Laboratori Nazionali di Frascati

LNF-69/61

C. Castagnoli, E. Etim and P. Picchi : IMPLICATIONS OF DEEP INELASTIC MUON-NUCLEON SCATTERING ON COSMIC-RAY PHENOMENA

Estratto da : Lett. Nuovo Cimento 2, 595 (1969)

Implications of Deep Inelastic Muon-Nucleon Scattering on Cosmic-Ray Phenomena.

C. CASTAGNOLI and E. ETIM (*)
Laboratorio di Cosmo-Geofisica del CNR - Torino

P. PICCHI
Laboratori Nazionali del CNEN - Frascati

(ricevuto il 5 Agosto 1969)

Previous analyses of lepton-nucleon scattering have made extensive use of the Weizsäker-Williams approximation and improvements thereof according to which the cross-section $\sigma_t(\nu, q^2)$ for the absorption of transverse photons on a nucleon can be expanded in a convergent series for small values of the invariant momentum transfer q^2 (1,2),

$$\sigma_t(\nu, q^2) = \sigma_\gamma(\nu)(1 + aq^2 + bq^4 + \ldots)\,. \tag{1}$$

where $\sigma_\gamma(\nu)$ is assumed finite and q^2-indipendent, and a is of the order of the nuclear size $\approx 1/M^2$.

In this letter we show, by analysis of cosmic-ray experiments, that for large values of the photon energy $\nu \to \infty$ the validity of eq. (1) is doubt and a different approach is to be preferred. We further show that the Bjorken (3) inelastic sum rule for asymptotic cross-sections is not saturated.

The cross-section for inelastic muon-nucleon scattering is given in terms of the Drell-Walecka (4) structure functions $W_1(\nu, q^2)$, $W_2(\nu, q^2)$ by

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\nu\,\mathrm{d}q^2} = \frac{4\pi\alpha^2}{q^4}\frac{1}{E^2}\left[\frac{1}{2}q^2 W_1(\nu, q^2) + \left(E^2 - E\nu - \frac{1}{4}q^2\right)W_2(\nu, q)^2\right]. \tag{2}$$

(*) On leave of absence from the Department of Physics, University of Ife, Ibadan.

(1) C. M. HOFFMAN, A. D. LIBERMAN, E. ENGELS, D. C. IMRIE, P. G. INNOCENTI, R. WILSON, C. ZAJDE, W. A. BLAMPIED, P. G. STAIRS and D. DRICKEY: *Phys. Rev. Lett.*, 22, 659 (1969); J. A. KIRK, M. COTTRELL, J. J. LORD and R. J. PISERCHIO: *Nuovo Cimento*, 40 A, 523 (1965); *Bull. Am. Phys. Soc.*, 12, 917 (1967).

(2) C. CASTAGNOLI, P. PICCHI and R. SCRIMAGLIO: *Nucl. Phys.*, 87, 641 (1967) and references therein.

(3) J. BJORKEN: *Phys. Rev. Lett.*, 16, 408 (1966); SLAC-PUB-510 (1968).

(4) S. DRELL and J. WALECKA: *Ann. of Phys.*, 28, 18 (1964).

The form factors W_1, W_2 are related to the total cross-sections σ_T, σ_L for the absorption of transverse and longitudinal photons on a nucleon; one has

$$(3)\qquad \begin{cases} W_1(\nu, q^2) = \dfrac{1}{4\pi^2\alpha} K\sigma_T(\nu, q^2)\,, \\ W_2(\nu, q^2) = \dfrac{1}{4\pi^2\alpha} K \dfrac{q^2}{q^2+\nu^2} (\sigma_T + \sigma_L)\,, \end{cases}$$

where M is the nucleon mass and

$$K = \nu - \frac{q^2}{2M}\,.$$

According to vector dominance for small values of $q^2 < m_\rho^2$,

$$(4)\qquad \begin{cases} \sigma_T(\nu, q^2) = \sigma_\gamma(\nu)\left(\dfrac{m_\rho^2}{m_\rho^2 + q^2}\right)^2, \\ \sigma_L(\nu, q^2) \simeq 0\,. \end{cases}$$

Substituting from (3) and (4) in (2) gives for $q^2 \ll \nu^2$

$$(5)\qquad \frac{d^2\sigma}{d\nu\, dq^2} = \frac{\alpha}{\pi q^2 \nu}\left(1 - \frac{\nu}{E} + \frac{\nu^2}{2E}\right)\sigma_\gamma(\nu)\left(\frac{m_\rho^2}{m_\rho^2 + q^2}\right)^2.$$

Equation (5) is of the same form as that obtained by means of the Weizsäcker-Williams method (5). In fact for $q^2 < m_\rho^2$ one can expand $\sigma_T(\nu, q^2)$ in eq. (4) in a series as in eq. (1).

