved; for the same reason no information about a contribution of E_{2-} can be given.

3. – Comparison between π^+ and π^0 photoproduction.

A qualitative comparison between π^- and π^0 photoproduction can be made through the coefficients $A_\perp$, $B_\perp$, $C_\perp$ and the corresponding A_0, B_0, C_0 deduced from the analysis of the angular distribution of π^0 cross-section [15-17].

The similarity of these quantities (see Fig. 3, 4, 5) indicates that, when the retardation term is absent (as in $\sigma_\perp$), the behaviour of the π^+ photoproduction is analogous to that of π^0.

However, a large contribution of the S-wave, via the E_{0+} multipole, causes the quantitative difference between $A_\perp$ and A_0, and between $B_\perp$ and B_0. We can subtract such a contribution from $A_\perp$ and compare the result with that coming from A_0. At $E_\gamma = 348$ MeV (where the phase δ_{33}, given in ref. [11], is equal to 90°) we can write from (1.1) (*)

$$\sigma_\perp(90°,\ E_\gamma = 348\ \text{MeV}) = \frac{q}{K} A_\perp \sim \frac{q}{K}\left(\frac{8}{9}|M_{33}|^2 + \Sigma\right)$$

and approximately

$$\Sigma \approx \frac{8}{9}|M_{33}^{\text{pole}}|^2 + |E_{0+}|^2 + 2\,\mathrm{Re}\, E_{0+}^*(E_{2-} + 3M_{2-}) . \tag{3.1}$$

Evaluating Σ from the theory [11] we obtain

$$|M_{33}|^2 \approx \frac{9}{8}[1.14(28.2 \pm 0.5) - 8.4] = (26.8 \pm 0.6)\ \mu\text{b/sr} . \tag{3.2}$$

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(15) G. FISHER *et al.*: *XIV Intern. Conf. on High-Energy Physics, Vienna, 1968.*

(16) B. B. GOVORKOV, S. P. DENISOV and E. V. MINARIK: *Sov. Journ. Nucl. Phys.*, **6**, 370 (1968).

(17) R. MORAND *et al.*: LAL-1201 (1968), preprint Orsay.

(*) At this energy $M_{1+} \sim -\frac{2}{3}(M_{33} - M_{33}^{\text{pole}})$.

Conversely, with the same approximations, we write for π^0

$$(3.3)\qquad \begin{cases} |M_{33}|^2 = \dfrac{9}{10}\left[\dfrac{K}{q'}A_0 - \Sigma'\right], \\ \Sigma' \approx |E_{0+}^{\pi^0}|^2 . \end{cases}$$

Inserting in (3.3) the experimental

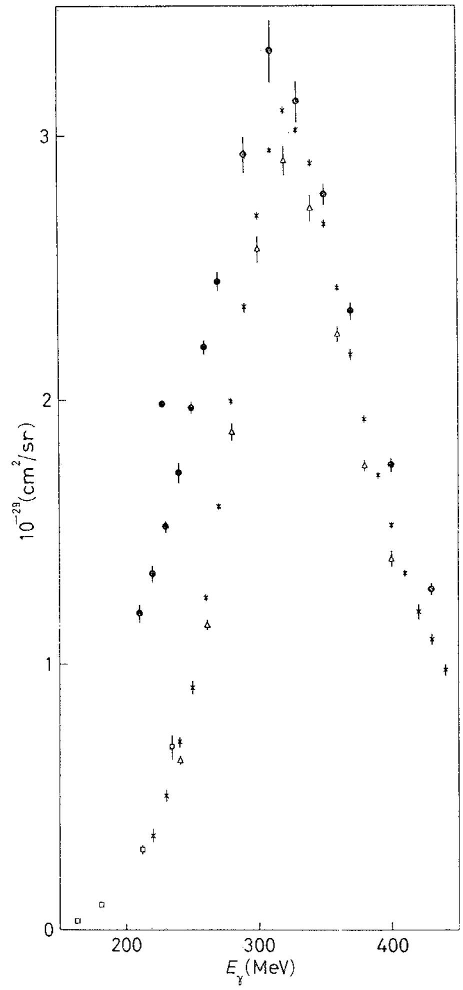

Fig. 3. – Coefficients of the polynomial fits of the experimental data: $A_\perp$ (•, q/k) from π^+ photoproduction by polarized γ-rays (polarization vector perpendicular to the production plane); A_0 (×, ref. [15]; □, ref. [16]; △, ref. [17]) from π^0 photoproduction by unpolarized γ-rays.

