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Regge-Pole Families and Toller Poles: $t \neq 0$ (*).

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Summary. — « Mass formulae » for $M = 0$ and $M = 1$ Toller families are derived using the standard tools of the S-matrix theory.

1. – Introduction.

In recent years, the properties of scattering amplitudes at $t = 0$ (***) have been widely studied. The origin of the interest in this particular kinematical point, which at high energy is very close to the physical region, is essentially twofold. In fact, at this point some kinematical complications, due to the existence of spins, occur in the form of constraints between different helicity amplitudes. These constraints are independent of any dynamical scheme used to describe the S-matrix but must be satisfied in any dynamical theory

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(***) For a more detailed exposition, see, *e.g.*, ref. (1).

(1) P. Di Vecchia and F. Drago: California Institute of Technology preprint CALT-68-172, hereafter referred to as I.

eventually used. They are particularly relevant from the point of view of the Regge-pole theory, which has proved, in the last few years, to be the most successful dynamical scheme for the description of the strongly interacting particles, in that they gave rise to the concept of « conspiracy » between different Regge trajectories.

Additional interest in the point $t = 0$ arises in connection with the Regge expansion itself: the somewhat surprising result of the study of the latter problem is that Regge poles must occur in families, generally infinite, with well-defined relations among the members of a family at $t = 0$.

In the present paper we will discuss some properties of these families: let us therefore remember how the concept of Regge-pole families entered in the game (*). There have been two approaches to this problem: an analytic one and a group-theoretical one. The former covers the unequal-mass situation, but is mute in the equal-mass case, the latter covers the pairwise equal-mass case, but its extension to the unequal-mass case is not very easy.

In the unequal-mass situations, one finds that the contribution of a single Regge pole to the scattering amplitude is not an analytic function at $t = 0$. The simplest way to avoid these unwanted singularities is to assume (2) that Regge trajectories occur in familes, with definite requirements on the spacing of the members of a family and on the behavior of the residue functions at $t = 0$.

In the pairwise equal-mass situation, on the other hand, it can be shown (3-7) that the scattering amplitude at $t = 0$ can be expanded in terms of the representations of the group $O_{3,1}$ (or O_4). The Reggeization of these expansions shows that a Toller pole, that is a pole in the « four-dimensional angular-momentum plane », leads to an infinite family of Regge poles, with definite relations between the trajectories and the residue functions at $t = 0$. The families of Regge poles so generated are characterized, apart from the internal quantum numbers, by a Lorentz quantum number M, which, for boson trajectories, can take all the integer values.

On the other hand, using analyticity arguments and the factorization theorem, it has been shown (8) how a classification of the Regge-pole families deduced from analyticity is also possible. The equivalence of the « group

(*) For a more detailed exposition, see, *e.g.*, ref. (1).

(2) D. Z. FREEDMAN and J. H. WANG: *Phys. Rev.*, **153**, 1596 (1967).

(3) M. TOLLER: *Nuovo Cimento*, **37**, 631 (1965); University of Rome reports No. 76 (1965) and No. 84 (1966) (unpublished).

(4) A. SCIARRINO and M. TOLLER: *Journ. Math. Phys.*, **7**, 1670 (1967).

(5) M. TOLLER: *Nuovo Cimento*, **53** A, 671 (1968); **54** A, 295 (1968).

(6) D. Z. FREEDMAN and J. H. WANG: *Phys. Rev.*, **160**, 1560 (1967).

(7) G. DOMOKOS: *Phys. Lett.*, **24** B, 293 (1967); *Phys. Rev.*, **159**, 1387 (1967).

(8) P. DI VECCHIA, F. DRAGO and M. L. PACIELLO: *Nuovo Cimento*, **56** A, 1185 (1968).

theoretical » and « analytical » families, and of their classification, has been proved in I.

Up to now we discussed only the point $t = 0$. However, the existence of the daughter trajectories raises some very interesting problems from the complementary point of view according to which a Regge trajectory is associated to a whole string of resonances, provided that its real part reaches the « physical » angular-momentum region for positive values of t.

The possibility that some of the recently discovered resonances could be associated with Toller families stimulated a study of the properties of these families for $t \neq 0$. Some work in this direction has been done on the basis of the group-theoretical formalism by DOMOKOS and collaborators [9-12]. This has been possible, in the group-theoretical formalism, by clearly distinguishing the « invariance group » of a scattering amplitude from the « classification group » [10] of the Regge bound states. The former depends on « external » properties of the amplitude, in particular the masses of the incoming and outgoing particles, whereas the latter does not. This means that, in the general mass situation, the « classification group » does not need to coincide with the « invariance group ». In this approach a Lie algebra, the « trajectory-generating algebra » (TGA), is associated with Regge poles, and it can be shown [10] that the only physically admissible algebra is the Lie algebra $SL_{2.C}$ of the homogeneous Lorentz group. The question whether the $SL_{2.C}$ classification can be extended to nonzero values of t, has been discussed particularly in ref. [11] and [12] where, using the Bethe-Salpeter equation, a perturbation theory in t has been developed. This approach, in particular, leads to the « mass formula » which describes the behavior of a whole Toller family near $t = 0$ in terms of a few parameters. The « mass formula » so obtained, leads to quite encouraging results in the analysis of the best-known Regge trajectories: the $I = \frac{1}{2}$ and $I = \frac{3}{2}$ nonstrange baryon trajectories [13].

The aim of the present paper is to use the analytical formalism, developed in I, to study the properties of the Toller families for small nonzero $|t|$. In fact, the requirement that a class of Regge trajectories belong to a single Toller family places very strong constraints on their behavior not only at $t = 0$, but also in the neighborhood of this point. Of course, these constraints become less stringent as $|t|$ grows.

There are two kinds of quantities that can be discussed in this context. These are the trajectories and the residue functions of the members of a

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(9) G. DOMOKOS: *Phys. Rev.*, **159**, 1387 (1967).
(10) G. DOMOKOS and G. L. TINDLE: *Phys. Rev.*, **165**, 1906 (1968).
(11) G. DOMOKOS and P. SURANYI: *Nuovo Cimento*, **56** A, 445 (1968).
(12) G. DOMOKOS and P. SURANYI: preprint KFI 4/1968.
(13) G. DOMOKOS, S. KÖVESI-DOMOKOS and P. SURANYI: *Nuovo Cimento*, **56** A, 233 (1968).