$a = 0$ is the Weizsäcker-Williams approximation and $a = -2/0.365$ that of DAIYASU *et al.* (6). Equations (4) and (5) are in agreement with experiment for ν not too large (1,2).

An entirely different approach to eq. (5) is suggested here by an extrapolation to very high energy transfer $\nu \geqslant 10^3$ GeV of the information supplied by fits to the SLAC data (7). At a scattering angle of 6° and ν up to 9 GeV the group at SLAC finds that

$$(6)\qquad \nu W_2(\nu, q^2) \simeq \text{const} \geqslant 0.3$$

when $\nu/q^2 > 5\,(\text{GeV})^{-1}$. If for $\nu \to \infty$ this holds for all $\nu/q^2 > 5\,(\text{GeV})^{-1}$ one finds from eq. (3) even for small q^2

$$(7)\qquad \sigma_T + \sigma_L = \frac{4\pi^2\alpha}{q^2}\, C\,,$$

where C is the constant limit in eq. (6).

(5) C. F. V. WEIZSÄCKER: *Zeits. f. Phys.*, **88**, 612 (1934); E. J. WILLIAMS: *Kgl. Dansk. Vid. Selsk.*, **13**, no. 4 (1935); D. KESSLER and P. KESSLER: *Nuovo Cimento*, **6**, 601 (1956).

(6) K. DAIYASU, K. KABAYAKAWA, T. MUROTA and T. NAKANO: *Journ. Phys. Soc. Japan*, **17**, Suppl. A-III, 344 (1962).

(7) E. BLOOM, D. COWARD, H. STAEBLER, J. DRESS, J. LITT, G. MILLER, L. MOO, R. E. TAYLOR, M. BREIDENBACH, J. I. FRIEDMAN, H. W. KENDALL and S. LAKEN: *Proc. XIV Intern. Conf. on High-Energy Physics* (Vienna, 1968).

Equation (7) is very different from eqs. (1) and (4) and would imply, if confirmed, a breakdown of the Weizsäker-Williams approximation and of vector dominance. These implications of eq. (7) are surprising and very far-reaching but, as will be shown below, cosmic-ray data on muons with $\nu \geqslant 10^3$ GeV and q^2 about 0.1 (GeV)^2 seem to be fitted only using the parametrization (6).

We have examined the data of the Japanese INS group [8] on large horizontal air showers and we find that the vector-dominance approximation eq. (5) is unable to fit them, while the parametrization (6) gives the only goodfit with $C = 0.3$. The situation is illustrated in Fig. 1, in which the integral flux

$$J(\nu_{\min}) = \int_{\nu_{\min}}^{\infty} \mathrm{d}E\, M(E) \int_{\nu_{\min}}^{E} \mathrm{d}\nu \frac{\mathrm{d}\sigma}{\mathrm{d}\nu} N_A \Delta X\,, \tag{8}$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\nu\,\mathrm{d}q^2} = \frac{4\pi\alpha^2 C}{q^4} \frac{1}{E^2\nu} \cdot \left[\frac{1}{2}\frac{\sigma_T}{\sigma_T+\sigma_L} + \left(E^2 - E\nu - \frac{1}{4}q^2\right)\right]$$

is plotted against the minimum energy transfer $\nu_{\min}$ in the two extreme cases