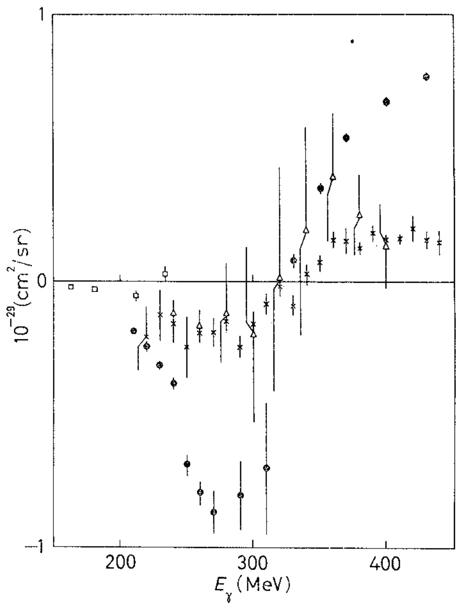

Fig. 4. – Coefficients of the polynomial fits of the experimental data: $B_\perp$ (•, q/k) from π^+ photoproduction by polarized γ-rays (polarization vector perpendicular to the production plane); B_0 (×, ref. [15]; □, ref. [16]; △, ref. [17]) from π^0 photoproduction by unpolarized γ-rays.

values of ref. [15,17] we obtain respectively

$$|M_{33}|^2 \approx \frac{9}{10}[1.13(27.5 \pm 0.1) - 0.2] = (27.3 \pm 0.1)\ \mu\text{b/sr}\,,$$

$$|M_{33}|^2 \approx \frac{9}{10}[1.13(25.8 \pm 0.5) - 0.2] = (26.1 \pm 0.5)\ \mu\text{b/sr}\,.$$

Such results, compared with the value (3.2) (*), confirm the hypothesis on the difference between π^+ and π^0 production. However we will emphasize that the numerical results for $|M_{33}|^2$ are strongly model-dependent. The quoted errors do not include the indeterminacy coming from the evaluation of Σ and Σ'.

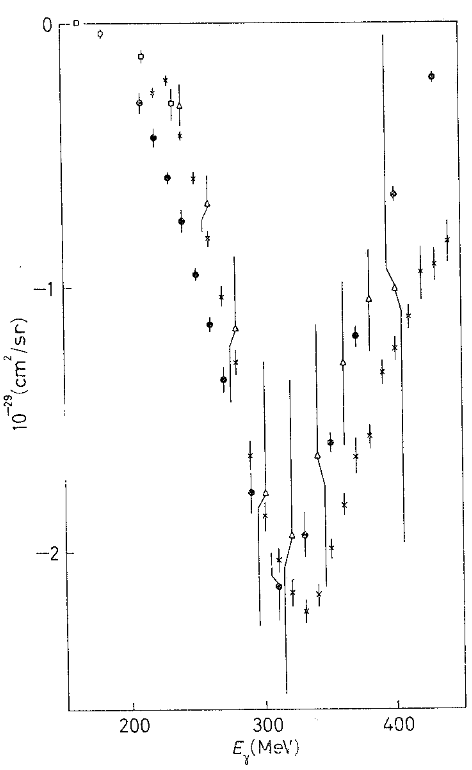

Fig. 5. – Coefficients of the polynomial fits of the experimental data: $C_\perp$ (•, q/k) from π^+ photoproduction by polarized γ-rays (polarization vector perpendicular to the production plane); C_0 (×, ref. [15]; □, ref. [16]; △, ref. [17]) from π^0 photoproduction by unpolarized γ-rays.

