Laboratori Nazionali di Frascati

R. Wilson : THE RHO-GAMMA COUPLING AND THE OPTICAL MODEL

The Rho-Gamma Coupling and the Optical Model.

R. WILSON (*)

Laboratori Nazionali del CNEN - Frascati (Roma)

Recent experiments using the electron-positron storage rings at Orsay [1] and Novosibirsk [2] have measured the coupling of the ρ-meson to the virtual photon field at the ρ mass. They find $(\gamma_\rho^2/4\pi) \approx 0.5$. If we assume that the coupling, and the decay of the ρ-meson, are independent of the virtual-photon mass, and also make appropriate corrections for the finite width of the ρ-meson [3,4], the contribution to the pion form factor at $q^2 = 0$ is found to be unity within 10%, showing that the ρ-meson dominates the pion form factor at $q^2 = 0$ also.

The isovector nucleon form factor presents a puzzle of long standing; the radius of the nucleon is larger than the value given by ρ-meson dominance. Although this problem is not definitively solved, several theorists [5-8] have given good reasons why the effective $\rho\mathcal{N}\mathcal{N}$ coupling should vary with q^2 to make the nucleon radius greater than the pion radius.

A comparison [9] of $\sigma_{tot}(\gamma p)$ and $(d\sigma/dt)_{t=0}$ $(\gamma p \to V p)$ for the vector mesons ω, ρ and φ also yields $\gamma_\rho^2/4\pi = \frac{1}{2}$.

It becomes appropriate, therefore, to consider all process near $q^2 = 0$ with the same value of $\rho\gamma$ coupling $\gamma_\rho^2/4\pi = \frac{1}{2}$. In particular, the photoproduction of ρ-mesons is believed to proceed by a diffraction model and a value of $\gamma_\rho^2/4\pi$ can be derived therefrom. A group at DESY [10] fit their data at a photon energy of 4.5 GeV with

[1] J. E. AUGUSTIN, J. C. BIZOT, J. BUON, J. HAISSINSKI, D. LALANNE, P. MARIN, H. NGUYEN NGOC, J. PEREZ-Y-JORBA, F. RUMPF, E. SILVA and S. TAVERNIER: *Phys. Lett.*, **28** B, 7, 508 (1969).
[2] V. L. AUSLANDER *et al.*: to be published (1968).
[3] G. J. GOUNARIS and J. J. SAKURAI: *Phys. Rev. Lett.*, **21**, 244 (1968).
[4] M. ROSS and J. PILSUT: CERN (unpublished) TH 1000 (1968).
[5] S. FURUICHI and K. WATANABE: *Progr. Theor. Phys. (Kyoto)*, **35**, 174 (1966).
[6] P. SIGNELL and J. W. DURSO: *Phys. Rev. Lett.*, **18**, 185 (1967).
[7] M. A. B. BEG, J. BERNSTEIN and M. TAUSNER: *Phys. Rev.*, **173**, 1523 (1968).
[8] S. D. DRELL and D. J. SILVERMAN: *Phys. Rev. Lett.*, **20**, 1325 (1968).
[9] Z. G. T. GUIRAGOSSIAN and A. LEVY: to be published.
[10] J. G. ASBURY, U. BECKER, W. K. BERTRAM, P. JOOS, M. ROHDE, A. J. S. SMITH, C. L. JORDAN and S. C. C. TING: *Phys. Rev. Lett.*, **19**, 865 (1965); J. G. ASBURY, U. BECKER, W. K. BERTRAM, M. BINKLEY, E. COLEMAN, C. L. JORDAN, M. ROHDE, A. J. S. SMITH and S. C. C. TING: *Phys. Rev. Lett.*, **20**, 227 (1968); S. C. C. TING: *XIV Intern. Conference on High-Energy Physics, 1968*.

$\gamma_\rho^2/4\pi = 0.5 \pm 0.1$; whereas groups at Cornell [11] and SLAC [12] find $\gamma_\rho^2/4\pi = 1.0$ to 1.2 at 6 and 9 GeV respectively.

