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C. Castagnoli[(+)], E. Etim[(+o)] and P. Picchi: IMPLICATIONS OF DEEP INELASTIC MUON-NUCLEON SCATTERING ON COSMIC-RAY PHENOMENA.

Previous analyses of lepton-nucleon scattering have made extensive use of the Weizsaker-Williams approximation and improvements there of according to which the cross-section $\sigma_t(\nu, q^2)$ for the absorption of transverse photons on a nucleon can be expanded in a convergent series for small values of the invariant momentum transfer q^2 (1, 2),

(1) $$\sigma_T(\nu, q^2) = \sigma_\gamma(\nu)\,(1 + aq^2 + bq^4 + \dots)$$

In this letter we show, by analysis of cosmic ray experiments, that for large values of the photon energy $\nu \longrightarrow \infty$ the validity of the Eq. (1) is very much in doubt.

We further show that the Bjorken (3) inelastic sum rule for asymptotic cross-sections is not saturated.

(+) - Laboratorio di Cosmo-Geofisica del CNR - Torino.
(o) - On leave of absence from the Department of Physics, University of Ife, Ibadan, Nigeria.

The cross-section for inelastic muon-nucleon scattering is given in terms of the Drell-Walecka[4] structure functions

$$W_1(\nu, q^2), \quad W_2(\nu, q^2)^{(4)}$$

by

$$\text{(2)} \qquad \frac{d^2\sigma}{d\nu\, dq^2} = \frac{4\pi\alpha^2}{q^4}\,\frac{1}{E^2}\,\frac{1}{2}\,q^2 W_1(\nu, q^2) + (E^2 - E\nu - \frac{1}{4}q^2)\, W_2(\nu, q^2)$$

The form factors W_1, W_2 are related to the total cross-sections σ_T, σ_L for the absorption of transverse and longitudinal photons on a nucleon; one has

$$W_1(\nu, q^2) = \frac{1}{4\pi^2\alpha}\, K\, \sigma_T(\nu, q^2)$$

(3)

$$W_2(\nu, q^2) = \frac{1}{4\pi^2\alpha}\, K\, \frac{q^2}{q^2 + \nu^2}\, (\sigma_T + \sigma_L)$$

where $K = \nu - \frac{q^2}{2M}$

and M is the nucleon mass.

According to vector dominance for small values of $q^2 < m_\rho^2$,

$$\sigma_T(\nu, q^2) = \sigma_\gamma(\nu) \left(\frac{m_\rho^2}{m_\rho^2 + q^2}\right)^2$$

(4)

$$\sigma_L(\nu, q^2) = 0$$

Substituting from (3) and (4) in (2) gives for $q^2 \ll \nu^2$

$$\text{(5)} \qquad \frac{d^2\sigma}{d\nu\, dq^2} = \frac{\alpha}{\pi q^2 \nu}\left(1 - \frac{\nu}{E} + \frac{\nu^2}{2E}\right) \sigma_\gamma(\nu) \left(\frac{m_\rho^2}{m^2 + q^2}\right)^2$$

Eq. (5) is of the same form as that obtained by means of the Weizsaker-Williams method[5]. In fact for $q^2 < m_\rho^2$ one can expand $\sigma_T(\nu, q^2)$ in eq. (4) in a series as in eq. (1).

$a = 0$ is the Weizsaker-Williams approximation and $a = -\frac{2}{0{,}365}$ that of Daiyasu et al. [6]. Eqs. (4) and (5) are in agreement with experiment for ν not too large[1, 2].

An entirely different approach to eq. (5) is suggested here by an extrapolation to very high energy transfer 10^3 GeV of the information supplied by fits to the SLAC data(7). At a scattering angle of 6^{o} and up to 9 GeV the group at SLAC finds that

$$(6) \qquad \nu W_2(\nu, q^2) \cong \text{const.} \geqslant 0.3$$

when $\frac{\nu}{q^2} > 5\ \text{GeV}^{-1}$. If for $\nu \longrightarrow \infty$ this holds for all $\frac{\nu}{q^2} > 5\ \text{GeV}^{-1}$ one finds from eq. (3) even for small q^2

$$(7) \qquad \sigma_T + \sigma_S = \frac{4\pi^2\alpha}{q^2}\, C$$

where C is the constant limit in eq. (6).
Eq. (7) is very different from eqs. (1) and (4) and would imply, if confirmed, a break-down of the Weizsaker-Williams approximation and of Vector dominance. These implications of eq. (7) are surprising and very far-reaching but, as will be shown below, cosmic ray data on muons with $\nu \geqslant 10^3$ GeV and q^2 about 0, 1 GeV^2 can only be fitted using the parametrization (6).