Toller family. In fact, at $t=0$ the intercept and the residue functions of the daughter poles can be expressed in terms of only one parameter: the parent trajectory intercept and residue function respectively. On the other hand, for $t \neq 0$, one can perform a Taylor expansion and study the restrictions imposed by the analyticity on the various coefficients of the expansion. It turns out that all the $t=0$ derivatives of the trajectories and residue function of the infinite members of a Toller family can be parameterized in a well-defined way. More definitely, one finds, for the simpler $M=0$ case, that

$$\alpha_m^{(k)} = f_k(m;\, \alpha_0^{(0)}, \alpha_0^{(1)}, \dots, \alpha_0^{(k)}; \dots, \alpha_k^{(1)}, \dots, \alpha_k^{(k)}) ,$$

where the superscript denotes the order of the derivative, evaluated at $t=0$, and f_k is a known function. This means that in the Chew-Frautschi approximation (the power series of $\alpha(t)$ broken off at the linear term), a generally infinite family of boson Regge trajectories that, kinematically, « conspire » to give a Toller pole at $t=0$, is described by only three independent parameters.

In a similar way, one can find a parameterization of the residue derivatives. In this case, however, the trajectory derivatives also enter as determining parameters. The general result, in the $M=0$ case, can be expressed as

$$\gamma_m^{(k)} = g_k(m;\gamma_0^{(0)},\gamma_0^{(1)},\dots,\gamma_0^{(k)};\dots;\gamma_k^{(0)},\gamma_k^{(1)},\dots,\gamma_k^{(k)};\alpha_0^{(0)},\alpha_0^{(1)},\dots,\alpha_0^{(k)};\dots;\alpha_k^{(0)},\alpha_k^{(1)},\dots,\alpha_k^{(k)}).$$

In the present paper we will limit ourselves to the study of the trajectories in the Chew-Frautschi approximation. In Sect. **2**, after a short review of the general idea of our method, we will deduce the mass formula for $M=0$ Toller families. The results turn out to be the same as in the group-theoretical approach. This completes the proof that all the results obtained using the group-theoretical formalism can also be derived by a careful use of the standard methods of the S-matrix theory. The $M=1$ case is discussed in Sect. **3**. This case has not been previously covered in the group-theoretical discussions and the new result is that the $M=1$ Toller family, that means in fact two families of Regge trajectories with all the same internal quantum numbers but with opposite natural parity, can be parameterized, in the Chew-Frautschi approximation, in terms of only four independent parameters.

For both $M=0$ and $M=1$ we discuss the unequal-unequal (UU) and the equal-unequal (EU) mass configurations. In our approach, the UU case has more information than the EU. This can be easily understood, since the singularity to be canceled is weaker in the EU than in the UU case, while our fundamental relations are just the conditions that express such a cancelation. The results deduced from the EU case, although less complete, are always in agreement with those deduced from the UU case.

An account of our work has already been published [14]; the $M=0$ case

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(14) P. DI VECCHIA and F. DRAGO: *Phys. Lett.*, **27** B, 387 (1968).

has also been discussed independently by BRONZAN, JONES, and KUO [15].

The parametrization of the residue functions near $t=0$ is not discussed here. In fact, as a detailed analysis shows, the structure of the resulting formulae is rather complicated and at present no convenient expressions are available for the purpose of phenomenological analyses. For an example of the results that can be obtained, see eq. (9) of ref. [15].

2. – The $M=0$ case.

Let us briefly recall our assumptions. A more detailed discussion can be found in I. We assume (*):

a) analyticity;

b) simplicity;

c) crossing symmetry.

The analyticity and simplicity require the existence of the daughter poles [2] and establish the relations between the parent and the daughter trajectories and residue functions at $t=0$, while the requirements of analyticity and crossing symmetry impose some constraints on the helicity amplitudes that must be satisfied by the Toller families.

We discuss in this Section both the class I and class II families. The discussion of the UU case is the same for both classes, while some differences appear in the EU case.

The notations used are the same as in I, to which the reader is also referred for some algebraic manipulations that we do not repeat here.

2'1. *The unequal-unequal mass case.* – We consider the s-channel reaction $1+2 \to 3+4$ and the t-channel reaction $\bar{4}+2 \to 3+\bar{1}$. The contribution of a Toller family to the nonflip-nonflip amplitude $(\lambda=\mu=0)$ $f(s,t)$ is given by

$$f(s,t)=\sum_{m=0}^{\infty}\left(\frac{t_0}{t}\right)^m\left[\frac{B(t)}{t_0}\right]^m\left(\frac{s}{s_0}\right)^{-m}\sum_{k=0}^{N(m)}\sum_{n=0}^{m-2k}d^m_{n,k}(t)\left(\frac{s}{s_0}\right)^{\alpha_n(t)+n}, \tag{2.1}$$

where

$$N(m)=\begin{cases}\dfrac{m}{2} & \text{if } (-1)^m=1\,,\\[2mm] \dfrac{m-1}{2} & \text{if } (-1)^m=-1\,,\end{cases}$$

([15]) J. B. BRONZAN, C. E. JONES and P. K. KUO: Massachusetts Institute of Technology preprint (1968).

