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E. Etim and P. Picchi : FIELD-CURRENT IDENTITY AND HADRONIC CONTRIBUTION TO THE MUON G-FACTOR.

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E. Etim[(x)] and P. Picchi: FIELD-CURRENT IDENTITY AND HADRONIC CONTRIBUTION TO THE MUON G-FACTOR. -

ABSTRACT. -

Hadronic contribution to the muon g-factor is calculated according to various mixing schemes using field-current identity techniques.

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In both static (e.g. measurement of g-2 factor of charged leptons) and dynamic (e.g. measurement of cross-sections of genuine electrodynamic processes) tests the exploration of the limit of validity of QED becomes more difficult when the break-down effects to be observed are comparable to those due to virtual strongly interacting particles. In a previous note[(1)] we had presented a simple method for calculating fairly accurately the corrections to be administered on cross sections of electron-positron colliding beam reactions as a result of the hadronic

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modification of the photon propagator. In the present one we apply the techniques developed in Ref. (1) to the calculation of the hadronic contribution to the muon "anomaly", $a_\mu = (1/2)(g-2)$, which is very sensitive to the behaviour of QED at small distances. g-2 experiments are important for yet another reason; they can be used to explore the possible existence of an as yet unknown coupling of the muon to another field which might perhaps explain the mass difference between the muon and the electron.

Hadrons contribute to the muon g- factor through their modification of the photon propagator as shown in Fig. 1. The modified photon propagator is given in the Källen-Lehmann form by

$$(1) \qquad D_{\mu\nu}(q^2) = \delta_{\mu\nu} + 4\pi\alpha\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\int_0^\infty \frac{S(-x)\,dx}{x^2(x+q^2-i\varepsilon)}$$

where the spectral function $S(q^2)$ is defined in terms of matrix elements of the hadronic electromagnetic current $j_\mu(x)$ by

$$(2) \qquad \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) S(q^2) = \sum_n (2\pi)^4\, \delta^4(q-P_n)\, \langle 0|\, j_\mu^{(0)}\, |n\rangle \langle n|\, j_\nu^{(0)}\, |0\rangle$$

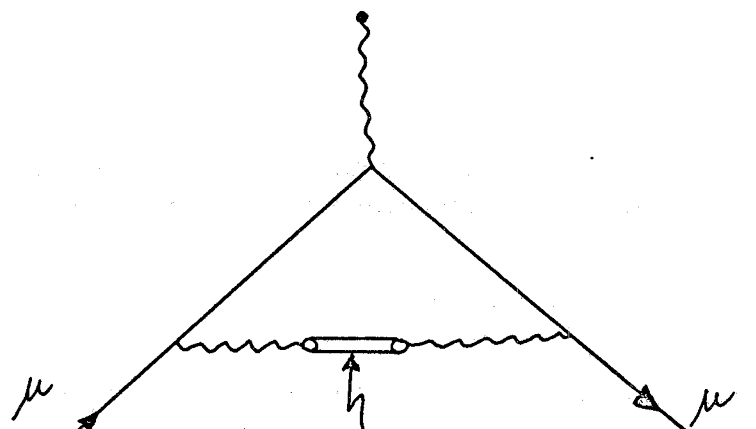

FIG. 1 - Hadronic contribution to muon magnetic moment.

The $|n\rangle$ are sets of hadronic states with 4-momentum P_n. The resulting modification of the muon g-factor is given by[2]

$$(3) \qquad a_\mu(H) = 4\alpha^2 \int_0^\infty \frac{S(-x)\cdot K(x)}{x^2}\, dx$$

where the function K(x) is defined by the integral

$$K(x) = \int_0^1 \frac{y^2(1-y)}{y^2 + \frac{x}{m^2}(1-y)}\, dy = \frac{1}{2} - \frac{x}{m^2} - \frac{x}{m^2} \ln\left(\frac{x}{m^2}\right) +$$

$$(4) \qquad + \frac{x^2}{2m^4} \ln \frac{x}{m^2} + \left[\frac{2x^2}{m^4} - \frac{x^3}{2m^6} - \frac{x}{m^2}\right] \cdot$$

$$\cdot \frac{1}{\sqrt{\frac{x}{m^2}\left(\frac{x}{m^2} - 4\right)}} \cdot \ln \frac{\frac{x}{m^2} + \sqrt{\frac{x}{m^2}\left(\frac{x}{m^2} - 4\right)}}{\frac{x}{m^2} - \sqrt{\frac{x}{m^2}\left(\frac{x}{m^2} - 4\right)}}$$

with m the mass of the muon. K(x) is a decreasing function of x and for very large values of the argument falls off as

$$(5) \qquad K(x) = \frac{m^2}{3x} + O\ \left(\frac{m^2}{x}\right) \log\left(\frac{x}{m^2}\right)$$

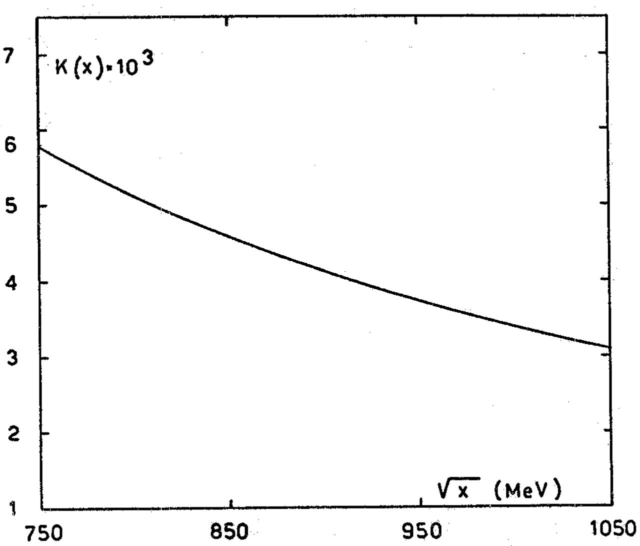

FIG. 2 - Plot of K(x) against $\sqrt{x}$.