We have examined the data of the Japanese INS group(8) on large horizontal air showers and we find that the vector dominance approximation eq. (5) is unable to fit them while the parametrization (6) gives the only good fit with C = 0, 3. The situation is illustrated in Fig. 1, in which the integral flux

$$(8) \qquad J(\nu_{min}) = \int_{\nu_{min}}^{\infty} dE\, M(E) \int_{\nu_{min}}^{E} d\nu \frac{d\sigma}{d\nu} N_A\, \Delta X$$

$$\frac{d^2\sigma}{d\nu\, dq^2} = \frac{4\pi\alpha^2 C}{q^4} \frac{1}{E^2\nu}\left[\frac{1}{2}\frac{\sigma_T}{\sigma_T + \sigma_S} + (E^2 - E\nu - \frac{1}{4} q^2)\right]$$

is plotted against the minimum energy transfer ν_{min} in the two extreme cases

$$\frac{\sigma_T}{\sigma_T + \sigma_L} \quad \text{equal to zero and one}$$

M(E) is the conventional differential muon spectrum, N_A Avogadro's number and ΔX = 400 gm cm^{-2} the mean distance from the starting point of the event to that of observation. The minimum and maximum values of q^2 used are

$$q^2_{min} = \nu^2 m^2 / \left[E(E - \nu)\right]$$

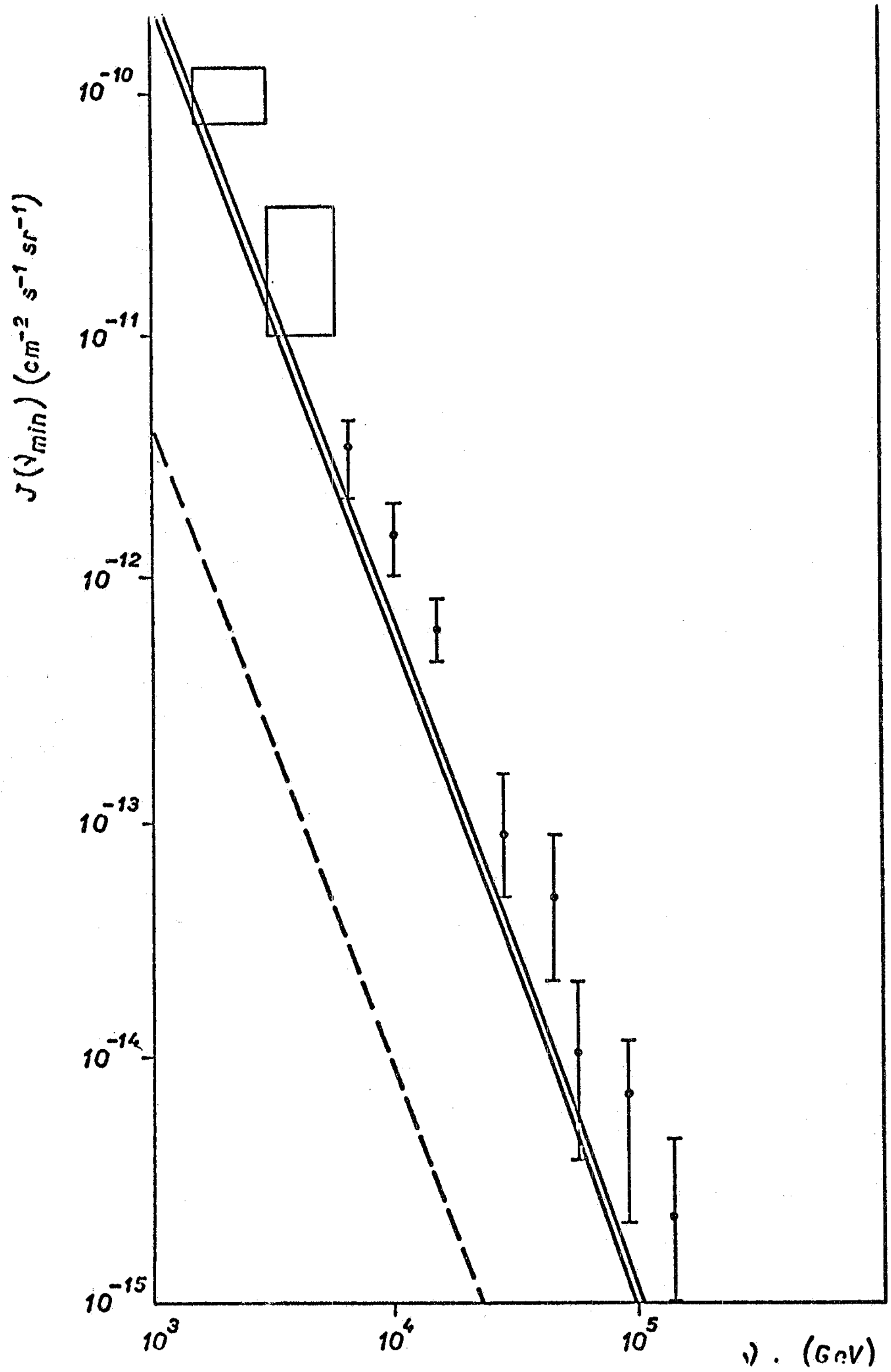

FIG. 1 - Plot of integral flux $J(\nu_{min})$ against the minimum energy transfer ν_{min}. Dashed curve is the result of vector dominance; full curve using the parametrization in eq. (6). The average zenith angle of the INS events is $\simeq 80^{o}$ and the muon spectrum M(E) is from ref. (13).

$$q^2_{max} = 2 M \nu$$

m is the muon mass.