(*) The factorization assumption used in I in order to reconstruct the Toller pole is not needed for the study of the mass formulae.

$$d^m_{n,k}(t) = g(\alpha_n)\gamma_m(t)\,a_k(\alpha_n)\left[\frac{D(t)}{B(t)}\right]^{2k}\left[\frac{B(t)}{t_0}\right]^{-n}\frac{\Gamma(\alpha_n-2k+1)}{(m-n-2k)!\,\Gamma(\alpha+1-m+n)},$$

$$g(\alpha) \;= -\frac{2\alpha+1}{\sin\pi\alpha}\,[1+\tau\exp[-i\pi\alpha]]\,\frac{\Gamma(\alpha+\frac{1}{2})\,2^{\alpha}}{\Gamma(\alpha+1)\sqrt{\pi}},$$

$$B(t) \;= \frac{t(t-\sum m_i^2)+(m_1^2-m_3^2)(m_2^2-m_4^2)}{2s_0},$$

$$D(t) \;= \frac{1}{2s_0}\{[t-(m_1+m_3)^2][t-(m_1-m_3)^2][t-(m_2+m_4)^2][t-(m_2-m_4)^2]\}^{\frac{1}{2}},$$

$$a_k(\alpha) \;= \frac{\Gamma(-\alpha/2+k)\,\Gamma(-\alpha/2+\frac{1}{2}+k)\,\Gamma(\frac{1}{2}-\alpha)}{\Gamma(-\alpha/2)\,\Gamma(-\alpha/2+\frac{1}{2})\,\Gamma(\frac{1}{2}-\alpha+k)\,k!},$$

and $\gamma_n(t)$ is the reduced residue function of the n-th daughter pole, from which the appropriate singular factor $(t_0/t)^n$ has been factored out; α is the intercept and τ the signature of the parent trajectory. Since the amplitude $f(s, t)$ has to be analytic at $t=0$, for any s, we must require that the following analyticity conditions hold:

$$\sum_{k=0}^{N(m)}\sum_{n=0}^{m-2k} d^m_{n,k}(t)\left(\frac{s}{s_0}\right)^{\alpha_n(t)+n} = O(t^m)\,, \qquad \text{for } m\geqslant 1. \tag{2.2}$$

Relations of this kind are the fundamental ones in our approach.

Relations (2.2) were used in I, as they stand, in order to express all the quantities $\gamma_n(0)$ in terms of $\gamma_0(0)$. The result

$$\gamma_n(0) = \frac{(-1)^n}{n!}\,\frac{\Gamma(n-1-2\alpha)}{\Gamma-1-2\alpha}\,\gamma_0(0) \tag{2.3}$$

has been found.

We will now show how a parametrization of the derivatives $\alpha'_n(0)$ can be deduced from (2.2). Differentiating the equation (2.2) with respect to t and evaluating it at $t=0$, we find

$$\sum_{k=0}^{N(m)}\sum_{n=0}^{m-2k}\frac{\mathrm{d}}{\mathrm{d}t}[d^m_{n,k}(t)]\bigg|_{t=0} + \left(\log\frac{s}{s_0}\right)\sum_{k=0}^{N(m)}\sum_{n=0}^{m-2k}\alpha'_n(0)\,d^m_{n,k}(0) = 0 \qquad m\geqslant 2. \tag{2.4}$$

Since those conditions must hold for any s, the two sums in (2.4) must go to zero separately. We must therefore have, at least

$$\sum_{k=0}^{N(m)}\sum_{n=0}^{m-2k}\alpha'_n(0)\,d^m_{n,k}(0) = 0\,, \qquad m\geqslant 2\,. \tag{2.5}$$

After some rearrangements, the equation (2.5) gives the following system (*):

$$\sum_{n=0}^{m} I_{mn}\zeta^n \alpha'_n(0) = 0\,, \qquad m \geqslant 2\,, \tag{2.6}$$

where the quantity I_{mn} and ζ^n are defined by

$$I_{mn} = \frac{1}{2^{2\alpha+1}\sqrt{\pi}}\,\frac{1}{(m-n)!}\,\frac{2^m \Gamma(m-\alpha)}{\Gamma(m+n-2\alpha)} \tag{2.7}$$

and

$$\zeta^n = [2(\alpha-n)+1]\gamma_n(0)\,. \tag{2.8}$$

Using (2.3), the system (2.6) turns out to be

$$\sum_{n=0}^{m}(-1)^n \binom{m}{n} \frac{\alpha'_n(0)}{\prod\limits_{\substack{k=1\\k\neq n}}^{m}(n+k-2\alpha-1)} = 0\,, \qquad m \geqslant 2\,, \tag{2.9}$$

and its solution is

$$\alpha'_n(0) = \frac{\alpha'_1(0)\,n(1+2\alpha-n)}{2\alpha} + \frac{\alpha'_0(0)(n-1)(n-2\alpha)}{2\alpha}\,. \tag{2.10}$$

In order to show that (2.10) is the solution of the system (2.9), one has to show that

$$\frac{\alpha'_0(0)-\alpha'_1(0)}{2\alpha}\sum_{n=0}^{m}(-1)^n\binom{m}{n}\frac{n}{\prod\limits_{\substack{k=1\\k\neq n}}^{m}(n+k-2\alpha-1)} + {} + \alpha'_0\sum_{n=0}^{m}(-1)^n\binom{m}{n}\frac{1}{\prod\limits_{\substack{k=0\\k\neq n}}^{m}(n+k-2\alpha-1)} = 0\,. \tag{2.11}$$

The second sum in (2.11) has been shown to be equal to zero in I (Appendix B). It is therefore enough to prove that

$$\sum_{n=1}^{m}(-1)^n\binom{m}{n}\frac{n}{\prod\limits_{\substack{k=1\\k\neq n}}^{m}(n+k-2\alpha-1)} = 0\,, \qquad m \geqslant 2\,. \tag{2.12}$$

(*) The choice $t_0 = D(0) = B(0)$ has been made.

This can be done, using the same method as in I (Appendix B), starting from the identity

$$\sum_{m=1}^{h}(-1)^n\binom{m}{n}\frac{n}{\prod\limits_{\substack{k=1\\k\neq n}}^{m}(n+k+x)}=\frac{(-1)^h m(m-2)!}{(m-h-1)!\,(h-1)!\prod\limits_{k=2}^{m}(h+k+x)}\,.$$

That can be proved by induction.

We have therefore found, in the Chew-Frautschi approximation, the following phenomenological expression for Regge trajectories that belong to an $M=0$ Toller family:

$$\alpha_n(t)=\alpha-m+[c_1+c_2(\alpha-n)(\alpha-n+1)]t+O(t^2)\,. \tag{2.13}$$

The meaning of the mass formula (2.13) is particularly clear from the group-theoretical point of view ([9]). In fact, if one starts from the equal-mass situation where the Lorentz symmetry applies at $t=0$, one can think that in going to $t\neq 0$ a symmetry breaking is introduced. In the absence of symmetry breaking, the daughter trajectories would be parallel to the parent one. In the presence of a Lorentz symmetry breaking, the trajectories are no longer parallel; however, since the rotational symmetry must be preserved, the symmetry-breaking term must be proportional to the Casimir operator of the group O_3, that is $l(l+1)$. The strength of the symmetry breaking is therefore measured by the constant c_2.

