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A. MALECKI, *et al.*
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Elastic Electron Scattering from ^{6}Li.

A. MALECKI (*)
Institute of Nuclear Physics - Krakow
International Centre for Theoretical Physics - Trieste

P. PICCHI
Laboratori Nazionali di Frascati del CNEN - Frascati

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It is well known that the harmonic-oscillator well model, which is successful in predicting the elastic electron scattering form factors of the light nuclei, fails for ^{6}Li.

A good fit to the experimental data might be obtained by allowing the s- and p-shell nucleons to move in different wells as suggested by ELTON (1). This would reflect the fact that in the ^{6}Li nucleus the p-nucleons are bound a good deal less firmly that in heavier nuclei in the p-shell.

The model with the two different oscillator parameters ($\alpha_s \neq \alpha_p$) holds, however, only at low momentum transfer q and does not reproduce the recent large-q experimental results (2).

Recently a nice interpretation of the data has been given by CIOFI DEGLI ATTI (3) who considered the s- and p-protons moving in different wells, and, in addition, took into account the effect of two-body correlations arising from repulsive core of the nucleon-nucleon interaction.

The influence of the short-range nucleon-nucleon correlations on the elastic charge form factor has been also investigated by the present authors (4).

In this note we make a comment on the calculation of the nuclear center-of-mass motion correction to the elastic form factor in the model with two different oscillator potential wells.

Usually one uses (1) $C_{\text{c.m.}} = \exp[q^2/4A\alpha^2] - \alpha = \alpha_s, \alpha_p$, A being the mass number, as the factor which multiplies the contributions to the form factor coming from s and p nucleons, respectively. This correction is taken, by analogy, from the model with a common oscillator well for all nucleons (5).

(*) Permanent address: Instytut Fizyki Jadrowey, Krakow.
(1) L. R. B. ELTON: *Nuclear Sizes* (Oxford, 1961), p. 25.
(2) L. R. SUELZLE, M. R. YEARIAN and M. CRANNELL: *Phys. Rev.*, **162**, 992 (1967).
(3) C. CIOFI DEGLI ATTI: *Phys. Rev.*, **175**, 1256 (1968).
(4) A. MALECKI and P. PICCHI: *Lett. Nuovo Cimento*, **1**, 81 (1969).
(5) L. J. TASSIE and F. C. BARKER: *Phys. Rev.*, **111**, 940 (1958).

One might suggest also another c.m. correction by using in $C_{\text{c.m.}}$ an average value of the oscillator parameter α obtained *e.g.* either by averaging α_s^2 and α_p^2 [4] or by averaging the r.m.s. radii for the two shells.

All such corrections seem to be plausible but in fact are chosen quite arbitrarily without an exact justification.

The exact way of calculating the c.m. motion effect on the electron scattering elastic form factor is to perform in the expression for the latter the so-called Gartenhaus-Schwartz [6] transformation.

Then, in a shell model yielding the nuclear wave function ψ_{SM}, the elastic form factor is given [*] by

$$(1)\quad \begin{cases} F_{\text{GS}} = f(q_\mu^2)\dfrac{1}{Z}\langle\psi_{\text{SM}}|\sum\limits_{j=1}^{A} e_j \exp[i\boldsymbol{q}(\boldsymbol{r}_j-\boldsymbol{R})]|\psi_{\text{SM}}\rangle\,, \\ \boldsymbol{R} = \dfrac{1}{A}\sum\limits_{k=1}^{A}\boldsymbol{r}_k\,, \qquad Z=\sum\limits_{k=1}^{A} e_k\,. \end{cases}$$

In eq. (1) one sums over all nucleons in the target nucleus and e_j, $\boldsymbol{r}_j$ are the charge and position operators for j-th nucleun; $f(q_\mu^2)$ is the correction due to finite nucleon sizes.

It may easily be shown that

$$(2)\quad \langle\psi_{\text{SM}}|\sum_{j=1}^{A} e_j \exp[i\boldsymbol{q}(\boldsymbol{r}_j-\boldsymbol{R})|\psi_{\text{SM}}\rangle = \sum_{j,k=1}^{A}\langle\alpha_j|\,e\exp\left[i\frac{A-1}{A}\,\boldsymbol{q}\cdot\boldsymbol{r}\right]|\alpha_k\rangle(-1)^{j+k}M_{jk}\,,$$

where M_{jk} is the determinant of order $A-1$ which is constructed from the determinant $\|\langle\alpha_j|\exp[-i\boldsymbol{q}\cdot\boldsymbol{r}/A]|\alpha_k\rangle\|$ by removing in the latter the j-th row and the k-th column; $|\alpha_j\rangle$, $|\alpha_k\rangle$ being single-particle states of the shell model.