The plot of K(x) against $\sqrt{x}$ for $750 \lesssim \sqrt{x} \lesssim 1050$ MeV is exhibited in Fig. 2.

To evaluate the integral in eq. (3) we appeal to the KLZ hypothesis[(3)] according to which $S(q^2)$ is related to two other spectral functions $S_\rho(q^2)$; $S_{\phi-\omega}(q^2)$. On these latter we shall make the usual assumptions of the vector dominance model. The decomposition of $S(q^2)$ into two terms is a consequence of the decomposition of the hadronic electromagnetic current into an isovector and isoscalar part. The relationship between these spectral functions is:

$$(6) \qquad S(q^2) = g_\rho^{-2} m_\rho^4 S_\rho(q^2) + \frac{1}{4}\left[g^{-1} M^2 S_{\phi-\omega}(q^2) M^2 \tilde{g}^{-1}\right]_{11}$$

$g_\rho = 2\gamma_\rho$ is the effective γ- ρ^o coupling constant, m_ρ the ρ^o mass and

(7) $$S_\rho(q^2) = \frac{1}{3}\sum_z \delta(q^2+m_z^2)\left|\langle 0|\,\vec{\rho}(0)\,|z\rangle\right|^2$$

with $\vec{\rho}(x)$ the spatial component of the ρ^o-meson field $\rho_\mu(x)$ and $|z\rangle$ is a spin one state of zero 3-momentum and energy m_z, g (with $\tilde{g}$ its transpose) M, $S_{\phi-\omega}(q^2)$ are 2x2 matrices generalising g_ρ, m_ρ, $S_\rho(q^2)$ respectively in the case of the ϕ-ω complex. $[\ldots\ldots]_{11}$ denotes the (1,1)th matrix element. A representation of these matrices is given in Ref. (3). The spectral function $S_\rho(q^2)$ and matrix $S_{\phi-\omega}(q^2)$ are Dirac singular at the masses of the resonances ρ^o, ω, ϕ and therefore have contributions of the form $Z_o\,\delta(q^2+m_R^2)$ where Z_o is a constant of order unity. In evaluating the integral in eq. (3) we retain only these delta function contributions and obtain

(8) $$a_\mu(H) = 4\alpha^2 Z_o\left[g_\rho^{-2}K(m_\rho^2) + \frac{g_y^{-2}}{4}\left\{\cos^2\theta_y\,K(m_\phi^2) + \sin^2\theta_y\,K(m_\omega^2)\right\}\right]$$

g_y is the hypercharge coupling constant and θ_y the corresponding mixing angle. According to the KLZ hypothesis[3] eq. (8) suffices to represent the contribution of all hadrons and not just those of the neutral vector mesons. This is so because KLZ implies an exact relationship in the strong interactions between the second order $O(e^2)$ hadronic contributions to the photon propagator and the propagator of the neutral vector mesons. To evaluate eq. (8) the following data have been used

$$\frac{g_\rho^2}{4\pi} = 2,3^{(3)}; \qquad \frac{g_y^2}{4\pi} = 1,4\cdot\frac{\cos^2(\theta_N-\theta_y)}{\cos^2\theta_N}^{(3)}$$

$$\frac{m_\omega}{m_\phi}\tan\theta_y = \frac{m_\phi}{m_\omega}\tan\theta_N = \tan\theta^{(3,4)}$$

$$\theta = 35^{o\,(5)}; \quad m_\rho = 760; \quad m_\omega = 780; \quad m_\phi = 1020;$$

θ_N is the baryon current mixing angle[3].

For $a_\mu(H)$ we find the value

(9) $$a_\mu(H) = 5,21\times 10^{-8}, \quad Z_o = 1$$
$$a_\mu(H) = 6,25\times 10^{-8}, \quad Z_o = 1,2$$

although the values of $a_\mu(H)$ in eq. (9) are below the present experimental accuracy ($\pm 3 \times 10^{-7}$)[6] it is interesting to find out how $a_\mu(H)$ varies with the mixing models. We have made use of the parameters in Ref. (3) and have found the results displayed in Table 1. It is clear from the table that $a_\mu(H)$ is not very sensitive to the mixing scheme its variation being about 6%.

TABLE I

Variation of $a_\mu(H)$ with the Mixing Scheme.

Mixing Scheme	$a_\mu(H) \times 10^{+8}$		θ_y	$\frac{g_y^2}{4\pi}$
	$Z_o=1$	$Z_o=1,2$		
Current Mixing Model	5,24	6,27	33^o	1,5
Mass Mixing Model	5,00	6,00	32^o	1,9
Variation of Mass Mixing Model	4,93	5,92	39^o	2,2

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