Since the properties of the Regge trajectories are independent of the properties of the ingoing and outgoing particles, the same mass formula, eq. (2.13), should be found starting from the EU case. We will see, in this case, that for class I families the mass formula can be derived only for the even-numbered members of the family, while for the class II families it can be derived for the even and odd members, one condition between them being provided by the conspiracy relation.

2'2. *The equal-unequal mass case.* – We will explicitly restrict ourselves to configurations in which the equal-mass vertex is provided by an $\mathcal{N}$-$\overline{\mathcal{N}}$ system. In the EU case, the kinematics is simpler. One has

$$D(t)=\sqrt{t\tilde{D}(t)}\,,$$

with

$$D(t)=\frac{(t-4m^2)[t-(m_1+m_3)^2][t-(m_1-m_3)^2]}{2s_0^2}\,,$$

and

$$B(t) = \frac{t\,\tilde{B}(t)}{s_0},$$

with

$$\tilde{B}(t) = \frac{t - \sum m_i^2}{2}.$$

At the equal-mass vertex, the selection rules due to parity and G-parity invariance must be taken into account. This implies that class I and class II Toller families must be discussed separately.

For class I, which we discuss first, one finds that the parent and even daughter trajectories couple to the amplitude $\tilde{f}^{(+)}_{c,c;\frac{1}{2},\frac{1}{2}} = \tilde{f}^{(+)}_{0,0}$, while the odd daughters are decoupled from the $\mathcal{N}$-$\bar{\mathcal{N}}$ system.

The contribution of a class I Toller family to the amplitude $\tilde{f}^{(+)}_{0,0}$ can be written as

$$f^{(+)}_{0,0} = \sum_{m=0}^{\infty} \left(\frac{t_1}{t}\right)^m \left[\frac{s + \tilde{B}(t)}{s_0}\right]^{2m} \sum_{n=0}^{m} b_{2n;\,m-n}(t) \left[\frac{s + \tilde{B}(t)}{s_0}\right]^{\alpha_{2n}(t)+2n}, \tag{2.14}$$

where

$$b_{n,k}(t) = g(\alpha_n)\gamma_n(t)\,a_k(\alpha_n)\left[\frac{\tilde{D}(t)}{t_1}\right]^k. \tag{2.15}$$

The $t = 0$ analyticity conditions are therefore given by

$$\sum_{n=0}^{m} b_{2n;m-n}(t)\left[\frac{s + \tilde{B}(t)}{s_0}\right]^{\alpha_{2n}(t)+2n} = O(t^m), \qquad m \geqslant 1. \tag{2.16}$$

Using the same method as in Subsect. 2·1, the following system (*) for the derivative $\alpha'_{2n}(0)$ is deduced from (2.16):

$$\sum_{n=0}^{m} \frac{[2(\alpha - 2n) + 1]\,\gamma_{2n}(0)\,\alpha'_{2n}(0)}{(m - n)!\,\Gamma(\frac{1}{2} - 2 + m + n)} = 0, \qquad m \geqslant 2. \tag{2.17}$$

Using the result

$$\gamma_{2n}(0) = \frac{(-1)^n}{n!}\,\frac{\Gamma(n - \frac{1}{2} - \alpha)}{\Gamma(-\frac{1}{2} - \alpha)}\,\gamma_0(0), \tag{2.18}$$

proved in I, it can be shown that the solution of the system (2.17) is given by

$$\alpha'_{2n}(0) = \frac{\alpha'_2(0)\,n(2\alpha + 1 - 2n)}{2\alpha - 1} - \frac{\alpha'_0(0)(n - 1)(2\alpha - 1 - 2n)}{2\alpha - 1}. \tag{2.19}$$

(*) The choice $t_1 = D(0)$ has been made.

A mass formula of the same kind as (2.13) has therefore been deduced from the EU case, but for the even members of the family.

We discuss now the class II families. Due to the charge-conjugation and parity-conservation rules at the nucleon vertex, one finds that the parent and the even daughters contribute to the amplitude $\tilde{f}^{(-)}_{c,c;\frac{1}{2},\frac{1}{2}} = \tilde{f}^{(-)}_{0,1}$, while the odd daughters contribute to the amplitude $\tilde{f}^{(-)}_{c,c;\frac{1}{2},\frac{1}{2}} = \tilde{f}^{(-)}_{0,0}$. The contribution of the whole family to the previous amplitudes is given by

$$\tilde{f}^{(-)}_{0,0} = \sum_{m=0}^{\infty} \left(\frac{t_1}{t}\right)^m \left[\frac{s+\tilde{B}(t)}{s_0}\right]^{-2m} \sum_{n=0}^{m} b_{2n+1;m-n}(t) \left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_{2n+1}(t)+2n}, \tag{2.20}$$

$$\tilde{f}^{(-)}_{0,1} = \sum_{m=0}^{\infty} \left(\frac{t_1}{t}\right)^m \left[\frac{s+\tilde{B}(t)}{s_0}\right]^{-2m} \sum_{n=0}^{m} b_{2n;m-n}(t) \frac{\alpha_{2n}(t)-2(m-n)}{\sqrt{\alpha_{2n}(\alpha_{2n}+1)}} \cdot \left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_{2n}(t)+2n-1}, \tag{2.21}$$

where $b_{n,k}(t)$ is defined by (2.15) with the appropriate residue functions. The $t=0$ analyticity conditions are therefore given by

$$\sum_{n=0}^{m} b_{2n+1;m-n}(t) \left[\frac{s+B(t)}{s_0}\right]^{\alpha_{2n+1}(\tilde{t})+2n} = O(t^m), \qquad m \geqslant 1, \tag{2.22}$$

$$\sum_{n=0}^{m} b_{2n;m-n}(t) \frac{\alpha_{2n}(t)-2(m-n)}{\sqrt{\alpha_{2n}(\alpha_{2n}+1)}} \left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_{2n}(t)+2n-1} = O(t^m), \qquad m \geqslant 1. \tag{2.23}$$