I will take the point of view that $\gamma_\rho^2/4\pi = 0.5$ and try to understand what changes must be made in the photoproduction process in order to understand all the data. I believe that the optical model is, in fact, being applied without taking account of its approximate nature. The apparent conflict between the DESY group on one hand, and the Cornell and SLAC groups on the other, is rendered complex by three factors. The minimum momentum transfer to the nucleus in photoproduction is given by $t_{\min} = -m^4/4k^2$. Yet in lead the variation with momentum transfer is $\exp[400t]$; so to find the value at $t = 0$ an extrapolation is required. At the DESY energies this is a factor of 6.

Secondly, the DESY group finds it convenient to compare its data directly with theory, using a Monte Carlo calculation of efficiency, just like bubble chamber analyses. A comparison is then hard to make.

Thirdly, the photoproduction from most nuclei is only measured at the peak of the ρ-meson spectrum. The cross-section then depends upon the width assumed; DESY and SLAC, with good resolution, measured widths of 145 MeV; the Cornell group finds for carbon a smaller width of 120 MeV, the value found in colliding-beam and leptonic-decay measurements, and uses this for subsequent analysis. The fact that the ρ width measured in $\pi^+\pi^-$ decay from photoproduction is larger than the « real » width from colliding beams is interesting in itself, but is not presently relevant. But to compare results from the 3 groups, we should probably raise the Cornell data by 20%.

At 5 GeV, the data on photoproduction from carbon appear to agree between SLAC and DESY, showing that normalization of γ-ray fluxes is probably not a problem [13]. It is not easy to compare the data from copper and lead, but the published SLAC data show extrapolated forward cross-sections for lead 0.75 times the uncorrected Cornell cross-sections and 0.6 times the Cornell cross-sections corrected as above. The energy variation should be much smaller than this.

Thus the SLAC and Cornell data are not consistent and this may account for some of the problem. Because the SLAC data are the hardest to fit with $\gamma_\rho^2/4\pi = 0.5$, these data will be emphasized below.

The basic formula for the analysis is the optical theorem, combined with ρ-meson dominance, and the assumption that the real part of the amplitude is small. Then

$$(1) \qquad \left(\frac{d\sigma}{dt}\right)_{t=0} (\gamma A \to \rho^0 A) = \frac{\alpha}{64\pi}\left(\frac{4\pi}{\gamma_\rho^2}\right)\sigma_{\text{tot}}^2(\rho A).$$

In turn $\sigma_{\text{tot}}(\rho A)$ is expressed in terms of nuclear radii and $\sigma_{\text{tot}}(\rho N)$. This may be written down at once using the optical model of FERNBACH, SERBER and TAYLOR [14]; it may also be derived, after approximation, using the multiple-scattering formulation of

[11] G. McLELLAN, N. MISTRY, P. MOSTEK, H. OGREN, A. SILVERMAN, J. SWARTZ, R. TALMAN, K. GOTTFRIED and A. I. LEBEDEV: Phys. Rev. Lett., 22, 374 (1969); G. McLELLAN, N. MISTRY, P. MOSTEK, H. OGREN, A. SILVERMAN, J. SWARTZ and R. TALMAN: Phys. Rev. Let., 22, 377 (1969).

[12] F. BULOS, W. BUSZA, R. GIESE, R. R. LARSEN, D. W. G. S. LEITH, B. RICHTER, V. PERE-MENDEZ, A. STETZ, S. H. WILLIAMS, M. BENISTON and J. RETTBERG: Phys. Rev. Lett., 22, 490 (1969).

['] S. C. C. TING: private communication.

[14] S. D. DRELL and J. S. TREFIL: Phys. Rev. Lett., 16, 552 (1966); these authors actually write a different form which can be reduced to the one written here.

GLAUBER [15]:

$$(2) \qquad \sigma_{\text{tot}}(\text{pA}) = 4\pi \int_{0}^{\infty} b \, db \left[1 - \exp\left[-\tfrac{1}{2} \sigma_{\text{tot}}(\text{p}\mathcal{N}) \right] \int_{-\infty}^{\infty} \varrho \, dz \right].$$

The angular distribution may also be written down at once.

