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On the Parallel Penetrating Particles Underground as E.A.S. Muonic Component.

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Summary. — A calculation on the nuclear cascade in high energy E.A.S. is performed by using a composite isobaric model, in order to study the muon density function $\Delta(N, R, > E_\mu)$, where R is the distance from the axis of the E.A.S., E_μ the muon energy and N the size of the E.A.S. The obtained expression

$$\Delta(N, R, > E_\mu) = \frac{N^{0.85} R^{-0.24} (R + 90)^{-2.4}}{4.6 \cdot 10^{-3} (R + 45)(E_\mu - 10) + 1} \left(\frac{E_\mu + 7}{10}\right)^{0.002R} \quad (\mathrm{m}^{-2})$$

shows a good agreement with experimental data on the muonic component of the E.A.S. The $\Delta(N, R, > E_\mu)$ is then used to interpret the results obtained up to now on the parallel penetrating particles underground as resulting from the very penetrating muonic component of the E.A.S. At depths greater than 200 m w.e. a disagreement is found between the calculations and the experimental data. Some discussion is given of the basis for this discrepancy.

1. – Introduction.

There have recently been some results (1,2) which have proposed again (3,4) the question concerning the properties of the underground cosmic radiation,

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(1) H. E. Bergeson, J. K. Keuffel, M. D. Larson, E. R. Martin and G. W. Mason: *Phys. Rev. Lett.*, **19**, 1487 (1967); **21**, 1089 (1968).

(2) M. Dardo, P. Penengo and K. Sitte: *Nuovo Cimento*, **58** A, 59 (1968); **54** B, 349 (1968).

(3) G. C. Callan and S. L. Glashow: *Phys. Rev. Lett.*, **20**, 779 (1968).

(4) C. Castagnoli, E. Etim and P. Picchi: *Lett. Nuovo Cimento*, **1**, 197 (1969).

renewing the interest about this research and generating a critical revision of the various results. The part concerning the μ showers underground, whose phenomenology is somewhat difficult, has assumed a particular importance.

Recently BARTON (5) has again examined a particular case of underground muon groups, namely the so-called « parallel penetrating particles (PPP) », which have been studied for many years (6-11). He finds an indication that the muon pairs are less strongly absorbed than it would be expected, and concludes that this may be due to the underestimation of the muon density near the shower axis given by Greisen's function.

In this paper we resume this problem by developing a model (Sect. **2**) of nucleonic cascade of the E.A.S. which is compared (Sect. **3**) with various experimental data and has been used to give a muon density function Δ in the E.A.S. This function allows us to calculate (Sect. **4**) the PPP frequency at various depths and then to compare our results with experimental data (Sect. **5**).

These comparisons show that there is a disagreement between the experimental and theoretical results for the PPP frequencies.

We suggest however that further experiments are needed to clarify this phenomenological question, which is to-day of great interest. As a matter of fact:

a) There are not very many physical facts in E.A.S. which permit us to distinguish between the several cascade models, and the PPP seem to be one of these. Such a hint has been recently given by KOSHIBA (12).

b) This question may be connected with the existence of new processes recently discussed in underground-muon physics.

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(5) J. C. BARTON: *Proc. Phys. Soc.*, A **1**, 43 (1968).

(6) E. AMALDI, C. CASTAGNOLI, A. GIGLI and S. SCIUTI: *Nuovo Cimento*, **9**, 969 (1952).

(7) D. KESSLER and R. MAZE: *Nuovo Cimento*, **5**, 1540 (1957).

(8) M. F. BIBILASHVILI, T. T. BARNAVILI, G. A. GRUBELASHVILI and N. A. MURADOVA: *Proc. IX Int. Conf. Cosmic Rays* (London, 1965), p. 956.

(9) N. CHAUDHURI and M. S. SINHA: *Proc. Int. Conf. on Cosmic Rays, Jaipur* (Bombay, 1953), p. 106.

(10) D. R. CREED, J. B. M. PATTISON, A. W. WOLFENDALE, C. V. ACHAR, V. S. NARASIMHAM and P. V. RAMANA MURTHY: *Proc. IX Int. Conf. Cosmic Rays* (London, 1965), p. 980.

(11) P. H. BARRET, L. M. BOLLINGER, G. COCCONI, Y. EISENBERG and K. GREISEN: *Rev. Mod. Phys.*, **24**, 133 (1952).

(12) M. KOSHIBA, Y. TOTSUKA and S. YAMADA: *Proc. X Int. Conf. on Cosmic Rays*, Part A (Calgary, 1968), p. 525.

2. – The model of the nuclear cascade in the E.A.S.

Any nuclear cascade model of the E.A.S. must rest on the following main experimental points: *a*) The inelasticity in the nucleon-nucleon collisions is small (~ 0.4) and varies little when the incident momentum increases. *b*) The transverse-momentum distribution of secondaries has a sharp peak with $\langle p_t \rangle = (300 \div 400)$ MeV/c. *c*) About 80% of secondaries are pions. *d*) The mean free paths of interaction, $\lambda_n = 80 \text{ g cm}^{-2}$ and $\lambda_\pi = 120 \text{ g cm}^{-2}$, are constants.