We have calculated (for more details see ref. [7]) F_{GS} for the ^{6}Li, ^{12}C and ^{16}O nuclei in the model with two different oscillator potential wells. We have compared the results with the expression of F obtained in this model by applying the usual c.m. correction as given above [1].

In Fig. 1 is presented the ratio $R=(F_{\text{GS}}/F)^2$ as a function of momentum transfer for the three nuclei.

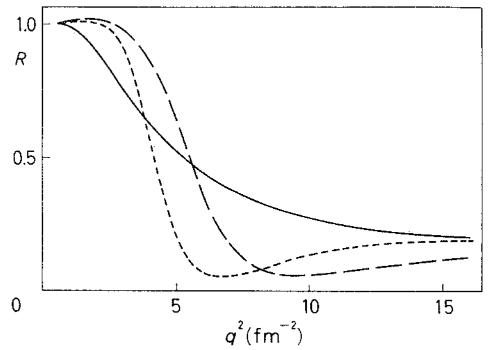

Fig. 1. – Comparison of form factors (shell model with two oscillator wells; no short-range correlations) calculated with two different nuclear c.m. motion corrections. F_{GS} = c.m. correction calculated exactly (by means of the Garthenhaus-Schwartz transformation), F = c.m. correction calculated as in ref. [1]; $R=(F_{\text{GS}}/F)^2$. The oscillator parameters are: for ^{6}Li: $\alpha_s = 130.517$ MeV, $\alpha_p = 98.588$ MeV; for ^{12}C: $\alpha_s = 170$ MeV, $\alpha_p = 130$ MeV; for ^{16}O: $\alpha_s = 170$ MeV, $\alpha_p = 120$ MeV. ——— ^{6}Li, — — — ^{12}C, - - - ^{16}O.

([6]) S. GARTENHAUS and C. SCHWARTZ: *Phys. Rev.*, **108**, 482 (1957).

([*]) We consider here only the charge monopole scattering as it would be the case for spin-zero nuclei. The ^{6}Li nucleus has a nuclear spin equal to 1. Note, however, that since the ^{6}Li quadrupole momentum is very small, the additional quadrupole scattering contribution may be neglected for this nuclid.

([7]) A. MALECKI and P. PICCHI: Frascati report LNF-69/10 (1969).

Evidently (and perhaps suprisingly) there is a remarkable difference between F_{GS} and F. The mathematical structure of the ratio $R(q)$ depends strongly on the parameters α_s, α_p one has chosen and may become quite complicated. The difference between the two form factors will be most accentuated at the values of q where either $F_{GS}(q)$ or $F(q)$ has a minimum. In general, one may say that the discrepancy between F_{GS} and F becomes larger as the difference between the oscillator parameters α_s, α_p increases (if $\alpha_s = \alpha_p$ one has $F_{GS} = F$).

The difference between F_{GS} and expressions for the factor obtained by using other c.m. motion corrections than those mentioned above (with a single, average value of α) is even bigger.

All of this indicates that one must be careful in taking into account the c.m. motion effect in shell-model calculations as various and apparently similar procedures may bring to very different results.

We have also made an attempt to fit the ^{6}Li elastic electron scattering data ([2]) in the model with the two different oscillator wells and with the short-range nucleon-nucleon correlations included.

Since nucleon-nucleon correlations affect only the intrinsic part of the nuclear wave function we have assumed the same c.m. motion correction as in the model without correlations. Precisely speaking we write the expression for the correlated « elastic form factor » as follows:

$$\tilde{F} = \frac{F_{GS}}{F_{SM}}(F_{SM} + \Delta F)\,, \tag{3}$$

where F_{SM} is the so-called shell-model form factor, *i.e.* eq. (1) with $f = 1$ qnd $R = 0$. ΔF is the correction to F_{SM} which accounts for nucleon-nucleon correlations.

The short-range dynamical correlation correction has been calculated as in ref. ([4]). The correlations are introduced into the nuclear wave function by means of a unitary transformation.

As we would like to consider the short-range correlations, *viz.* those arising from the hard-core repulsion between nucleons, we are allowed to take into account only the two-particle correlations. We neglect then the probability of simultaneous modification of the wave functions of more than two particles as the probability for three and more nucleons to come close together should be small.

