F. Drago: CHARGED PION PHOTOPRODUCTION VENEZIANO
REPRESENTATION AND THE PROBLEM OF PION CONSPIRACY.

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F. Drago[x]: CHARGED PION PHOTOPRODUCTION VENEZIANO RE
PRESENTATION AND THE PROBLEM OF PION CONSPIRACY[+].

1. - SUMMARY AND CONCLUSIONS -

We study the Veneziano representation for charged pion photo-
production near the forward direction. The simplest (i. e. without sa-
tellites) representation that contains the gauge invariant Born term re
quires the existence of an M = 1 pion trajectory. The model predicts
correctly some of the peculiar features of the parity doublet which a-
re necessary to fit the data.

Due to the difficulties associated with the M = 1 assignment,
we studied the possibility of constructing solutions with an M = 0 pion
trajectory. In the framework of the model this can be done in an es-
sentially arbitrary way, by adding suitably chosen satellites.

2.

2. - INTRODUCTION AND NOTATIONS -

The mechanism of the reaction $\gamma + p \rightarrow \pi^+ + n$ near the forward direction is still unclear. The most economical interpretation of the now well known narrow forward peak is in terms of a parity doublet conspiracy of the pion (that means that the pion trajectory belongs to a Toller family with $M = 1$). However the $M = 1$ assignment for the pion leads to both theoretical and phenomenological difficulties. If on the other hand the pion trajectory has $M = 0$ its contribution to the photoproduction cross section vanishes at $t = 0$ and models with poles and cuts are necessary to fit the data[1].

We will discuss here an application of the Veneziano model[2] to the photoproduction of $\pi^\pm$ near the forward direction. Only two amplitudes (for each isospin state) are important near the forward direction. We will mainly use t-channel helicity amplitudes free from kinematical singularities: $F_2(s,t)$ and $F_3(s,t)$ are defined in Ref. 3[x]. The amplitude $F_2^{(-)}(s,t)$ receives asymptotic contributions from t-channel exchanges with the quantum numbers of the pion and $F_3^{(-)}(s,t)$ from the exchange of trajectories with all the same quantum numbers of the pion, but with opposite natural parity (natural parity=parityxsignature).

At $t = 0$ the two amplitudes are related by the constraint equation[3]

$$(1) \qquad F_2^{(-)}(s, t=0) = -\frac{m_\pi^2}{2M} F_3^{(-)}(s, t = 0)$$

3. - THE MODEL -

Let us start with the amplitude $F_2^{(-)}(s,t)$. We assume that it receives contibution from the nucleon and Δ trajectories in the s and u channel and from the pion trajectory in the t-channel. The simplest Veneziano amplitude that contains the gauge invariant Born term is

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(x) - These amplitudes are related to the CGLN invariant amplitudes[4] in the following way:

$$\frac{F_2^{(-)}(s, t)}{t - m_\pi^2} = A_1^{(-)}(s, t) + t A_2^{(-)}(s, t)$$

$$F_3^{(-)}(s, t) = 2MA_1^{(-)}(s, t) - tA_4^{(-)}(s, t)$$

$$\frac{F_2^{(-)}(s,t)}{t-m_\pi^2} = -\beta \left\{ \frac{\Gamma\left[-\alpha_\pi(t)\right]\Gamma\left[1/2-\alpha_N(s)\right]}{\Gamma\left[3/2-\alpha_\pi(t)-\alpha_N(s)\right]} - (s \to u) + \right.$$

(2)

$$\left. +p\left[\frac{\Gamma\left[1-\alpha_\pi(t)\right]\Gamma\left[\frac{1}{2}-\alpha_\Delta(s)\right]}{\Gamma\left[3/2-\alpha_\pi(t)-\alpha_\Delta(s)\right]}-(s\to u)+\frac{\Gamma\left[\frac{1}{2}-\alpha_N(s)\right]\Gamma\left[\frac{1}{2}-\alpha_\Delta(u)\right]}{\Gamma\left[1-\alpha_N(s)-\alpha_\Delta(u)\right]}-(s\to u)\right]\right\}$$

We do not discuss here the problem of the baryon parity doublets. We only note that for the parity doublet of the nucleon there is no trouble, since ot does not contribute to this amplitude; moreover since no parity doublet of the Δ has been observed, we have put its residue equal to zero in (2). The parameters β and p are obtained by comparing the residues at $\alpha_\pi(t = m_\pi^2) = 0$ and $\alpha_N(s = M^2) = 1/2$ in (2) with those obtained from pion and nucleon poles in the Born approximation. We get

(3)
$$\beta = e\,g\,\alpha'^2 m_\pi^2$$

and

(4)
$$p = -\frac{1}{2\alpha' m_\pi^2}$$

where $e^2/4\pi = 1/137$, $g^2/4\pi = 14.7$ and α' is the common slope of the linear Regge trajectories involved.