The following systems for the derivatives of the trajectories are now easily obtained from (2.22) and (2.23):

$$\sum_{n=0}^{m} \frac{[2(\alpha-2n)-1]\gamma^{2n+1}_{0,0}(0)\alpha'_{2n+1}(0)}{(m-n)!\,\Gamma(\frac{3}{2}-\alpha+m+n)} = 0\,, \qquad m \geqslant 2, \tag{2.24}$$

$$\sum_{n=0}^{m} \frac{[2(\alpha-2n)+1]\gamma^{2n}_{0,1}(0)\alpha(0)}{(m-n)!\,\Gamma(\frac{1}{2}-\alpha+m+n)\sqrt{(\alpha-2n)(\alpha-2n+1)}} = 0\,, \qquad m \geqslant 2. \tag{2.25}$$

The constraint equation

$$i\tilde{f}^{(-)}_{0,1} - \tilde{f}^{(-)}_{0,0} = O(t)$$

gives the following $t=0$ analyticity conditions:

$$i\sum_{n=0}^{m} b_{2n;m-n}(t) \frac{\alpha_{2n}(t)-2(m-n)}{\sqrt{\alpha_{2n}(\alpha_{2n}+1)}} \left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_{2n}(t)+2n-1} - \sum_{n=0}^{m} b_{2n+1;m-n}(t) \left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_{2n}(t)+2n} = O(t^{m+1}). \tag{2.26}$$

These conditions provide the following system for the first derivatives of the even and odd daughter trajectories:

$$(2.27)\qquad i\sum_{n=0}^{m}\frac{[2(\alpha-2n)+1]\,\gamma_{0,1}^{2n}(0)\,\alpha'_{2n}(0)}{(m-n)!\,\Gamma(\frac{1}{2}-\alpha+n+m)\sqrt{(\alpha-2n)(\alpha-2n+1)}}-\\ -\sum_{m=0}^{m}\frac{[2(\alpha-2n)-1]\,\gamma_{0,0}^{2n+1}(0)\,\alpha'_{2n+1}(0)}{(m-n)!\,\Gamma(\frac{3}{2}-\alpha+m+n)}=0\,,\qquad m\geqslant 1\,.$$

Using the following expressions, derived in I:

$$(2.28)\qquad \gamma_{0,1}^{2n}(0)=\frac{(-1)^n}{n!}\left[\frac{(\alpha-2n)(\alpha-2n+1)}{\alpha(\alpha+1)}\right]^{\frac{1}{2}}\frac{\Gamma(n-\frac{1}{2}-\alpha)}{\Gamma(-\frac{1}{2}-\alpha)}\,\gamma_{0,1}^{n=0}(0)\,,$$

$$(2.29)\qquad \gamma_{0,0}^{2n+1}(0)=\frac{(-1)^n}{n!}\,\frac{\Gamma(n+\frac{1}{2}-\alpha)}{\Gamma(\frac{1}{2}-\alpha)}\,\gamma_{0,0}^{n=1}(0)\,,$$

$$(2.30)\qquad \gamma_{0,0}^{n=1}(0)=-i\,\frac{2\alpha+1}{\sqrt{\alpha(\alpha+1)}}\,\gamma_{0,1}^{n=0}(0)\,,$$

the solution of the systems (2.24) and (2.25) are

$$(2.31)\qquad \alpha'_{2n}(0)=\frac{\alpha'_2(0)\,n(2\alpha+1-2n)}{2\alpha-1}-\frac{\alpha'_0(0)(n-1)(2\alpha-1-2n)}{2\alpha-1}\,,$$

$$(2.32)\qquad \alpha'_{2n+1}(0)=\frac{\alpha'_3(0)\,n(2\alpha-1-2n)}{2\alpha-3}-\frac{\alpha'_1(0)(n-1)(2\alpha-3-2n)}{2\alpha-3}\,.$$

The system (2.27) provides the supplementary condition

$$(2.33)\qquad \big(\alpha'_0(0)-\alpha'_2(0)\big)-\frac{2\alpha-1}{2\alpha-3}\big(\alpha'_1(0)-\alpha'_3(0)\big)=0\,.$$

This means that we obtained

$$(2.34)\qquad \begin{cases}\alpha'_{2m}(0)=c_1+c_2(\alpha-2n)(\alpha-2n+1)\,,\\ \alpha'_{2n+1}(0)=d_1+c_2(\alpha-2n-1)(\alpha-2n)\,,\end{cases}$$

in agreement with the mass formula (2.13).

At this point it is also apparent how our method can be used to parametrize all the derivatives of the trajectories and of the residue functions at $t=0$. It is also clear how the general structure of the parameterization, given in the Introduction, appears. However, as the order of the derivative increases, the resulting systems become more and more involved, the constraints less stringent, and the final expressions more cumbersome.

3. – The $M=1$ case.

The discussion of this family is much more involved, due essentially to the fact that σ (the natural parity) is not diagonalizable with M. Here, for the first time, the parity doubling phenomenon appears. This means that every family contains two subfamilies of Regge poles, with all the same quantum numbers, but with opposite parities (and with the same α at $t=0$, $\alpha=\alpha^+(0)=\alpha^-(0)$).

The algebra is more involved than in the previous Section and our discussion will rest heavily on the results derived in I.

3·1. *The unequal-unequal mass case.* – The main complication is due to the fact that the amplitudes $\tilde{f}^{(\pm)}_{\mu,\lambda}$ are not dominated by the exchange of definite parity when $\min\{|\lambda|, |\mu|\}>0$. Since the amplitudes to be studied here are $\tilde{f}^{(\pm)}_{1,1}$, it follows that the two subfamilies with opposite natural parity must be studied simultaneously.