In this approximation the correlated elastic form factor can be expressed in terms of the matrix elements between the two-particle states. In order to obtain the correlated state of a nucleon-nucleon pair we perform first the so-called Moshinsky ([8]) transformation, *i.e.* the transformation from single-particle co-ordinates to relative and center-of-mass co-ordinates for the two nucleons. The nucleon-nucleon correlations are introduced then by modifying (in a unitary way: preserving normalizations and orthogonalities) the shell-model radial wave function $R_{nl}(r)$ of the relative two-nucleon motion:

$$\left\{\begin{aligned} \tilde{R}_{nl}(r) &= \frac{g(r)}{\sqrt{N_{nl}}}R_{nl}(r)\,,\\ N_{nl} &= \int_0^\infty \mathrm{d}r^2 r R_{nl}^2 g^2(r)\,, \end{aligned}\right. \tag{4}$$

where $g(r)$ is the « correlation » function.

([8]) M. Moshinsky: *Nucl. Phys.*, **13**, 104 (1959).

Employing the Moshinsky [8] technique one obtains the following two-particle correlation correction to the elastic form factor:

$$(5)\qquad Z\Delta F = 6\exp[-t_s]\,\Delta\langle 000, 000; q; s\rangle + (Z-2)\exp[-t_p]\Big[(3-4t_p+t_p^2)\,\Delta\langle 000, 000;\; q; p\rangle + \frac{1}{2}\Delta\langle 100, 100; q; p\rangle - \left(\frac{2}{3}\right)^{\frac{1}{2}} t_p\Delta\langle 100, 000; q; p\rangle + \frac{5}{3}\Delta\sum_m\langle 01m, 01m; q; p\rangle - \frac{10}{3}t_p\Delta\langle 011, 011; q; p\rangle + \frac{1}{2}\Delta\sum_m\langle 02m, 02m; q; p\rangle + \left(\frac{4}{3}\right)^{\frac{1}{2}} t_p\Delta\langle 020, 000; q; p\rangle\Big] + \frac{1}{2}(Z-2)\exp[-v_s v_p t_{sp}]\Big\{(3-2v_s v_p t_{sp})v_s\big[\Delta\langle 000, 000; qv_s; sp\rangle + \Delta\langle 000, 000; qv_p; sp\rangle\big] + \frac{5}{3}v_p\Big[\Delta\sum_m\langle 01m, 01m; qv_s; sp\rangle + \Delta\sum_m\langle 01m, 01m; qv_p; sp\rangle\Big]\Big\},$$

where

$$t_c = \frac{q^2}{8\alpha_c^2}, \qquad c = s, p, sp,$$

$$\alpha_{sp} = \left(\frac{2\alpha_s^2\alpha_p^2}{\alpha_s^2+\alpha_p^2}\right)^{\frac{1}{2}}, \qquad v_s = \left(\frac{\alpha_{sp}}{\alpha_s}\right)^2, \qquad v_p = \left(\frac{\alpha_{sp}}{\alpha_p}\right)^2,$$

$$\langle nlm, n'l'm'; q; c\rangle = \langle (nlm)_c|\exp\left[\frac{iqZ}{\sqrt{2}}\right]|(n'l'm')_c\rangle .$$

In eq. (5) $\Delta(...)$ denotes the difference between correlated and uncorrelated magnitudes, $|(nlm)_c\rangle$ is an harmonic-oscillator state with the oscillator parameter α_c.

Formula (5) is valid for nuclei with two protons in the s-shell, and $Z-2$ protons in the p-shell. The oscillator parameters for the two shells are assumed to be different.

There is an improvement in formula (5) (*) as compared to that in ref. [4], namely the Moshinsky transformation for two nucleons from the different shells (terms with $c = sp$) has been performed in the exact way. Moreover some mistakes have been corrected. These modifications in the formula for ΔF yield, however, only very small changes of the numerical results.

We have calculated the ^{6}Li elastic charge form factor using the following form of $g(s)$:

$$(6)\qquad g^2(s) = 1 - \exp\left[-\frac{1}{2}\Lambda^2 s^2\right].$$

Such a correlation function represents the soft-core repulsion between nucleons at small

(*) Let us call attention to the fact that in our preliminary work [9] on the short-range correlation the formula for ΔF should be read without a factor $2Z-1$. The mistake has been already corrected in ref. [4].