The representation (2) has also been discussed in a recent Letter[5] by Ahmad, Fayyazuddin and Riazuddin. However the interpretation that they gave to (2) is completely different fro the one presented below.

We will now show that the leading singularity in the t-channel angular momentum plane is a Regge pole corresponding to a pion with M = 1. Moreover the model naturally embodies some of the peculiar properties of the residue functions that are needed to fit the data.

From Eq. (2) we get, for large s and small t

(5)
$$F_2^{(-)}(s,t) = -\frac{\beta}{2\alpha' m_\pi^2}(t+m_\pi^2)\frac{\pi\alpha_\pi(t)}{(\alpha_\pi(t)+1)}\frac{1+e^{-i\pi\alpha_\pi(t)}}{\sin\pi\alpha_\pi(t)}(\alpha's)^{\alpha_\pi(t)-1}$$

This amplitude is Regge-behaved and non zero at t=0. Due to Eq. (1) this means that another trajectory $\alpha_c(t)$, with the same quantum numbers of the pion, but with opposite parity, such that $\alpha_c(0) = \alpha_\pi(0)$,

4.

must be present in the model: due to the universality of the slope α' the two trajectories will actually be degenerate in the present model.

We stress once again that Eq. (1) is not at all trivial, even if, expressed in terms of invariant amplitudes it reads $A_1(s, t=0)=A_1(s, t=0)$. This only means in the framework of pure Regge pole models, that if $A_1(s, t = 0)$ is Regge behaved and non zero at $t = 0$, then we have a parity doublet exchange in the t-channel.

We can now easily construct a representation for $F_3^{(-)}(s, t)^{(x)}$ that satisfies Eq. (1):

$$F_3^{(-)}(s,t) = -\gamma \left\{ \frac{\Gamma\left[1-\alpha_c(t)\right]\Gamma\left[1/2-\alpha_N(s)\right]}{\Gamma\left[\frac{3}{2}-\alpha_c(t)-\alpha_N(s)\right]} - (s \to u) + \right.$$

$$\left. + q \left[\frac{\Gamma\left[1-\alpha_c(t)\right]\Gamma\left[\frac{1}{2}-\alpha_\Delta(s)\right]}{\Gamma\left[\frac{3}{2}-\alpha_c(t)-\alpha_\Delta(s)\right]} -(s \to u) + \frac{\Gamma\left[\frac{1}{2}-\alpha_N(s)\right]\Gamma\left[\frac{1}{2}-\alpha_\Delta(u)\right]}{\Gamma\left[1-\alpha_N(s)-\alpha_\Delta(u)\right]} - (s \to u) \right] \right\} +$$

+ non leading terms for large s and small t

with $\gamma = 2Meg\alpha'$ and $q = -1/2$.

From Eq. (6) we obtain for large s and small t

(7) $$F_3^{(-)}(s,t) = \frac{\gamma}{2} \frac{\pi\alpha_c(t)}{\Gamma(\alpha_c(t)+1)} \frac{1+e^{-i\pi\alpha_c(t)}}{\sin\pi\alpha_c(t)} (\alpha's)^{\alpha_c(t)-1}$$

We will now briefly discuss some very interesting results which are automatically contained in the model:

1) as also remarked by Ahmad, Fayyazuddin and Riazuddin, in the limit $s \to \infty$, $\alpha_\pi(t) = 0$ the model reduces to the electric Born term, which is known to give a resonable fit to the forward cross section.

2) the pion residue function is proportional to $(1+1/2(t-m^2)/m^2)$ and therefore varies very rapidly in the interval $-m_\pi^2 \leq t \leq m_\pi^2$. A rapid variation of this kind is known to be necessary to fit both the magnitude of the differential cross section and the data with polarized photons with

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(x) - Note that $F_3^{(-)}(s,t)$ being a nonsense-nonsense amplitude at $\alpha(t)=0$ cannot contain a pole at $\alpha(t) = 0$.

an M = 1 pion. This behavior has to be compared with the one obtained from high energy[3] and continuous moment sum rules[6] fits:
$(1 + \lambda (t-m_\pi^2)/m_\pi^2)$, typical values of λ being $\lambda = 0.42$.