$$\frac{\sigma_T}{\sigma_T+\sigma_L} \text{ equal to zero and one}\,.$$

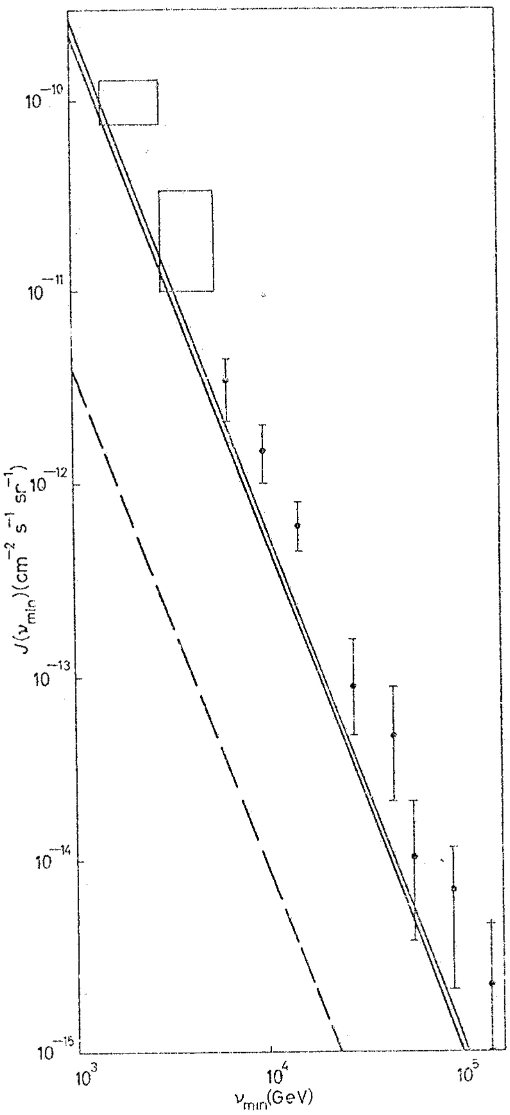

Fig. 1. – Plot of the integral flux $J(\nu_{\min})$ against the minimum energy transfer $\nu_{\min}$. The dashed curve is the result of vector dominance; the full curve is drawn using the parametrization in eq. (6). The average zenith angle of the INS events is $\simeq 80°$ and the muon spectrum $M(E)$ is taken from ref. [9].

---

[8] T. Matano, M. Nagano, S. Shibata, K. Suga, T. Kameda, Y. Toyoda, T. Maeda and H. Hasegawa: *Proc. Intern. Conf. Cosmic Rays* (London, 1965), p. 1045; *Proc. Intern. Conf. Cosmic Rays* (Calgary, 1967), M4 IV-22. Originally the disagreement of vector dominance with experiment triggered speculations about an anomalous muon-nucleon coupling or that the showers observed were of strongly interacting parents. Cf. C. Castagnoli, E. Etim and P. Picchi: *Lett. Nuovo Cimento*, **1**, 197 (1969). A different approach based on the bremsstrahlung effect calculated according to the modified Utah muon spectrum is given by K. Sitte: *Lett. Nuovo Cimento*, **1**, 252 (1969).

[9] M. G. K. Menon and P. V. Ramana Murthy: *Progress in Cosmic-Ray and Elementary-Particle Physics*, vol. 9 (Amsterdam, 1967); J. L. Osborne, A. W. Wolfendale and N. S. Palmer: *Proc. Phys. Soc.*, **84**, 911 (1964).

$M(E)$ is the conventional differential muon spectrum, N_A Avogadro's number and $\Delta X = 400\ \text{g cm}^{-2}$ the mean distance from the starting point of the event to that of observation. The minimum and maximum values of q^2 used are

$$q^2_{\min} = \nu^2 m^2/[E(E-\nu)]\,, \qquad q^2_{\max} = 2M\nu\,,$$

m is the muon mass.

Figure 1 confirms eq. (7) and gives a clear indication of the nonvalidity of eq. (5). We have also used the parametrization (6) to compute the integral spectrum of the transfer q^2 for $\nu_{\min} \geqslant 5$ GeV. After normalization at $q^2 = 2\ (\text{GeV})^2$ we compare our results with the data of HIGASHI *et al.* (9) (Table I). Agreement is again good.

TABLE I. – *Integral spectrum of the momentum transfer q^2 for $\nu_{\min} > 5$ GeV.*

q^2 $((\text{GeV})^2)$	$\sigma_T = 0$	$\sigma_L = 0$	Vector dominance	Experimental (Higashi)
2	10	10	10	10 ± 2.0
5	3	3.16	0.97	5 ± 1.4
10	0.75	0.9	0.098	2.3 ± 1.1