The approximation turns out to be quite good for two reasons.

1) Although it is a small-angle approximation, it is unitary and gives correctly the total cross-section by use of the optical theorem. Thus even the large-angle scattering cannot be far wrong.

2) For transparent nuclei, when the particle-nucleon cross-sections are small, it reduces, correctly, to the Born approximation independent of A. Thus the condition $\lambda \ll R$ may not be important.

I emphasize that the measurement is most simply understood as a measurement of $\sigma_{\text{tot}}(\text{pA})$ with an uncertain systematic normalization $(\gamma_{\varrho}^2/4\pi)$. I am therefore inverting the procedure of the SLAC group. When we so view the data, we can draw upon the wealth of data and experience of neutron and proton nucleus scattering.

Thus for nucleon scattering we have available, at several energies between 150 and 400 MeV, angular distributions for scattering of both isobaric states; polarization measurements, a total cross-section for neutrons, and Coulomb interference for protons. For p-nucleus scattering we have the scattering by one isobar only and then we must extrapolate to the forward direction, and normalize to find $(\gamma_{\varrho}^2/4\pi)$.

TABLE I.

Target	A	ϱ (9 GeV)		$\mathcal{N}$ ((180÷400) MeV) ref. [16]	$\mathcal{N}$ (27 GeV) ref. [17]	p [a] (21 GeV) ref. [18]
		$\gamma_{\varrho}^2/4\pi = \tfrac{1}{2}$	$\gamma_{\varrho}^2/4\pi = 1$			
p	1	0.025	0.035	0.034	0.039	0.039
n	1	[b]	[b]	0.023	0.035	0.038
Be	9	0.14	0.20	0.23	0.255	0.280
C	12	0.20	0.28	0.30	0.308	0.335
Al	27	0.40	0.57	0.59	0.585	0.690
Cu	64	0.75	1.03	1.20	1.14	1.36
Ag	118	0.98	1.38	—	—	—
Pb	208	1.90	2.7	2.90	2.8	3.30

(a) For high Z the cross-sections derived from protons may be high due to Coulomb scattering effects.

(b) Expected to be approximately equal to the numbers in the row above.

[15] K. S. KÖLBIG and B. MARGOLIS: Nucl. Phys., B 6, 85 (1968).
[16] V. A. NEDZEL: Phys. Rev., 94, 174 (1954).
[17] M. L. LONGO: CERN Conference on Hadron Interactions, CERN 68-7 (Geneva, 1968), p. 523.
[18] G. BELLETTINI, G. COCCONI, A. N. DIDDENS, E. LILLETHUN, G. MATTHIAE, J. P. SCANLON and A. M. WETHERELL: Nucl. Phys., 79, 609 (1966).
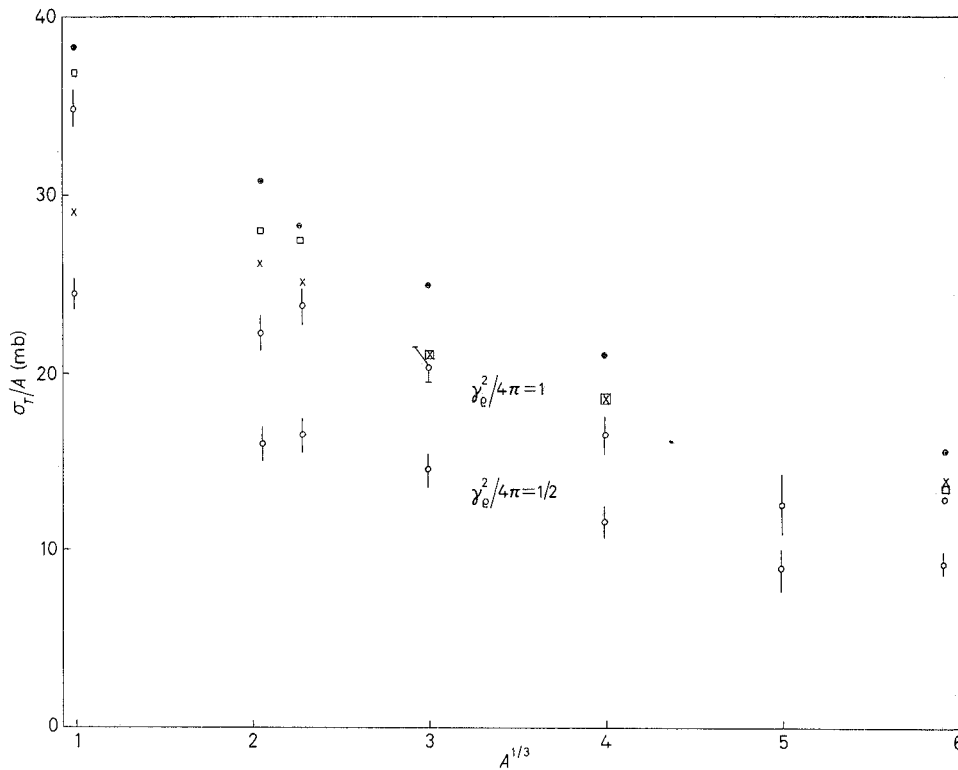
Fig. 1. — ○ ρ, 9 GeV, SLAC, $\gamma_\rho^2/4\pi = \frac{1}{2}$ or 1; × $\mathcal{N}$, (150÷400) MeV; □ $\mathcal{N}$, 27 GeV; ● p, 21 GeV.