Among the two models prevailing today, *i.e.* the two-fireball and the isobar pionization model, we choose the second one. On the other hand, the usual hypothesis on the inelasticity, transverse momentum and multiplicity make ultimately the two models rather similar, as recently pointed out by KOSHIBA (12).

These main assumptions are: 1) production of fireballs and 2) excitation of one or both of the nucleons to isobar levels. The evaporation of the fireball produces not only mesons but also nucleons (13,14), antinucleons and kaons. We assume: *a*) the nonpionic particles are all baryon pairs ($n\bar{n}$); *b*) their average number n_h is 20% of the total number of secondaries for primary energies $E_0 > 200$ GeV. *c*) $n_h = 2$ for $E_0 < 200$ GeV (15,16).

Investigations of the angular distribution of secondaries in emulsions give some information on a possible motion of the fireballs relative to the c.m. system of the nn collision. The angular distribution in the laboratory system is assumed Gaussian in 60% of the cases or alternatively doubly peaked with a c.m. distribution featured by $\cos^n \theta \, d(\cos \theta)$, with $1.3 < n < 2$.

The transverse-momentum distribution is supposed of Boltzmann type with $\langle P_t \rangle = 350$ MeV/c.

At high energies (17,18) the isobar of isospin $I = \frac{1}{2}$ seems to be the only one excited, the excitation of the isobar $I = \frac{3}{2}$ decreasing steeply with in-

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(13) T. V. DANILOVA, E. V. DENISOV, S. I. NIKOLSKY and A. A. PAMANSKY: *Phys. Soc. Jap.*, **17**, *Suppl. III-A*, 205 (1964).

(14) B. K. CHATTERSEE, C. T. MURTHY, S. NARANAN, B. V. SREEKANTAM, M. V. SRINIVASARAO and S. C. TONWAR: *Proc. Int. Conf. Cosmic Rays* (1965), p. 802.

(15) V. S. BARASHENKOV, V. M. MALTSEV and I. PATERA: preprint JINRP-1577 (1966).

(16) V. S. BARASHENKOV and I. PATERA: *Forth. Phys.*, **41**, 469 (1963).

(17) I. M. BLAIR, A. E. TAYLOR, W. S. CHAPMAN, P. I. P. KALMUS, J. LITT, M. C. MILLER, D. B. SCOTT, H. J. SHERMAN, A. ASTBURY and T. G. WALKER: *Phys. Rev. Lett.*, **17**, 789 (1966).

(18) E. W. ANDERSON, E. J. BLESER, G. B. COLLINS, T. FUJII, J. MENES, F. TURKOT, R. A. CARRIGAN jr., R. M. EDELSTEIN, N. C. HIEN, T. J. McMAHON and I. NADELHAFT: *Phys. Rev. Lett.*, **19**, 198 (1967).

creasing E_0. This may be attributed to the tendency of the processes with isospin exchange, or more generally with any quantum-number exchange, to disappear at high energies.

The isobars taken into consideration are $\mathcal{N}^*_{\frac{1}{2}}$ of mass 1410 MeV (spin $\frac{1}{2}$); $\mathcal{N}^*_{\frac{1}{2}}$ of mass 1688 MeV (spin $\frac{5}{2}$). The $\mathcal{N}^*_{\frac{1}{2}}$ isobar (1410) decays to its ground state mainly through $\mathcal{N}^* \to \pi + \mathcal{N}$ while $\mathcal{N}^*_{\frac{1}{2}}$ (1688) has two alternative channels: 1) $\mathcal{N}^* \to \pi + \mathcal{N}$; 2) $\mathcal{N}^* \to 2\pi + \mathcal{N}$ with a branching ratio of 0.7 and 0.3 respectively.

The probabilities of decaying into the many multiple-pion states and nucleon-pion states are fixed by the Clebsh-Gordon coefficients, taking into account that in process 2) the intermediate boson can be neither $\mathcal{N}^*_{\frac{5}{2}}$ (isospin conservation) nor $\mathcal{N}^*_{\frac{1}{2}}$ (the mass difference is too small for disintegration). The intermediate boson must then be the $\mathcal{N}^*_{\frac{3}{2}}$ isobar.

For the total average number n of particles created in a nn collision we assume: $n_t = 2.0 + 1.54 E_0^{0.296}$ in agreement with some experimental data (19,20).

To complete the description of the nn collision we assume that the transverse momentum of the target baryon has a Boltzmann distribution with $\langle p_t \rangle = 500$ MeV/c.

In the pion-nucleon collisions at very high energies we neglect a possible excitation of the target nucleon and assume the average number of created pions has the form

$$v_t = v_0 E_0^{(t-1)/t} ,$$

with $t = 1$ for truly elastic processes and $t = 2$ for truly inelastic processes. Experimental measurements (21) near sea level are in good agreement with the truly inelastic process $t = 2$ and $v_0 = 0.7$.