The following $t=0$ analyticity conditions were derived in I:

$$\sum_{k=0}^{N(m+1)}\sum_{n=0}^{m-2k+1}\tilde{d}^{m+1(+)}_{n,k}(t)\left(\frac{s}{s_0}\right)^{\alpha_n^++n}-\sum_{k=0}^{N(m)}\sum_{n=0}^{m-2k}\tilde{c}^{m(-)}_{n,k}(t)\left(\frac{s}{s_0}\right)^{\alpha_n^-+n}=O(t^{m+1})\,,\qquad m\geqslant 0\,, \tag{3.1}$$

$$\sum_{k=0}^{N(m+1)}\sum_{n=0}^{m-2k+1}\tilde{d}^{m+1(-)}_{n,k}(t)\left(\frac{s}{s_0}\right)^{\alpha_n^-+n}-\sum_{k=0}^{N(m)}\sum_{n=0}^{m-2k}\tilde{c}^{m(+)}_{n,k}(t)\left(\frac{s}{s_0}\right)^{\alpha_n^++n}=O(t^{m+1})\,,\qquad m\geqslant 0\,, \tag{3.2}$$

where

$$\tilde{c}^{m(\pm)}_{n,k}(t)=\tilde{g}(\alpha_n^\pm)\,\gamma^{(\pm)n}_{1,1}(t)[\alpha_n^\pm(t)-2k][\alpha_n^\pm(t)-2k-1]a_k(\alpha_n^\pm)\cdot$$
$$\cdot\left[\frac{D(t)}{B(t)}\right]^{2k+1}\left[\frac{B(t)}{t_0}\right]^{-n}\frac{\Gamma\big(\alpha_n^\pm(t)-2k-1\big)}{(m-2k-n)!\,\Gamma\big(\alpha_n^\pm(t)-m+n-1\big)}\,,$$

$$\tilde{d}^{m(\pm)}_{n,k}(t)=\frac{\alpha_n^\pm(t)-2k}{\alpha_n^\pm(t)-m+n-1}\left[\frac{B(t)}{D(t)}\right]\tilde{c}^{m(\pm)}_{n,k}(t)\,,$$

$$\tilde{g}(\alpha^\pm)\;=-\frac{g(\alpha^\pm)}{\alpha^\pm(\alpha^\pm+1)}\,.$$

In order to completely satisfy the analyticity requirements, we must also take into account the $t=0$ constraint

$$\tilde{f}^{(+)}_{1,1}+\tilde{f}^{(-)}_{1,1}=O(t)\,, \tag{3.3}$$

that, explicitly written, gives rise to

$$\tilde{d}^{0+}_{0,0}(t)\left(\frac{s}{s_0}\right)^{\alpha_0^+(t)-1}+\tilde{d}^{0(-)}_{n,k}(t)\left(\frac{s}{s_0}\right)^{\alpha_0^-(t)-1}=O(t)\,, \tag{3.4}$$

$$(3.5)\qquad \sum_{k=0}^{N(m+1)} \sum_{n=0}^{m+1-2k} \left[\tilde{d}_{n,k}^{m+1(+)}(t) \left(\frac{s}{s_0}\right)^{\alpha_n^+(t)+n} + \tilde{d}_{n,k}^{m+1(-)}(t) \left(\frac{s}{s_0}\right)^{\alpha_n^-(t)+n} \right] - \\ - \sum_{k=0}^{N(m)} \sum_{n=0}^{m-2k} \left[\tilde{c}_{n,k}^{m(+)}(t) \left(\frac{s}{s_0}\right)^{\alpha_n^+(t)+n} + \tilde{c}_{n,k}^{m(-)}(t) \left(\frac{s}{s_0}\right)^{\alpha_n^-(t)+n} \right] = O(t^{m+2}) .$$

Defining

$$a_{n,k}^m = \frac{\alpha - n - 2k}{(\alpha - n)(\alpha - n + 1)} \frac{1}{2^{2k+n} k! \Gamma(\frac{1}{2} - \alpha + n + k)(m - 2k - n)!} ,$$

$$b_{n,k}^m = \frac{a_{n,k}^m}{\alpha - n - 2k} ,$$

the following systems are derived from the analyticity conditions (3.1) and (3.2):

$$(3.6)\qquad \sum_{k=0}^{N(m+1)} \sum_{n=0}^{m+1-2k} a_{n,k}^{m+1} \zeta_{1,1}^{+n} \alpha_n'^{+}(0) - \sum_{k=0}^{N(m)} \sum_{n=0}^{m-2k} b_{n,k}^m \zeta_{1,1}^{-m} \alpha_n'^{-}(0) = 0 , \qquad m \geqslant 1 ,$$

$$(3.7)\qquad \sum_{k=0}^{N(m+1)} \sum_{n=0}^{m+1-2k} a_{n,k}^{m+1} \zeta_{1,1}^{-n} \alpha_n'^{-}(0) - \sum_{k=0}^{N(m)} \sum_{n=0}^{m-2k} b_{n,k}^m \zeta_{1,1}^{+n} \alpha_n'^{+}(0) = 0 , \qquad m \geqslant 1 ,$$

where $\zeta_{1,1}^{\pm n}$ is defined as in (2.8) with the appropriate residue functions. These systems must be solved taking into account the relation

$$(3.8)\qquad \alpha_1^{+\prime}(0) - \alpha_1^{-\prime}(0) = \frac{\alpha - 1}{\alpha + 1} [\alpha_0^{+\prime}(0) - \alpha_0^{-\prime}(0)]$$

that can be obtained from the conditions (3.5).

Remembering from I that $\zeta_{1,1}^{+n} = -\zeta_{1,1}^{-n}$, the systems (3.6) and (3.7) can be shown to be equivalent to the following:

$$(3.9)\qquad \sum_{n=0}^{m+1} J_{m,n} \zeta_{1,1}^{+n} \alpha_n^{+\prime}(0) = 0 , \qquad m \geqslant 1 ,$$

$$(3.10)\qquad \sum_{n=0}^{m+1} J_{m,n} \zeta_{1,1}^{+n} \alpha_n^{-\prime}(0) = 0 , \qquad m \geqslant 1 ,$$

where

$$(3.11)\qquad \left\{ \begin{array}{l} J_{m,n} = - \dfrac{\Gamma(m - \alpha)}{2^n (m - n + 1)! \Gamma((m + n + 1)/2 - \alpha) \Gamma((m + n)/2 + 1 - \alpha)} \\ \text{and} \\ \zeta_{1,1}^{+n} = \dfrac{[2(\alpha - n) + 1]}{2\alpha + 1} \dfrac{(-1)^n}{n!} \dfrac{\Gamma(n - 1 - 2\alpha)}{\Gamma(-1 - 2\alpha)} \zeta_{1,1}^{+n=0} . \end{array} \right.$$

The solution of the systems (3.9) and (3.10), taking into account the relation (3.8), is

$$\alpha_n^{\pm\prime}(0) = c_1 + [c_2 \pm c_3](\alpha - n)(\alpha - n + 1)\,, \tag{3.12}$$

as can be seen by using the identities given in Sect. **2**.