In other words the model predict a very rapid variation of the unnatural parity contribution and its vanishing at $t = - m_\pi^2$. This zero is extremely important to explain the behavior of the polarized photon data: the predictions are compared to the data[7] in Fig. 1 and the a-greement is very good in the extremely forward direction. Of course for $-t \gtrsim m_\pi^2$ other trajectories like the ρ, A_2 and B, which are not im portant near t=0, will come into the game will change considerably the predictions shown in the figure.

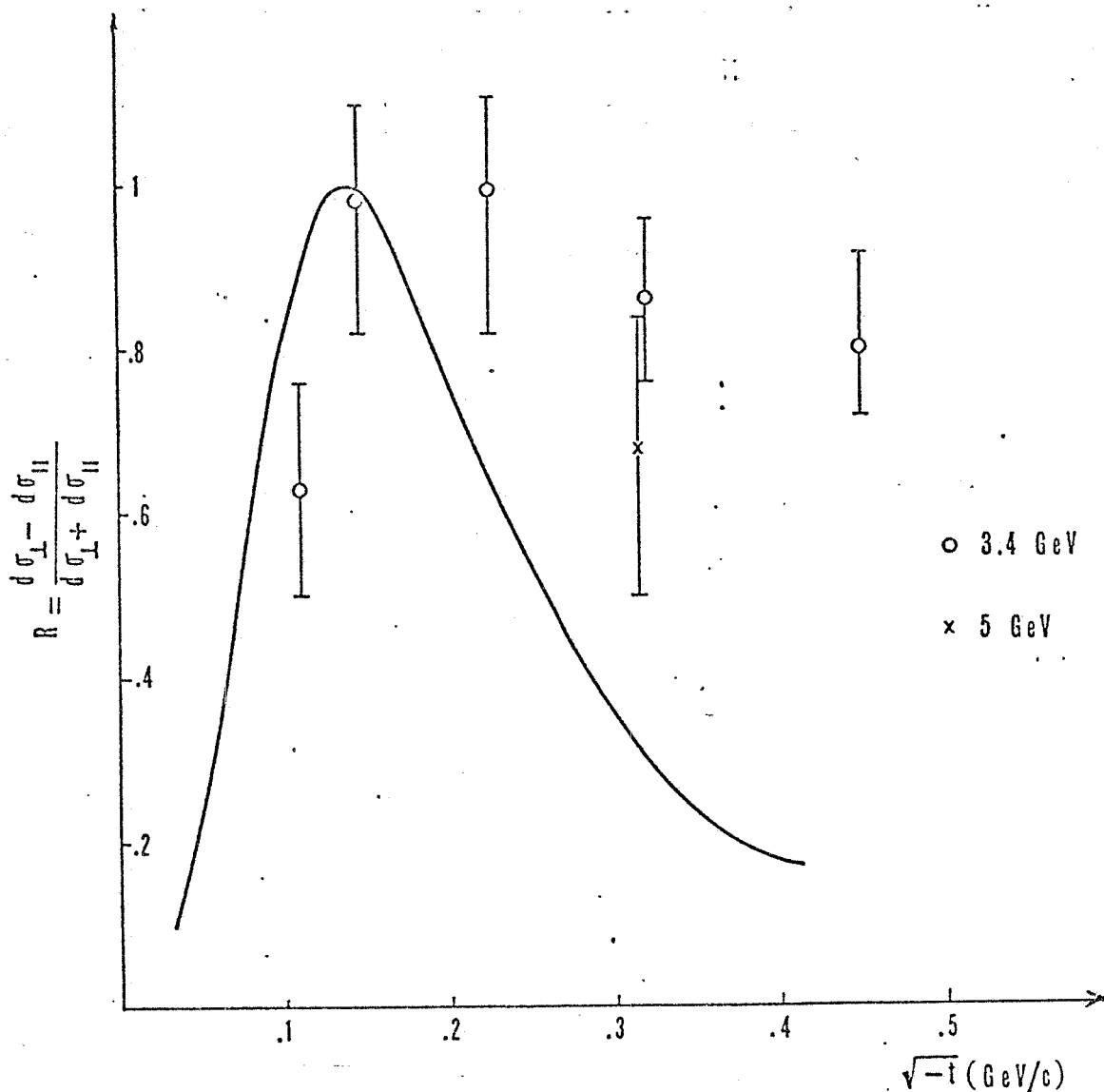FIG. 1 --The quantity R = $(d\sigma_\perp - d\sigma_\parallel)/(d\sigma_\perp + d\sigma_\parallel)$ is compared with the prediction of the model with a conspiring pion. The data are from Ref. 7.