The description of the kinematics of the created pions makes use of an uncorrelated jet model; momentum and energy conservation are taken into account, but any other correlation is neglected.

The pion angular distribution in the c.m.s. is assumed to be of the form $\cos^m \theta^* \mathrm{d}(\cos \theta^*)$, where the parameter m depends on the incident energy E_0; through a Monte Carlo calculation we found that it varies from $m = 1$ at $E_0 = 10$ GeV up to $m = 10$ at $E_0 = 10^3$ GeV and then attains a constant value. The transverse-momentum distribution is of the Boltzmann type, with $\langle p_t \rangle = 350$ MeV/c.

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(19) F. ABRAHAM, J. KIDD, M. KOSHIBA, R. LEVISETTI, C. H. TSAO, W. WOLTER, C. L. DENEY, R. L. FRICKEN and R. W. HUGGETT: preprint (1963).

(20) A. S. PEAK and R. L. S. WOOLCOTT: *Nuovo Cimento*, **42** A, 856 (1966).

(21) Y. PAL and B. PETERS: *Mat. Fys. Medd. Dan. Vid. Selsk.*, **33**, n. 15 (1964).

3. – Calculated method and experimental check of the physical model.

The method used for the simulation of the cascade is the following. In the storage of the computer we reserve three blocks of suitable size in which to store neutrons, protons and charged pions, whenever they are generated, each one with proper position, direction, energy and time. From these blocks we then take one at a time first the neutrons, then the protons and last the pions, and each particle is followed through the various interactions it causes.

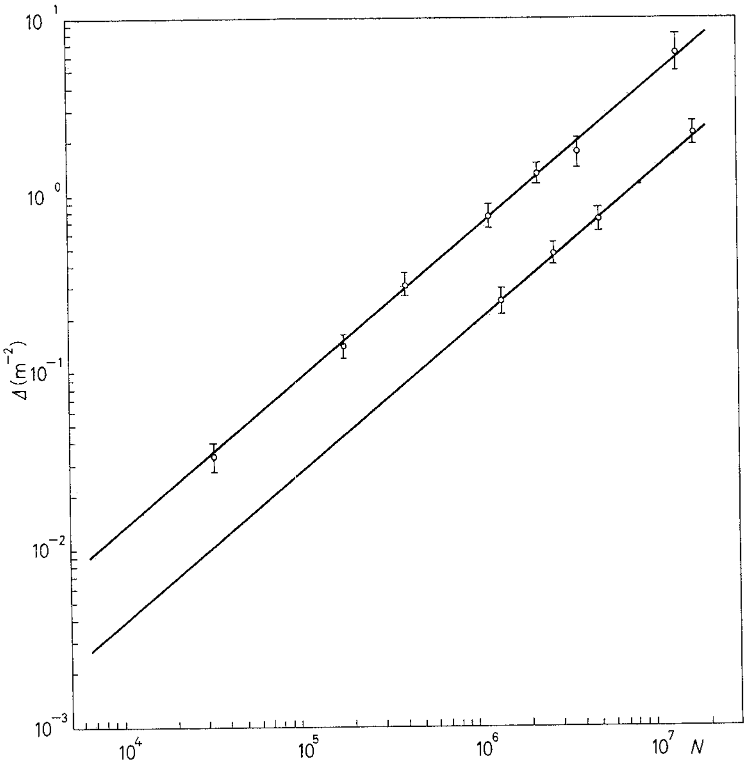

Fig. 1. – Dependence of the muons $E > E_\mu = 10$ GeV flux density Δ on the total number of particles N in the E.A.S. for a given distance from the shower axis ($20 \leqslant R \leqslant 40$; $60 \leqslant R \leqslant 100$). The full lines are obtained with (1). The experimental points are due to KHRENOV (22).

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(22) B. A. KHRENOV: *Sov. Phys. JETP*, **14**, 1001 (1961).

Whenever the considered particle has an interaction, all the reaction products with their features are transferred in the reserved storage blocks, joining the previously stored particles. All the particles arriving at the fixed quota are instead stored on a magnetic tape with all their characteristics. These data are next suitably analysed. The study of the shower is fulfilled when even the last pion has been analysed.

The density $\Delta(N, R, > E_\mu)$ of the muons with energy $> E_\mu$, as a function of the size N of the shower and of the distance R from the axis, is the most interesting result obtained with our model, and we shall now discuss it.