Therefore, in the Chew-Frautschi approximation all the members of an $M=1$ Toller family can be parametrized in terms of only four parameters. We stress that the constraint equation (3.3) played an essential role in reducing the number of independent parameters from five to four.

3'2. *The equal-unequal mass case.* – Owing to the parity and G-parity selection rules, the parent and even daughters of the subfamily with $\sigma=-1$ couple to the partial-wave amplitudes $F^{J(-)}_{1;\frac{1}{2},\frac{1}{2}} \equiv F^{J(-)}_{1,0}$ and the odd daughters to the amplitude $F^{J(-)}_{1;\frac{1}{2}-\frac{1}{2}} \equiv F^{J(-)}_{1,1}$. The parent and the even daughters of the subfamily with $\sigma=+1$ couple to the amplitude $F^{J(+)}_{1;\frac{1}{2}-\frac{1}{2}} \equiv F^{J(+)}_{1,1}$, while the $\sigma=+1$ odd daughters are decoupled from the $\mathcal{N}$-$\bar{\mathcal{N}}$ system.

The conditions that can be deduced from the analyticity of the amplitude $\tilde{f}^{(-)}_{1,0}$ have already been studied in Subsect. **2**'2. We will discuss here the constraints imposed by the analyticity requirements on the amplitudes $\tilde{f}^{(\pm)}_{1,1}$.

The analyticity of $\tilde{f}^{(+)}_{1,1}$ imposes the following conditions:

$$\begin{aligned}\sum_{n=0}^{m+1} \tilde{b}^{+}_{2n;m+1-n}(t)[\alpha^{+}_{2n}-2m+2n-2]^2\left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha^{+}_{2n}+2n-1} - \\ -\sum_{n=0}^{m}\left[\frac{\tilde{D}(t)}{t_1}\right]^{\frac{1}{2}}[\alpha^{-}_{2n+1}-2m+2n][\alpha^{-}_{2n+1}-2m+2n-1]\cdot \\ \cdot\tilde{b}^{-}_{2n+1;m-n}(t)\left[\frac{s+B(t)}{s_0}\right]^{\alpha^{-}_{2n+1}+2n} = O(t^{m+1})\,,\end{aligned} \tag{3.13}$$

while the analyticity of $\tilde{f}^{(-)}_{1,1}$ requires

$$\begin{aligned}\sum_{n=0}^{m} \tilde{b}^{-}_{2n+1;m-n}(t)[\alpha^{-}_{2n+1}-2m+2n]^2\left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha^{-}_{2n+1}+2n} + \\ +-\sum_{n=0}^{m}\left[\frac{\tilde{D}(t)}{t_1}\right]^{\frac{1}{2}}[\alpha^{+}_{2n}-2m+2n][\alpha^{+}_{2n}-2m+2n-1]\cdot \\ \cdot\tilde{b}^{+}_{2n;m-n}(t)\left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha^{+}_{2n}+2n-1} = O(t^{m+1})\,,\end{aligned} \tag{3.14}$$

which must hold for any $m\geqslant 0$. Here $\tilde{b}^{\pm}_{n,k}(t)$ is defined as in (2.15) with the appropriate residue function and with $\tilde{g}(\alpha^{\pm}_n)$ in place of $g(\alpha_n)$.

Starting from the analyticity conditions (3.13) and (3.14), we arrive, after some manipulations, at the following systems for the trajectory derivatives:

$$\sum_{n=0}^{m+1} \frac{\zeta_{1,1}^{+2n}\, \alpha_{2n}^{+\prime}(0)}{(m+1-n)!\,\Gamma(\frac{3}{2}-\alpha+m+n)} = 0\,, \qquad m \geqslant 1\,, \tag{3.15}$$

$$\sum_{n=0}^{m+1} \frac{\zeta_{1,1}^{-2n+1}\, \alpha_{2n+1}^{-1}(0)}{(m+1-n)!\,\Gamma(\frac{5}{2}-\alpha+n+m)} = 0\,, \qquad m \geqslant 1\,, \tag{3.16}$$

$$\frac{(\alpha-3)(2\alpha-1)}{(\alpha+1)(2\alpha-3)}\left\{\frac{\alpha_1^{-\prime}(0)}{\alpha(\alpha-1)} - \frac{\alpha_3^{-\prime}(0)}{(\alpha-2)(\alpha-3)}\right\} - \\ - \frac{\alpha_0^{+\prime}(0)}{\alpha(\alpha+1)} + \frac{\alpha_2^{+\prime}(0)}{(\alpha-1)(\alpha-2)} = 0\,, \tag{3.17}$$

where

$$\zeta_{1,1}^{+2n} = \frac{2(\alpha-2n)+1}{2\alpha+1}\,\frac{(-1)^n}{n!}\,\frac{\Gamma(n-\frac{1}{2}-\alpha)}{\Gamma(-\frac{1}{2}-\alpha)}\,\zeta_{1,1}^{+n=0}\,, \tag{3.18}$$

$$\zeta_{2,1}^{-2n+1} = \frac{2(\alpha-2n)-1}{2\alpha-1}\,\frac{(-1)^n}{n!}\,\frac{\Gamma(n+\frac{1}{2}-\alpha)}{\Gamma(\frac{1}{2}-\alpha)}\,\zeta_{1,1}^{-n=1}\,. \tag{3.19}$$

The systems (3.15) and (3.16) turn out to have the same structure as the one discussed in Sect. 2. Their solutions are, therefore, given by

$$\alpha_{2n}^{+\prime}(0) = a_1 + a_2(\alpha-2n)(\alpha-2n+1)\,, \tag{3.20}$$

$$\alpha_{2n+1}^{-\prime}(0) = b_1 + b_2(\alpha-2n-1)(\alpha-2n)\,. \tag{3.21}$$

The constants a_1 and a_2 can be expressed in terms of $\alpha_0^{+\prime}(0)$ and $\alpha_2^{+\prime}(0)$, and b_1 and b_2 in terms of $\alpha_1^{-\prime}(0)$ and $\alpha_3^{-\prime}(0)$.