3) from the behavior of the residue of the "conspirator" pole at $\alpha_c(t) = 0$ in the nonsense-nonsense amplitude $F_3^{(-)}$ we see that the "conspirator" trajectory chooses nonsense at $\alpha_c(t) = 0$. That means that no new 0^+ particle is expected. This result is also in agreement with the study of finite energy sum rules[8], once that the M = 1 assignement has been assumed.

We have thus shown that the simplest (i.e. without satellites) Veneziano representation requires an M = 1 pion trajectory and correctly predicts some of the peculiar properties required to fit the data.

However it has all the troubles of the M = 1 models[1] and it is therefore interesting to try to construct an amplitude that contains an M = 0 pion trajectory. In this case of course absorptive corrections are crucial to fit the data.

It is obviously impossible to construct a Veneziano amplitude without satellites and with an M = 0 pion trajectory, since we want it to contain the Born term which has a non zero residue at t = 0.

For simplicity of writing let us just consider the first term in Eq. (2): the discussion of the second term is very similar. We consider the amplitude

$$
-\beta \left\{ \frac{\Gamma\left[-\alpha_\pi(t)\right] \Gamma\left[\frac{1}{2}-\alpha_N(s)\right]}{\Gamma\left[\frac{3}{2}-\alpha_\pi(t)-\alpha_N(s)\right]} + \sum_{r=1}^{N} \gamma_r \times \right.
$$

(8)

$$
\left. \times \frac{\Gamma\left[r-\alpha_\pi(t)\right]\Gamma\left[\frac{1}{2}+r-\alpha_N(s)\right]}{\Gamma\left[\frac{1}{2}+2r-\alpha_\pi(t)-\alpha_N(s)\right]} - (s \to u) \right\}
$$

and we expand it in power series using the method of Ref. 9. We can now choose the coefficients γ_r in such a way that at t = 0 the amplitude (8) behaves as $(\alpha's)^{\alpha_\pi(0)-N}$: that means that we have removed the first (N-1) Toller poles with M = 1 present in the original amplitude.

It can be easily seen that to do their job the γ_r must satisfy the system

$$
A_{n-1}(\alpha'M^2, 1+\alpha'M^2-\alpha_\pi(0)) + \sum_{r=1}^{n} \gamma_r \frac{\Gamma\left[r-\alpha_\pi(0)\right]}{\Gamma\left[-\alpha_\pi(0)\right]} \times
$$

(9)

$$
\times A_{n-r}(r+\alpha'M^2, 2r+\alpha'M^2-\alpha_\pi(0)) = 0
$$

for $1 \leq n \leq N$, where the coefficients $A_n(\lambda, \mu)$ are defined and discussed in Ref. 9. The system (9) is equivalent to a recurrence formula that gives γ_s if γ_1, if γ_1, γ_2 .... γ_{s-1} are known. However it should be remarked that the cancellation mechanism introduced in (8) is highly arbitrary and it is presumably possible to find many other satellite contributions that work equally well. Moreover it is not very clear what happens if we let $N \rightarrow \infty$: in this case in fact the series in (8) must develop a pole at $\alpha_N = 1/2$ which is not present in the single terms and therefore will change the Born term.

By modifing in a similar way the second term in (2) we obtain the asymptotic expansion

$$F_2^{(-)}(s,t) = - eg\alpha't \; \frac{\alpha'(t+m_\pi^2) + 2}{2(\alpha'm_\pi^2 + 1)} \; \frac{\pi\alpha_\pi(t)}{\Gamma(\alpha_\pi(t)+1} \; \times$$

(10)

$$\times \; \frac{1 + e^{-i\pi\alpha_\pi(t)}}{\sin \pi \alpha_\pi(t)} \; (\alpha's)^{\alpha_\pi(t)-1}$$

that obviously corresponds to the exchange of an $M = 0$ pion trajectory.

In conclusion we have shown that the simplest Veneziano representation for charged pion phtoproduction requires pion conspiracy, with all its defects. Moreover there is an essentially infinite freedom in the construction of evasive solutions.

8.

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