Figure 1 shows the experimental results of KHRENOV (22), concerning the dependence of the density $\Delta(\text{m}^{-2})$ on the underground flux of muons with energy $E_\mu \geqslant 10$ GeV, and on the total number N of E.A.S. particles at two ranges of distance R from the E.A.S. axis (namely $20 \leqslant R \leqslant 40$ m and $60 \leqslant R \leqslant 100$ m). Figures 2, 3, 4 show the experimental results on the lateral distribution of muons in E.A.S. for several values of E_μ and N which, although

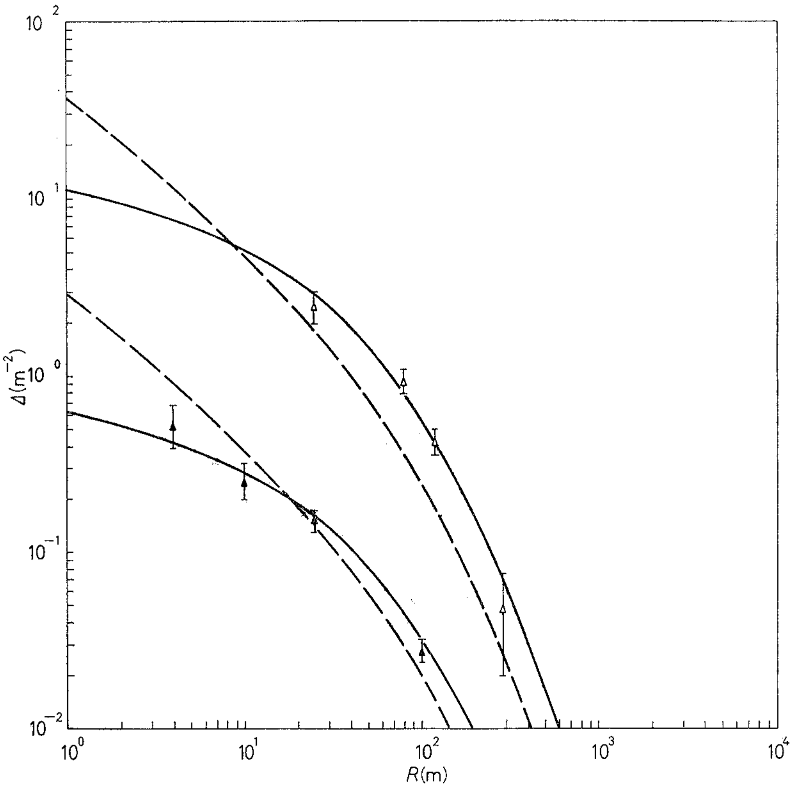

Fig. 2. – The lateral distribution of muons in E.A.S., with energy $E > E_\mu = 10$ GeV. The experimental points are from KHRENOV (22), the full lines are obtained using (1), and the dashed lines using (2). △ $N = 6 \cdot 10^6$; ▲ $N = 2 \cdot 10^5$.

obtained by different authors (22-25), are in good agreement with each other. The full lines plotted in Fig. 1-4 come from our Monte Carlo calculation which

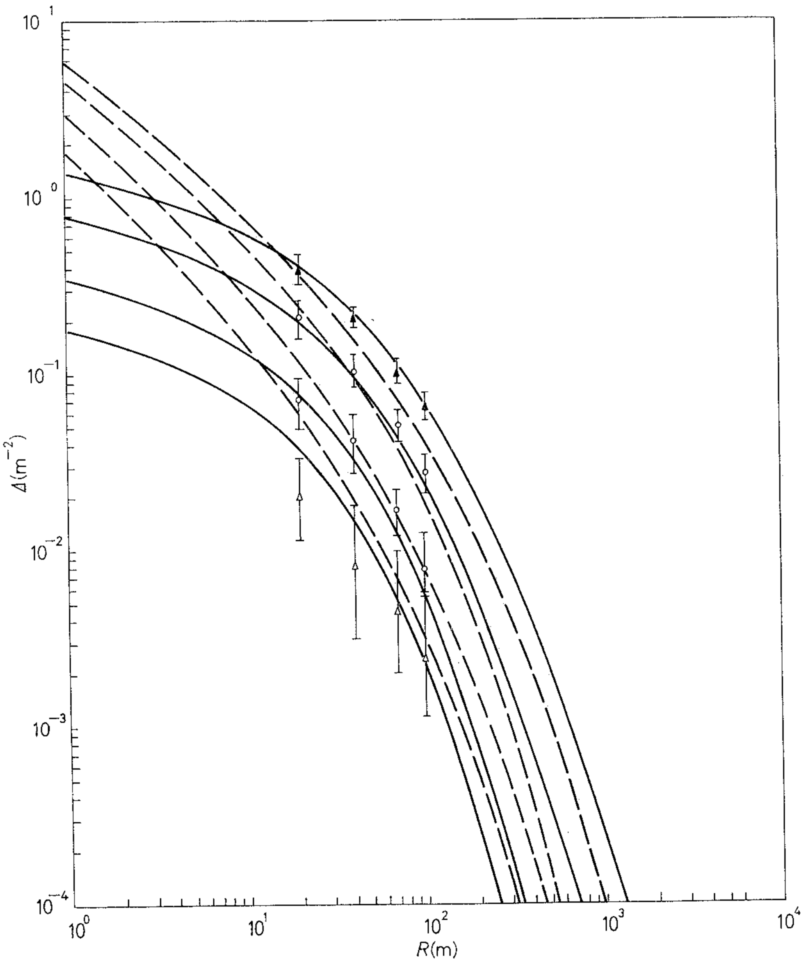

Fig. 3. – The lateral distribution for muons with different energies E_μ. The experimental points are due to EARNSHAW *et al.* (23); the full lines are obtained using (1) and the dashed lines using (2). ▲ >10 GeV; ○ >20 GeV; ● >50 GeV; △ >100 GeV. $N = 5\cdot10^5$.