From the condition (3.17), the relation $a_1 = b_1$ can be derived.

The analyticity of the amplitude $\tilde{f}_{1,0}^{(-)}$, studied in Sect. 2, gives the following expression for the derivatives of the even daughters of the subfamily with $\sigma = -1$:

$$\alpha_{2n}^{-1}(0) = c_1 + c_2(\alpha-2n)(\alpha-2n+1)\,. \tag{3.22}$$

Now, in order to completely satisfy the analyticity requirements, we must take into account the $t=0$ constraint

$$i\tilde{f}_{1,1}^{(+)} - \tilde{f}_{1,0}^{(-)} = O(t)\,, \tag{3.23}$$

that, explicitly written, gives rise to

$$i\,\tilde{b}_{0,0}^{+}(t)[\alpha_0^{+}(t)]^2\left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_0^{+}(t)-1} + b_{0,0}^{-}(t)\,\frac{\alpha_0^{-}(t)}{\sqrt{\alpha_0^{-}(\alpha_0^{-}+1)}}\left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_0^{-}-1} = O(t)\,, \tag{3.24}$$

$$(3.25)\qquad i\left\{\sum_{n=0}^{m+1}(\alpha_{2n}^{+}-2m+2n-2)^2\tilde{b}_{2n;m+1-n}(t)\left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_{2n}^{+}+2n-1}-\right.$$

$$-\sum_{n=0}^{m}\left[\frac{\tilde{D}(t)}{t_1}\right]^{\frac{1}{2}}[\alpha_{2n+1}^{-}-2m+2n][\alpha_{2n+1}^{-}-2m+2n-1]\cdot$$

$$\left.\cdot b_{2n+1;m-n}^{-}(t)\left[\frac{s+\tilde{B}(t)}{s_0}\right]^{\alpha_{2n+1}^{-}+2n}\right\}+\sum_{n=0}^{m}\frac{\alpha_{2n}^{-}-2(m+1-n)}{\sqrt{\alpha_{2n}^{-}(\alpha_{2n}^{-}+1)}}\cdot$$

$$\cdot\tilde{b}_{2m;m+1-n}^{-}\left[\frac{s+B(t)}{s_0}\right]^{\alpha_{2n}^{-}+2n-1}=O(t^{m+2})$$

for any $m\geqslant 0$.

The only new result is found deriving (3.25) and taking $m=0$. All the other conditions have already been obtained from the analyticity requirements in the simple amplitudes. Remembering (3.18), (3.19), and the relations

$$\gamma_{1,1}^{(-)m=1}(0)=-\frac{2\alpha+1}{\alpha+1}\gamma_{1,1}^{(+)n=0}(0)\,,$$

$$\gamma_{1,0}^{(-)m=0}(0)=i\sqrt{\frac{\alpha}{\alpha+1}}\gamma_{1,1}^{(+)n=0}(0)\,,$$

proved in I, the new relation turns out to be

$$(3.26)\qquad \frac{\alpha-2}{2[\alpha(\alpha+1)]^{-1}}\left\{\frac{\alpha_0^{+\prime}(0)}{\alpha(\alpha+1)}-\frac{\alpha_2^{+\prime}(0)}{(\alpha-1)(\alpha-2)}\right\}+$$

$$+\frac{2\alpha-1}{\alpha-1}\alpha_1^{-\prime}(0)-\frac{\alpha}{2}[\alpha_0^{-\prime}(0)-\alpha_2^{-\prime}(0)]=0\,,$$

which implies $b_2=c_2$.

In summary, from the study of the $M=1$ EU case, we have obtained the following relations:

$$(3.27)\qquad \alpha_{2n}^{+1}(0)\;=a_1+a_2(\alpha-2n)(\alpha-2n+1)\,,$$

$$(3.28)\qquad \alpha_{2n+1}^{-1}(0)=a_1+b_2(\alpha-2n-1)(\alpha-2n)\,,$$

$$(3.29)\qquad \alpha_{2n}^{-1}(0)\;=c_1+b_2(\alpha-2n)(\alpha-2n+1)\,.$$

Although we were not able to derive from the EU case the general formula (3.12), we see that the results, at which we arrived, are not in contradiction with it.

4. – Conclusions.

In this paper we have completed the work begun in ref. [1,8]. We have shown, using the standard tools of the S-matrix theory, that essentially all the results derived in the group-theoretical approach can be explicitly obtained without any recourse to the $t=0$ algebraic structure of the scattering amplitude.

The main advantage of our approach lies in its simplicity; in fact, a rudimentary knowledge of Regge-pole theory is enough to follow our work. Nevertheless our approach has been proved so powerful that all the results of the exact Lorentz symmetry (the existence of the Toller poles, their classification, and conspiracy properties) and of the broken Lorentz symmetry (the mass formulae) could be derived.

Although most of our results were already known from the group-theoretical work, the $M=1$ mass formula is a new result, giving an example of the power of our method even in the most complicated kinematical situations.

Of course, the discussion presented here could be continued to further study of the properties of the Toller families and their possible parameterization. It is, however, clear that the calculations become more and more cumbersome and that the results are not sufficiently stringent to have direct application to the analysis of the present experimental data.

As far as the present work is concerned, we stress once again that essentially the same mass formula has been obtained both in the EU and UU mass configurations. Although this is not a general proof of the consistency of the Regge-pole theory and the $t=0$ analyticity, it is certainly quite an encouraging result.

RIASSUNTO

Usando i metodi standard della teoria della matrice S, si derivano « formule di massa » per famiglie di Toller con $M=0$ e $M=1$.

Семейство полюсов Редже и полюса Толлера: $t \neq 0$.

Резюме (*). — Используя стандартные методы теории S-матрицы, выводятся « массовые формулы » для семейств Толлера $M=0$ и $M=1$.

(*) *Переведено редакцией.*