Fig. 1 confirms eq. (7) and gives a clear indication of the non-validity of eq. (5). We have also used the parametrization (6) to compute the integral spectrum of the momentum transfer q^2 for $\nu_{min} \geqslant 5$ GeV. After normalization at $q^2 = 2$ GeV^2 we compare our results with the data of Higashi et al.[9] (table 1). Agreement is again good.

TABLE 1

Integral spectrum of the momentum transfer q^2 for ν_{min} 5 GeV

q^2 (GeV)^2	$\sigma_T = 0$	$\sigma_S = 0$	Vector Dominance	(Higashi) experimental
2	10	10	10	10 ± 2.0
5	3	3.16	.97	5 ± 1.4
10	.75	.9	.098	2.3 ± 1.1

The fact that the parametrization of (6) is valid up to energy transfers $\nu \geqslant 10^3$ GeV calls for a comment on the usefulness of the Bjorken inelastic sum rule[3]:

(9) $$\int d\nu \; W_2 \,(\nu, q^2) \geqslant \frac{1}{4}$$

That eq. (6) is valid for $\nu \geqslant 10^3$ GeV means that there is no finite at which the sum rule saturates; its usefulness is therefore very much in doubt. Written in terms of the differential cross-section d /dq^2 eq. (9) reads

(10) $$\frac{d\sigma}{dq^2} = \int d\nu \, \frac{d^2\sigma}{d\nu \, dq^2} \geqslant \frac{1}{4} \left(\frac{d\sigma_\beta}{dq^2}\right)$$

where $d\sigma_\beta / dq^2$ is the cross-section for scattering off a point nucleon. If the Bjorken sum rule were non-trivially satisfied eq. (10) would imply quite dramatically the existence of point-like constituents (partons) within the nucleon[10]. There is in fact a proliferation of attempts to explain inelastic electron-proton scattering with a parton model[11]. Despite such a proliferation we wish to argue that the point-like behaviour of the inelastic cross-section $d\sigma_{inel}/dq^2$ can be very easily understood.

Indipendently of what happens at the nucleon vertex during an electron-proton collision the invariant momentum transfer q^2 is well defined; therefore from the conservation of probability

$$\frac{d\sigma_{tot}}{dq^2} = \frac{d\sigma_{el}}{dq^2} + \frac{d\sigma_{inel}}{dq^2} \tag{11}$$

where σ_{tot}, σ_{el}, σ_{inel} are respectively the total, elastic and inelastic cross-section. Defining the probability $P(E, q^2)$ for a proton not to break up in a collision with momentum transfer q^2 and laboratory energy E by the ratio:

$$P(E, q^2) = \left(\frac{d\sigma_{el}}{dq^2}\right)/\left(\frac{d\sigma_{tot}}{dq^2}\right) \tag{12}$$

and remembering that for E, $q^2 \longrightarrow \infty$

$$\frac{d\sigma_{el}}{dq^2} = \frac{d\sigma_{\beta}}{dq^2} G_M^2 (q^2) = \frac{4\pi\alpha^2}{q^4} G_M^2 (q^2) \tag{13}$$

with $G_M (q^2)$ the proton magnetic form factor we have from (11), (12) and (13) in the limit E, $q^2 \longrightarrow \infty$

$$\frac{d\sigma_{inel}}{dq^2} = \frac{d\sigma_{\beta}}{dq^2} \frac{G_M^2 (q^2)}{P(E, q^2)} \tag{14}$$

$P (E, q^2)$ by its very definition is a form factor and therefore when E, q^2 tend to infinity one experts $G_M^2(q^2)/P(E, q^2)$ to tend to a costant of the order of unity or at most a function which is slowly varying. That this is so can be seen by looking at high energy elastic proton-proton scattering at large momentum transfers. Since two protons are involved here the elastic cross-section is given by

$$\begin{aligned} \frac{d\sigma_{pp}}{dq^2} &= \left(\frac{d\sigma_{pp}}{dq^2}\right)_{q^2=0} P^2 (E, q^2) \\ &= \left(\frac{d\sigma_{pp}}{dq^2}\right)_{q^2=0} G_M^4(q^2) \left(\frac{P(E, q^2)}{G_M^2(q^2)}\right)^2 \end{aligned} \tag{15}$$

The Wu-Yang[12] asymptotic behaviour follows from (15) if $G_M^2(q^2)/P(E, q^2)$ tends to one or at most slowly varying. The point-like nature of $d\sigma_{inel}/dq^2$ in eq. (14) then follows from the above property of the ratio $G_M^2(q^2)/P(E, q^2)$. According to the above argument the remarkably large cross-section $d\sigma_{inel}/dq^2$ oberserved in the deep inelastic region does not necessarily imply the existence of point-like constituents of the nucleon but is related to the fact that the inelastic form factor (in our notation $G_M^2(q^2)/P(E, q^2)$ does not vary with q^2.

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