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(23) J. C. EARNSHAW, K. J. ORFORD, G. D. ROCHESTER, A. J. SOMOGYI, K. E. TURVER and A. B. WALTON: *Proc. Phys. Soc.*, **90**, 91 (1967).

(24) T. T. BARNAVELI, M. F. BIBILASHVILI, G. A. GEUBELASHVILI, A. K. DZHAVRISHVILI, R. E. KAZAROV, R. V. KURIDZE and I. V. KHALDEEVA: *Izv. Akad. Nauk SSSR*, **28**, 1894 (1964).

(25) B. A. KHRENOV: *Žurn. Èksp. Teor. Fiz.*, **41**, 1402 (1961).

gives for the muon density function $\Delta(m^{-2})$ the expression

$$(1) \qquad \Delta(N, R, > E_\mu) = \frac{N^{0.85} R^{-0.24} (R+90)^{-2.4}}{4.6 \cdot 10^{-3}(R+45)(E_\mu - 10) + 1} \left(\frac{E_\mu + 7}{10}\right)^{0.002R},$$

valid in the distance range of $1 \ll R < 500$ m.

It is seen that this expression describes satisfactorily the experimental results of different authors. Our Δ function is compared in Fig. 2, 3 with an empirical expression for the muon structure function obtained by GREISEN *et al.* (26).

$$(2) \qquad \Delta(n, R, > E) = \frac{14.4 R^{-\frac{3}{4}}}{(1 + R/320)^{2.5}} \cdot$$

$$\cdot \left(\frac{N}{10^6}\right)^{\frac{3}{4}} \frac{51}{(E+50)} \left(\frac{3}{E+2}\right)^{0.14 R^{0.37}} \quad (\mathrm{m}^{-2}),$$

valid for $10 \leqslant R \leqslant 500$ m.

As can be seen, in the given range of R the expression (2) also agrees with experiment, except for low E_μ and high R where it is rather smaller.

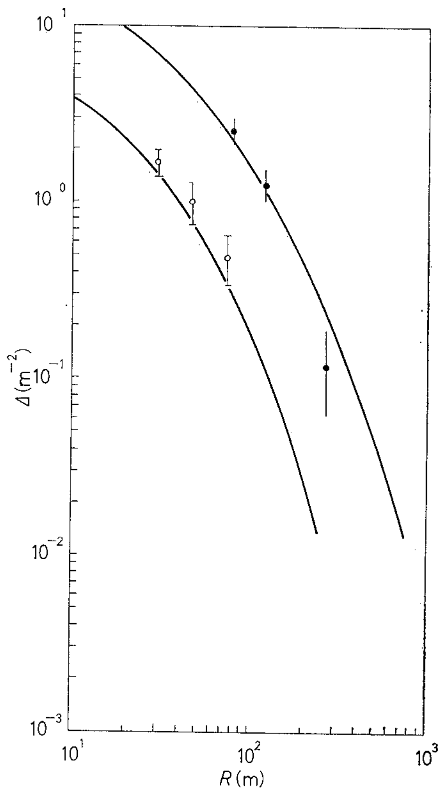

Fig. 4. – The lateral distribution for muons with different energies E_μ. The experimental points are due: • to KHRENOV (22) and ○ to BARNAVELI (24) *et al.*; the full lines are obtained using (1). $N = 2 \cdot 10^7$.

4. – PPP frequency estimate in E.A.S.

The calculated frequency D_{th} of events in which at least two muons of energy $> E_\mu$ cross an area of 1 m² is given by (6)

$$(3) \qquad D_{\text{th}}(E_\mu) = \int_N \int_R \{1 - (1 + \Delta)\exp[-\Delta]\} f(N) 2\pi R \, \mathrm{d}R \, \mathrm{d}N,$$

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(26) S. BENNETT and K. GREISEN: *Phys. Rev.*, **124**, 1982 (1961).