[9] A. MALECKI and P. PICCHI: Frascati report LNF-68/27 (1968); *Phys. Rev. Lett.*, **21**, 1395 (1968).

relative distance s. The form (6) of $g(s)$ enables us to perform all the integrations in (5) analytically.

The nucleon structure correction has been taken the same as in ref. [4].

Our results are presented in Fig. 2, where we compare the correlated elastic charge form factor (full line) and the uncorrelated one (dashed line) with the experimental data.

In fig. 3 we have presented the nuclear charge density distribution for the ^{6}Li nucleons, calculated as the Fourier transform of the form factor. The introdution of the

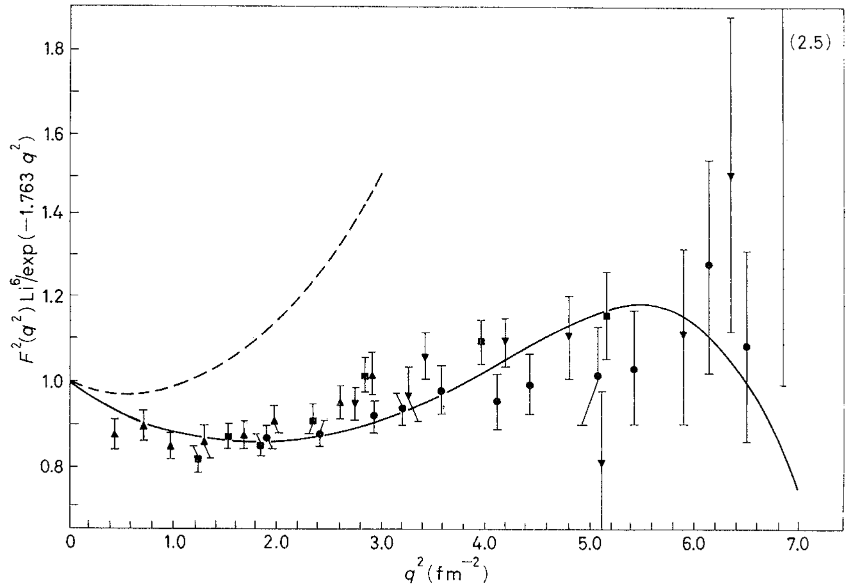

Fig. 2. – ^{6}Li charge form factor. Full line: short-range correlations included; dashed line: no correlations. The values have been divided by $\exp[-1.763q^2]$, which corresponds to the square of the form factor for a Gaussian charge distribution of r.m.s. radius 2.3 fm. The parameters are $\alpha_s = 130.517$ MeV, $\alpha_p = 98.588$ MeV, $\Lambda = 1.678\ \mathrm{fm}^{-1}$. Experimental points are taken from ref. [2].

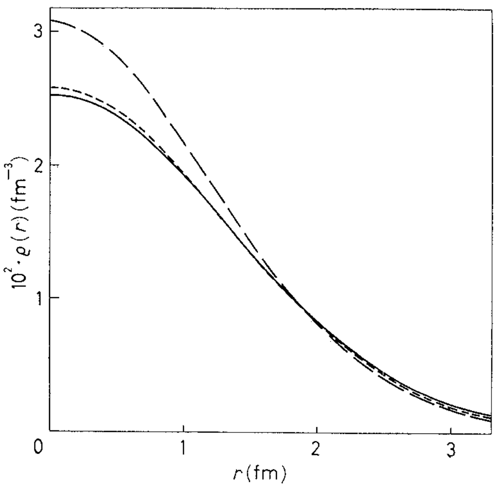

Fig. 3. – ^{6}Li charge density distribution. Full line: short-range correlations included; broken line: no correlations. The parameters are $\alpha_s = 130.517$ MeV, $\alpha_p = 98.588$ MeV, $\Lambda = 1.678\ \mathrm{fm}^{-1}$. The dashed line represents the phenomenological density found in ref. [2].

repulsion between nucleons at small distances leads to an appreciable decrease of the central part of charge density, and to an increase of the outer density.

We can summarize this letter as follows:

a) One can obtain a good fit to the ^{6}Li data over the large range of momentum transfer q in the model with the two different oscillator wells provided one includes the short-range correlations between nucleons.

b) It is very important to calculate the center-of-mass motion correction for this model in the exact way, *i.e.* via the Gartenhaus-Schwartz transformation.

* * *

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