Table I and Figure 1 show plots of $\sigma_{tot}(\rho A)$ vs. A derived from the SLAC data using a small extrapolation, for the two values of $\gamma_\rho^2/4\pi$; and for comparison $\sigma_{tot}(\mathcal{N}A)$ for (200÷400) MeV neutrons [16], 27 GeV neutrons [17] and 20 GeV protons [18]. Included are values of $\sigma_{tot}(\rho p)$ derived from the hydrogen data of ref. [10].

Other data exist for neutron cross-sections at 27 GeV [19] and 10 GeV [20]. These are not consistent with the data of ref. [17] making a very confusing situation. The proton-neutron total cross-sections of ref. [18] can be criticized [21] because in extrapolating to 0° it was assumed that there was no interference between the real Coulomb and the, predominantly imaginary, nuclear amplitudes. The real part of the nuclear amplitude is now known to be repulsive, leading to an over-estimate of the total cross-section by perhaps 20%. Nonetheless I plot these data as they stand.

Some theorists have refused to consider the $\sigma(\rho\mathcal{N})$ together with $\sigma(\rho A)$. This refusal is not consistent with the analyses of the neutron-nucleus data and I object to it. If a

[19] J. ENGLER, K. HORN, J. KÖNIG, F. MÖNNIG, P. SCHLUDECKER, H. SCHOPPER, P. SIEVERS, H. ULLRICH and K. RUNGE: Phys. Lett., 28 B, 64 (1968).

[20] V. S. PANTUEV: Sov. Phys. JETP, 15, 626 (1962).

[21] A. DAR has found the same problem in a private communication. This problem, with the opposite sign, has been found before at 150 MeV and 350 MeV proton scattering.

discontinuity of $\sigma(\rho A)$ vs. A is expected from hydrogen to beryllium, there are no tests of the optical model to help us and I have no trust in the model.

The question arises, are these data consistent with reasonable parameters? The problem can be seen most easily with the lead cross-section. Using the electromagnetic radius $\gamma_\rho^2/4\pi = \frac{1}{2}$ and $\sigma_{tot}(\rho p) = 25$ mb, formula (2) predicts $\sigma_{tot}(\rho\ Pb) = 2.6$ barns, 30% higher than experiment. If $\gamma_\rho^2/4\pi = 1$, $\sigma_{tot}(\rho p) = 35$ mb and the theory is more satisfactory. The problem does not appear in the DESY analysis. In order to find $(d\sigma/dt)_{t=0}$ they extrapolate using a large nuclear radius to find a large «experimental» $\sigma_{tot}(\rho\ Pb)$.