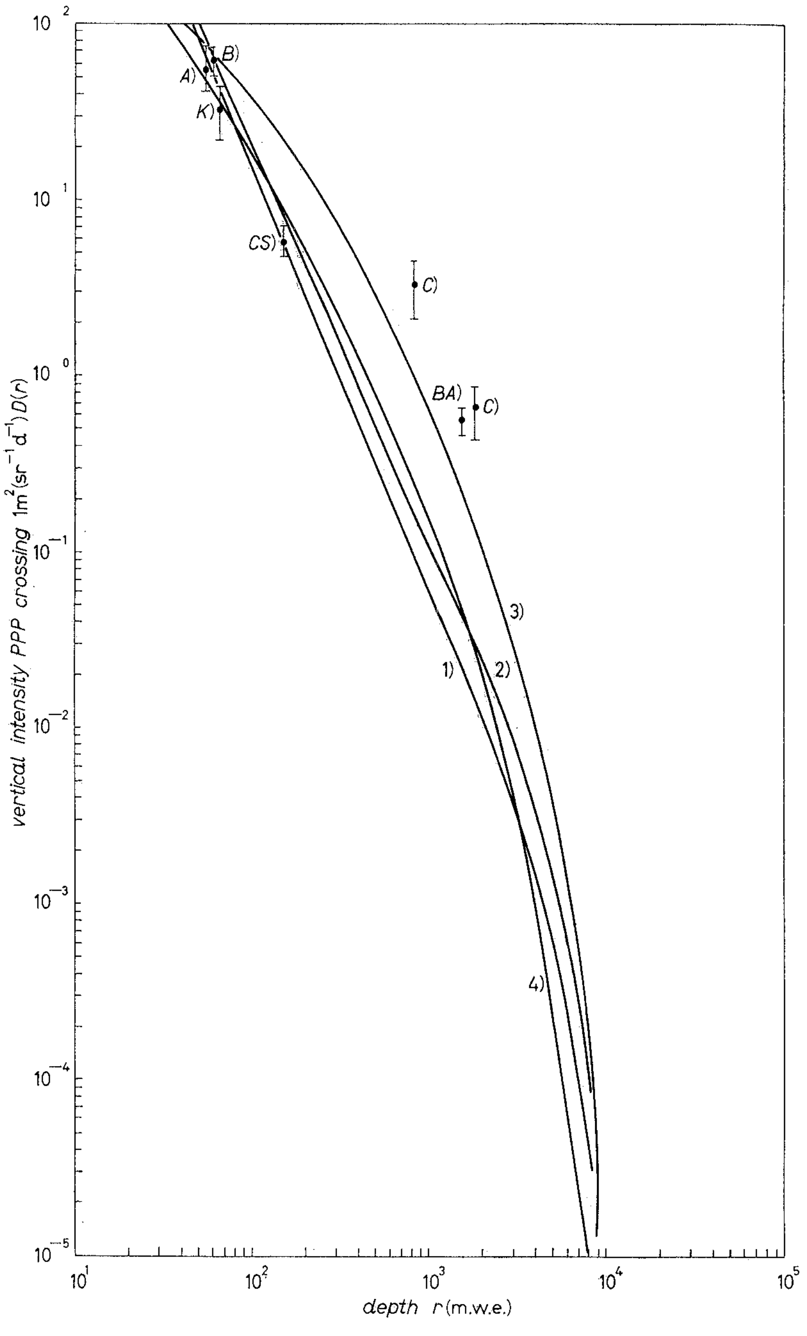

Fig. 5. – Vertical intensity of parallel penetrating particles crossing 1 m² (sr⁻¹ d⁻¹) below sea level. Curve 1) and 2) have been obtained with our expression of Δ for $R > 1$ m and $R > 10$ m respectively. Curves 3) and 4) are the corresponding curves given by Greisen's Δ. Experimental data: *A*) AMALDI *et al.* (⁶); *B*) BARTON (⁵); *K*) KESSLER and MAZE (⁷); *CS*) CHAUDHURI and SINHA (⁹); *C*) CREED *et al.* (¹⁰); *BA*) BARRET *et al.* (¹¹).