The fact that the parametrization of (6) seems to be valid up to energy transfers $\nu \geqslant 10^3$ GeV calls for a comment on the Bjorken inelastic sum rule (3)

$$\int \mathrm{d}\nu\, W_2(\nu, q^2) \geqslant \frac{1}{4}\,. \tag{9}$$

That eq. (6) is valid for $\nu \geqslant 10^3$ GeV means that there is no finite ν at which the sum rule saturates; its usefulness is therefore in doubt. Written in terms of the differential cross-section $\mathrm{d}\sigma/\mathrm{d}q^2$ eq. (9) reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2} = \int \mathrm{d}\nu \frac{\mathrm{d}^2\sigma}{\mathrm{d}\nu\,\mathrm{d}q^2} \geqslant \frac{1}{4}\frac{\mathrm{d}\sigma_\beta}{\mathrm{d}q^2}\,, \tag{10}$$

where $\mathrm{d}\sigma_\beta/\mathrm{d}q^2$ is the cross-section for scattering off a point nucleon. If the Bjorken sum rule were nontrivially satisfied, eq. (10) would imply (quite dramatically) the existence of pointlike constituents (partons) within the nucleon (11). There is in fact a proliferation of attempts to explain inelastic electron-proton scattering with a parton model (12). Despite of this we wish to argue that the pointlike behaviour of the inelastic cross-section $\mathrm{d}\sigma_{\text{inel}}/\mathrm{d}q^2$ can be easily understood considering that, indipendently of what happens at the nucleon vertex during an electron-proton collision, the invariant momentum transfer q^2 is well defined; therefore from the conservation of

(10) S. HIGASHI, T. KITAMURA, Y. MISHIMA, S. MITANI, S. MIYAMOTO, T. OSHIO, H. SHIBATA, K. WATANABE and Y. WATASE: *Journ. Phys. Soc. Japan*, **17**, Suppl. A-III, 362 (1962).
(11) V. WEISSKOPF: *Comm. Nucl. Part. Phys.* (Jan.-Feb. 1969), p. 1.
(12) S. DRELL, D. J. LEVY and T. M. YAN: SLAC PUB 55 6 (1969).

probability

$$\frac{d\sigma_{tot}}{dq^2} = \frac{d\sigma_{el}}{dq^2} + \frac{d\sigma_{inel}}{dq^2}, \tag{11}$$

where σ_{tot}, σ_{el}, σ_{inel} are respectively the total, elastic and inelastic cross-sections. Defining the probability $P(E, q^2)$ for a proton not to break up in a collision with momentum transfer q^2 and laboratory energy E by the ratio

$$P(E, q^2) = \frac{d\sigma_{el}}{dq^2} \Big/ \frac{d\sigma_{tot}}{dq^2} \tag{12}$$

and remembering that for E, $q^2 \to \infty$,

$$\frac{d\sigma_{el}}{dq^2} \simeq \frac{d\sigma_\beta}{dq^2} G_M^2(q^2) = \frac{4\pi\alpha^2}{q^4} G_M^2(q^2) \tag{13}$$

with $G_M(q^2)$ the proton magnetic form factor, we have from (11), (12) and (13) in the limit $E, q^2 \to \infty$,

$$\frac{d\sigma_{inel}}{dq^2} = \frac{d\sigma_\beta}{dq^2} \frac{G_M^2(q^2)}{P(E, q^2)}. \tag{14}$$

$P(E, q^2)$ by its very definition is a form factor and therefore when E, q^2 tend to infinity one expects $G_M^2(q^2)/P(E, q^2)$ to tend to a constant of the order of unity or at most to a function which is slowly varying. That this is so can be seen by looking at high-energy elastic proton-proton scattering at large momentum transfers. Since two protons are involved here the elastic cross-section is given by

$$\frac{d\sigma_{pp}}{dq^2} = \left(\frac{d\sigma_{pp}}{dq^2}\right)_{q^2=0} P^2(E, q^2) = \left(\frac{d\sigma_{pp}}{dq^2}\right)_{q^2=0} G_M^4(q^2) \left(\frac{P(E, q^2)}{G_M^2(q^2)}\right)^2. \tag{15}$$

The Wu-Yang [13] asymptotic behaviour follows from (15) if $G_M^2(q^2)/P(E, q^2)$ tends to one or at most is slowly varying. The pointlike nature of $d\sigma_{inel}/dq^2$ in eq. (14) then follows from the above property of the ratio $G_M^2(q^2)/P(E, q^2)$. According to the above argument the remarkably large cross-section $d\sigma_{inel}/dq^2$ observed in the deep inelastic region does not necessarily imply the existence of pointlike constituents of the nucleon but is related to the fact that the inelastic form factor (in our notation $G_M^2(q^2)/P(E, q^2)$) does not vary (or vary slowly) with q^2.

* * *

We would like to thank Prof. R. WILSON for very useful conversations.

---

[13] T. T. WU and C. N. YANG: *Phys. Rev.*, **137**, B 708 (1965); M. GRECO: *Phys. Lett.*, **27** B, 578 (1968).