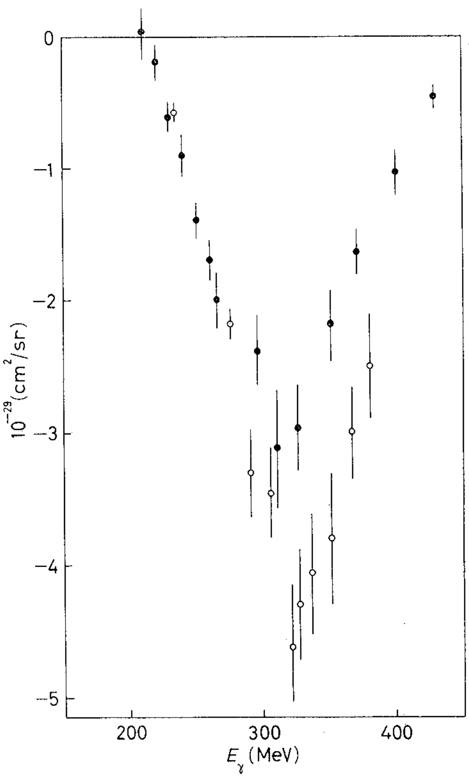

Fig. 6. – Comparison of α (•) from π^+ photoproduction (see formula (1.4)) with α_0 (○, ref. [18]) from π^0 photoproduction (formula (3.4)).

The function $|\Delta|^M$ gives an independent indication for this similarity; in $|\Delta|^M$, see eq. (1.3), (1.4) the coefficient α has the same meaning of α_0 in π^0

(*) Only for reference, the theoretical value of $|M_{33}|^2$ is 24.4 μb/sr.

photoproduction with polarized γ-rays:

$$\alpha_0 = \frac{1}{\sin^2\theta}(\sigma_\perp - \sigma_\parallel)^{\pi^0} = -\frac{2}{\sin^2\theta}\langle\sigma^{\pi^0}\rangle\left(\frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel}\right)^{\pi^0}. \tag{3.4}$$

In Fig. 6 α and α_0 are shown (α_0 is deduced from ref. ([18])).

As α, α_0 have a functional dependence on the multipoles very close to that of $C_\perp$, C_0, the similarity between α and α_0 reflects that of $C_\perp$, C_0 (compare Fig. 5, 6).

4. – Conclusion.

From the preceding discussion we can deduce the following general conclusions:

i) In the energy ranges $(200 \div 240)$ MeV and $(330 \div 430)$ MeV a very simplified model, in which the total production amplitude contains only S-, P-wave multipoles and the higher waves are introduced in the pole approximation, gives the correct functional dependence of the experimental data.

ii) In the energy range $(250 \div 310)$ MeV, a contribution of the magnetic amplitude M_{2-}, different from that coming from the pole approximation, must be included (see for discussion Sect. **2**).

The structure of the equations and the absence of discriminating measurements, see Sect. **2**, prevents us from outlining a analogous contribution of E_{2-}.

We remark here that the numerical results concerning M_{33} (see Sect. **3**) and M_{2-} are not conclusive. In fact the procedure for isolating some partial contribution in the total amplitude is strongly model-dependent and prevents us from verifying the accuracy of the hypothesis.

A more nearly correct procedure would be to obtain the whole set of the dominant amplitudes from the experimental data by means of a simplified model such as that described here. This kind of analysis, including as input all the measured quantities, such as cross-section, recoil polarization and symmetry, for both the π^+ and π^0 photoproduction, is now in progress.

([18]) G. BARBIELLINI, G. BOLOGNA, G. CAPON, J. DE WIRE, G. DE ZORZI, G. DIAMBRINI, F. L. FABBRI, G. P. MURTAS and G. SETTE: *Phys. Rev.*, in press.

RIASSUNTO

Si riportano le conclusioni di un'indagine fenomenologica dei dati della fotoproduzione di π^+ con fotoni polarizzati linearmente. Per mezzo di una analisi polinomiale delle sezioni d'urto si valutano, in particolare, alcune ampiezze di multipolo. Si discutono criticamente la validità del procedimento ed i limiti del metodo seguito. Si presenta inoltre un confronto fra i risultati della fotoproduzione di pioni carichi e neutri.

**Фоторождение положительных пионов когерентным тормозным излучением.
II. Анализ результатов.**

Резюме авторами не представлено.