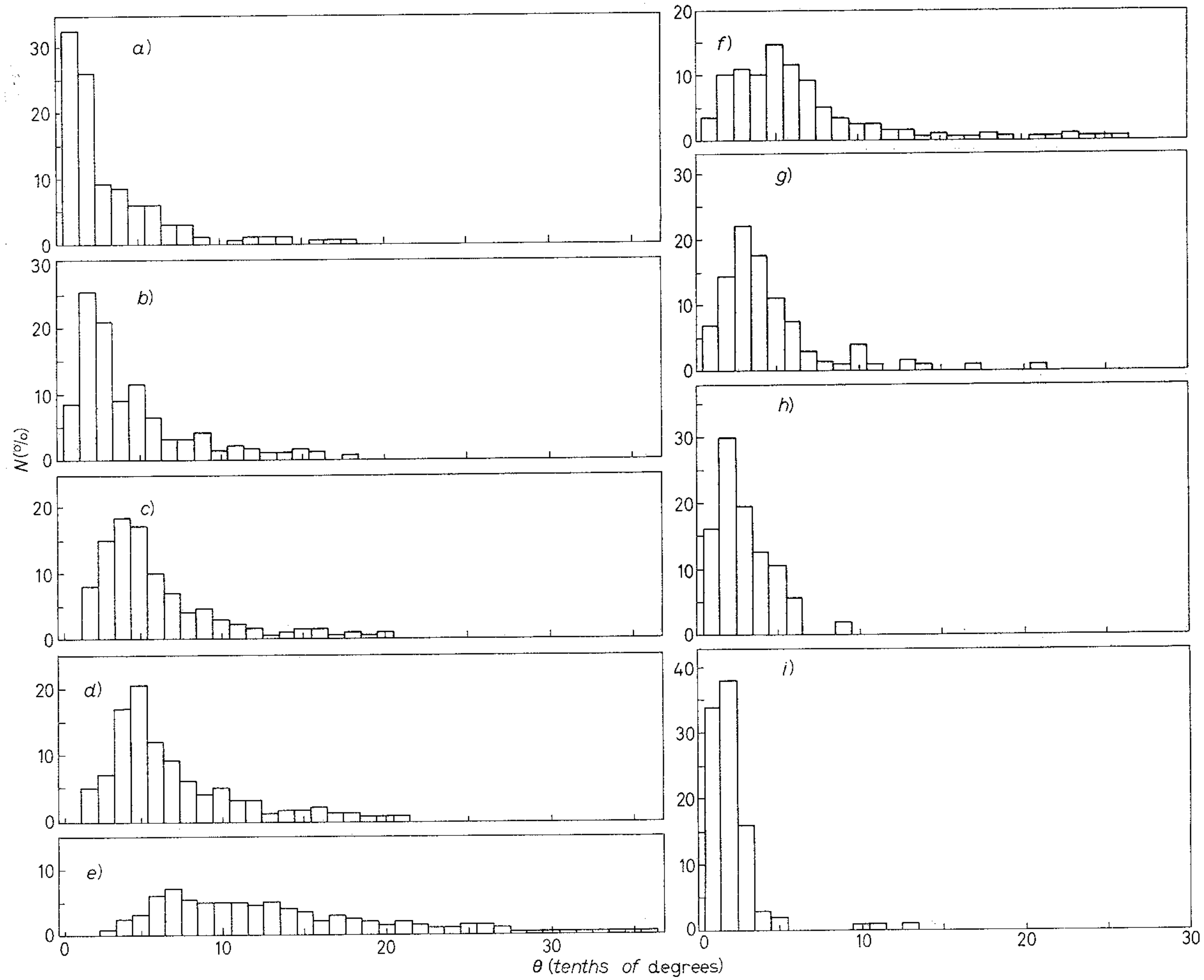

Fig. 6. – The angular distributions of muons in extensive air showers of size $N = 6.5 \cdot 10^3$ for a given distance R from the showers axis and for a given energy E_μ. *a*) $0 \leqslant R < 30$ m; *b*) $30 \leqslant R < 60$ m; *c*) $60 \leqslant R < 90$ m; *d*) $90 \leqslant R < 120$ m; *e*) $R > 120$ m; *f*) $30 \leqslant E < 60$ GeV; *g*) $60 \leqslant E < 90$ GeV; *h*) $90 \leqslant E < 120$ GeV; *i*) $E > 120$ GeV.

where the $f(N)$ is obtained from (²⁷)

$$(4)\qquad F(>N) = 3\cdot 10^{-8}\left(\frac{N}{10^6}\right)^{-1.66-0.06\log(N/10)^6} (\mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1})\,, \qquad 10^3 \leqslant N \leqslant 10^9\,.$$

We used for Δ the expression (1) and then for a comparison also Greisen's semi-empirical formula (2).

The above-mentioned $D_{\text{th}}(E_\mu)$ has been transformed into a $D_{\text{th}}(r)$, where r (in m w.e.) is the underground depth, by means of our range-energy relation which takes into account also the fluctuations (²⁸).

The results are shown in Fig. 5.

Curves 1) and 2) have been obtained with our expression of Δ for $R > 1$ m and $R > 10$ m respectively. Curves 3) and 4) are the corresponding curves obtained from (2). Curve 3) is practically the same as that obtained by BARTON (⁵).

In the calculation of $D_{\text{th}}(r)$, muons crossing the 1 m² area must be parallel in order to be identified as PPP. The $\Delta(N, R, > E_\mu)$, which is integrated over all the angular distribution, must be carefully used. For this purpose we obtained from our Monte Carlo calculations Fig. 6, which shows the angular distribution of muons in vertical E.A.S. of size $N = 6.5\cdot 10^3$ for a given distance R from the shower axis and for a given energy $E > E_\mu = 12$ GeV. As one can see, muons have a very narrow angular distribution near the E.A.S. shower axis. However they appear experimentally as parallel particles.

5. – Comparison with experimental data and conclusion.

The experimental results here analysed have been obtained at various depths > 50 m.w.e. by different authors (⁶⁻⁸,¹⁰⁻¹²).

The comparison with theoretical data is very difficult because of 1) local penetrating and electrophotonic showers in rock, 2) showers in lead or in the metals of the experimental arrangements, 3) diversity of the detector geometrical factors Γ and of triggering requirements, 4) diversity of energy needed by muons to cross different detectors, which is important in estimating the background due to local showers.