The plot of $\sigma_{tot}(\rho A)$ vs. A is similar in shape to that for 21 GeV protons without correlation [18]. The data for 21 GeV have been analysed [19], and agree with the electromagnetic radii [22]. They fall almost on top of each other if $\gamma_\rho^2/4\pi = 1.2$. It is therefore tempting to choose this value and a correspondingly large $\sigma_{tot}(\rho\mathcal{N})$. The shape for $(150 \div 400)$ MeV neutrons [16] is different; there is a smaller change $A = 1$ to $A = 9$, attributed to the attractive real part of the scattering amplitude. Although an attempt to make a fit to this shape would also need $\gamma_\rho^2/4\pi \simeq 1.0$, it shows the need for caution. The large drop in the ρ-meson case may be due to other causes; a smaller neutron cross-section [18] or a spin-flip term in the forward hydrogen cross-section which should not be included in the optical theorem.

Many of the errors introduced by the approximations in the optical model can be corrected by choosing a nuclear radius, or nuclear cross-section to use in eq. (2) which is not the correct one [22]. But we must beware of glib statements that we must use the «strong radius». The «strong radius» depends upon the process as we shall now show.

The following effects have both been considered theoretically and found experimentally in nucleon scattering; particularly between 150 MeV and 400 MeV where already $\lambda \ll R$ and much data exists. In considering these, Glauber's formulation, or that of KERMAN, McMANUS and THALER, is superior to the simple optical model. However, precise corrections have only been calculated for the deuteron [23].

1) In the Glauber formulation [23-24], the full amplitude for small-angle scattering becomes

(3) $F(\mathbf{k} - \mathbf{k}') =$

$$= \frac{k}{2\pi i} \int \exp\left[i(\mathbf{k} - \mathbf{k}')\mathbf{b}\right] \left[\exp\left[-\frac{1}{2\pi i k}\int f(q)\exp\left[-i\mathbf{q}\mathbf{b}\right]\varrho(q)\,\mathrm{d}^2(q)\right] - 1\right]\mathrm{d}^2 b\,,$$

$\varrho(q)$ is the nuclear form factor $= \int \exp\left[i\mathbf{q}\mathbf{r}\right]\varrho(r)\,\mathrm{d}^3 r$,

$f(q)$ is the particle-nucleus scattering amplitude.

Equation (2) is derived by approximating $f(q) = $ constant, putting $k - k' = 0$ and using the optical theorem.

For light transparent nuclei, and scattering angles within the first diffraction minimum, this becomes a fold of scattering amplitudes [25]

(4) $F_{\rho A}(q^2) = F_A(q^2)\,F_{\rho\mathcal{N}}(q^2)\,,$

[22] R. HOFSTADTER: Ann. Rev. Nucl. Sci., 4, 231 (1951).

[23] R. J. GLAUBER and G. MATHIAE: Istituto di Sanità, report ISS-67/16,

[24] V. FRANCO and R. J. GLAUBER: Phys. Rev., 142, 1195 (1966).

[25] R. WILSON: Phys. Rev., 114, 260 (1959).

where $F_A(q^2)$ is derived in turn from electron scattering

(5)
$$F_{eA}(q^2) = F_A(q^2) F_{ep}(q^2) ,$$

yielding

(6)
$$\langle r^2 \rangle_{pA} = \langle r^2 \rangle_{eA} - \langle r^2 \rangle_{ep} + \langle r^2 \rangle_{p\mathcal{N}} .$$

For nucleons of $(150 \div 400)$ MeV, $\langle r^2 \rangle_{\mathcal{N}\mathcal{N}}$ has different values for real, imaginary and spin-orbit amplitudes and is large. This shows the complexity of the problem. For p-mesons we find $(d\sigma/dt)_{p\mathcal{N}} = \exp[8.5t]$, giving $\langle r^2 \rangle_{p\mathcal{N}} = 2.0$ fm^2 which is smaller than the nucleon case. For the φ-meson the effect is even smaller $(d\sigma/dt)_{\varphi\mathcal{N}} = \exp[4t]$, $\langle r^2 \rangle_{\varphi\mathcal{N}} = 1.0$ fm^2.