For these reasons we selected for our comparison the results (like those performed in cloud chambers or with hodoscoped counters) which are less likely

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(²⁷) K. GREISEN: *Am. Rev. Nucl. Sci.*, **10**, 63 (1960).

(²⁸) C. CASTAGNOLI, P. PICCHI, R. SCRIMAGLIO and G. VERRI: *Acc. Naz. Lincei*, **41**, 56 (1966).

to be affected by local showers. The experimental results are also shown in Fig. 5.

It may be seen that at low depths (up to $\simeq 200$ m w.e.) the experimental data are in agreement with our results (curves 1) and 2)) but the agreement disappears at greater depths (namely > 1500 m w.e.).

The disagreement can be ascribed to two reasons at least:

A) The existence of events other than the E.A.S., for instance pair production by muons. This type of event becomes in fact significant at high energies. Also the results of DARDO *et al.* (^{2}) may be interesting for this purpose.

B) The insufficiency of our model of the nuclear cascade, which gives a low value of $\Delta(N, R, > E)$ near the axis ($1 < R < 10$ m). Actually the contribution to the calculated PPP frequency of the muons with R in this range is considerable, as can be seen from the comparison between curves 1) and 2), and especially between curves 3) and 4). The curve 3) shows a lower discrepancy between $D_{\text{exp}}(r)$ and $D_{\text{th}}(r)$, but some comments can be made on this curve: 1) it has been obtained by extrapolation of formula (2) up to 1 m, namely out of the range fixed by the authors ($10 < R < 500$ m); 2) the extrapolation is very critical because of the different behaviour of (1) and (2) for $R < 10$ m; 3) in the range of R up to 10 m our expression (1) seems to be better than the extrapolation of (2) as can be seen from the experimental data shown in Fig. 2-3. But these data are still scanty.

It seems however difficult to increase (1) in order to have agreement between $D_{\text{exp}}(r)$ with the $D_{\text{th}}(r)$.

To decide between *A*) and *B*) it is necessary:

1) to measure in a better way Δ for $R < 10$ m to obtain a selection criterion between (1) and the extrapolation of (2).

2) To compute the contribution of muon pairs produced by muons and the contribution of the events discussed by DARDO *et al.* (^{2}).

3) To improve the experimental data on the $D_{\text{exp}}(r)$ by using the same apparatus at various depths. We are now studying these points at 70 and 5 600 m w.e.

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We wish to thank Prof. AGENO, Director of the Physical Department of the Istituto Superiore di Sanità for granting us the use of the computer and Mr. A. DI SALVO for the help given during this investigation.

RIASSUNTO

Si sviluppa un calcolo della cascata nucleare negli E.A.S. di alta energia, usando un modello isobarico composto, per studiare la funzione densità di muoni $\Delta(N, R, > E_\mu)$, over R è la distanza dall'asse dell'E.A.S., E_μ l'energia minima dei μ ed N il numero totale di particelle dello sciame. Si ottiene l'espressione

$$\Delta(N, R, > E_\mu) = \frac{N^{0.85} R^{-0.24} (R + 90)^{-2.4}}{4.6 \cdot 10^{-3} (R + 45)(E_\mu - 10) + 1} \left(\frac{E_\mu + 7}{10}\right)^{0.002R} (\mathrm{m}^{-2}) ,$$

che risulta in buon accordo con i dati sperimentali noti sulla componente muonica dell'E.A.S. La $\Delta(N, R, > E_\mu)$ viene quindi usata per interpretare i risultati sperimentali finora ottenuti sulle particelle parallele sottoterra come dovuti alla componente muonica degli E.A.S. Risulta che a profondità $r > 200$ m a.e. non vi è accordo tra il calcolo ed i risultati sperimentali. Si discute questa discrepanza.

О параллельных проникающих частицах под землей, как результат E.A.S. мюонной компоненты.

Резюме (*). — Используя составную изобарическую модель, проводится вычисление ядерного каскада при высоких энергиях E.A.S., с целью изучить функцию мюонной плотности $\Delta(N, R > E_\mu)$, где R представляет расстояние от оси E.A.S., E_μ — мюонная энергия и N размер E.A.S. Полученное выражение

$$\Delta(N, R > E_\mu) = \frac{N^{0.85} R^{-0.24} (R + 90)^{-2.4}}{4.6 \cdot 10^{-3} (R + 45)(E_\mu - 10) + 1} \left(\frac{E_\mu + 7}{10}\right)^{0.002R} (\mathrm{m}^{-2}) ,$$

хорошо согласуется с экспериментальными данными для мюонной компоненты E.A.S. Затем $\Delta(N, R > E_\mu)$ используется для интерпретации результатов, полученных до сих пор по параллельным проникающим частицам под землей, как результат мюонной компоненты E.A.S., обладающей большой проникающей способностью. Наблюдается расхождение между вычислениями и экспериментальными данными для глубин больше, чем 200 m w.e. Проводится некоторое обсуждение причин этого расхождения.

(*) *Переведено редакцией.*