Now if we examine the carbon data, the SLAC group derives a radius from the shape of the forward peak. (It must be a r.m.s. radius, and their discussion of equivalent square-well radii and Woods-Saxon shapes is unnecessary and confusing.) They find $\sqrt{\langle r^2 \rangle} = 2.47$ fm. If we correct, as we must, for the $p\mathcal{N}$ radius we find $\sqrt{\langle r^2 \rangle} = 2.0$ fm which is a little less than the value 2.27 fm derived from electron scattering. For beryllium the difference is greater. Thus the problem may be present in light nuclei also and we may have a case where radii are smaller than theory.

2) For heavy nuclei the first effect is small (about 2%). But, if the nucleus is not very transparent, $V(r)$ is no longer proportional to $\varrho(r)$ but saturates for large $\varrho(r)$. This leads to a large effective radius. This effect should be more important for nucleons where $\sigma_{(\mathcal{N}\mathcal{N})} \approx 35$ mb than for p-mesons $\sigma_{(p\mathcal{N})} \approx 25$ mb or φ mesons $\sigma_{(\varphi\mathcal{N})} \approx 12$ mb.

3) Due to the exclusion principle, or other correlations, σ_{tot} $(p\mathcal{N})$ in eq. (2) may be either more or less than the free value. Moreover real amplitudes may also vary [26]. This effect is not included in eq. (2).

Two candidates arise to explain the low $\sigma_{tot}(p$ Pb). Firstly the effective radius may be wrong. Yet the effects noted above all lead to larger radii. Secondly we can reduce $\sigma_{tot}(p\mathcal{N})$ from 25 mb to 15 mb in nuclei. Such a large reduction is unusual, but a reduction for $\mathcal{N}\mathcal{N}$ scattering at 150 MeV of 25% has been found, in a detailed analysis [27].

The literature of nucleon scattering from nuclei has been fitted, usually incorrectly, with a variety of parameters R, σ, etc. The proton scattering at 150 MeV was first fitted with parameters which predicted forward $\mathcal{N}$-C scattering one half of its measured value, due to improper computations of plural scattering and Coulomb interference. At 350 MeV, neglect of the radius corrections also allowed a good fit to proton experiments but at the expense of ignoring the more precise neutron total cross-sections. The problems of p-nucleus scattering may be less because the p-nucleon cross-section is less, but may be worse, because the p has spin 1.

To summarize I believe some or all of the following effects can bring the data into agreement with $\gamma_p^2/4\pi = \frac{1}{2}$.

1) The Cornell data may be increased by 20% by correction for a different Γ_p.

2) Only the radii of the light nuclei must be appreciably increased due to the range of forces. The lead radius, for example, shoud not be increased more than 2 to 5%. But it is incorrect to take $R = \text{const } A^{\frac{1}{3}}$.

---

[26] J. DABROWSKY and J. SAWICKI: Nucl. Phys., 13, 621 (1959).
[27] J. PALMIERI and A. CROMER: Ann. of Phys., 30, 32 (1964).

3) The SLAC lead cross-sections may have to be increased to match those of Cornell.

4) The σ_{tot} from BELLETTINI et al. ([18]) must be reduced, when proper allowance is made for the real part of the amplitude. The direct comparison with the ρ cross-sections is then less obvious.

5) The σ_{tot} inside the nucleus may be lower than the free value by 20%. This last would be an interesting nuclear-physics result.

Since there are so many possible causes of the problem, including discrepancies in the data, it is hard to make a definitive analysis. The data should not give cause for alarm, but be treated as the first of many data in the nuclear physics of ρ-mesons. Nothing less sophisticated than eq. (3) should be used, with the correct radii.

Meanwhile, simultaneous extraction of coupling constants and cross-sections from photoproduction data on nuclei will have large errors, and agreement such as that of ref. ([10]) is accidental.

* * *