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M. Greco: POMERANCHON EXCHANGE AND p-p ELASTIC  
SCATTERING. -

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ABSTRACT. -

A representation for the Pomeron exchange and the multiple scattering corrections is suggested, which contains crossing symmetry, asymptotic Regge behaviour, and obeys the duality requirement of a non resonating background. A very good description of the proton-proton elastic scattering is obtained in the diffraction, intermediate and large angle regions.

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The Pomeron trajectory has been for a long time kind of mystery, since it was first introduced to get asymptotic non-zero constant cross sections. Recently, however, some light has been thrown on the dynamical role of this trajectory, which dominates the high energy behaviour of all elastic and some inelastic scattering amplitudes. First of all the Horn-Schmid<sup>(1+3)</sup> duality between Regge poles and low energy resonances, together with the Harari suggestion<sup>(4)</sup> to associate the Pomeron contribution with non resonating background at low energy, have led to a clear distinction between "ordinary" trajectories and the Pomeron. A further step in this direction is the idea that the leading vacuum singularity can be generated from ordinary trajec

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tories in a scheme of successive approximations to unitarity<sup>(5, 6)</sup>. All these arguments have prevented to present rigorously the Pomeron contribution by means of a Veneziano-like amplitude<sup>(7)</sup>, which seems to be so approximately true in nature, and this term had to be added by hand, with evident lack of crossing symmetry.

In this note we suggest a representation for the Pomeron exchange which satisfies the requirement of Regge asymptotic behaviours and crossing symmetry, without however any direct channel resonances. This representation can be extended to incorporate the multiple scattering corrections in a way strictly connected to the eikonal formalism<sup>(8)</sup>. The resulting amplitude is applied to the high energy p-p elastic scattering, giving a very good description of the differential cross section, in the full range of  $t$ . It predicts moreover the right ratio  $\text{Re}A(0)/\text{Im}A(0)$  for the forward non flip amplitude, and a total cross section which rises from 40 mb at 20 GeV to  $\approx 50$  mb at infinite energy, in the limit  $\alpha_p(0)=1$ . In our treatment a slope  $\alpha_p = 1(\text{GeV})^{-2}$  is required for the Pomeron trajectory.

More explicitly let us consider an elastic process with no direct channel resonances and with the  $t$  and  $u$  channels identical, for example  $\pi^+ \pi^+$  or p-p scattering. Let  $\alpha(t) = \alpha(0) + \alpha' t$  be the Pomeron trajectory and let put first  $\alpha(0)=1$ . The amplitude

$$(1) \quad A_{\text{pole}}(u, t) = - \frac{\beta}{\Gamma(1+i)} \frac{\Gamma[1-\alpha(u)+i] \Gamma[1-\alpha(t)+i]}{\Gamma[1-\alpha(u)-\alpha(t)+i]}$$

where  $\beta$  is a constant, has the following properties:

- i) No resonances in the  $s$  channel, or in the zero-width approximation, no poles for low  $s$  positive values.
- ii) Asymptotically it behaves:

$$(2) \quad \lim_{\substack{s \rightarrow \infty \\ t \text{ fixed}}} A_{\text{pole}}(u, t) = - \beta [-\alpha(u)+i]^{\alpha(t)} \frac{\Gamma[1-\alpha(t)+i]}{\Gamma(1+i)}$$

and asymmetrically for  $s \rightarrow \infty$  and  $u$  fixed. The eq. (2), neglecting terms of order  $1/s$ , shows clearly Regge behaviour and a completely imaginary amplitude in the forward direction. For small  $t$ , furthermore, it holds approximately

$$(3) \quad \frac{\Gamma[1-\alpha(t)+i]}{\Gamma(1+i)} \approx e^{-i \frac{\pi}{2} \alpha(t)}$$

This gives the right "reduced" signature factor, and explains why the unity has been chosen as the coefficient of the imaginary part of the argument of the Gamma function.

iii) In the  $t(u)$  channel, where  $t(u)$  now is the energy variable, the presence of a non-zero imaginary part in the argument of the Gamma function, will prevent the amplitude to blow up. This smooth behaviour does differentiate the Pomeron from the ordinary trajectories represented by a Veneziano formula.

Before comparing formula (1) with experiments, we must extend it in order to incorporate the multiple scattering corrections, which do contribute significantly at small  $t$ <sup>(8)</sup> and furthermore, are essential for the right large angle behaviour<sup>(9)</sup>. In the framework of the Glauber-type eikonal approximation it has been shown<sup>(8)</sup> how, starting from a single  $t$ -channel Pomeron exchange in the form

$$(4) \quad A_{\text{pole}}^t(s, t) \underset{\substack{\text{large } s \\ \text{small } t}}{\propto} \zeta \left[ \frac{s}{s_0} e^{-i \frac{\pi}{2}} \right] \alpha(t)$$

where at high energies  $\zeta$  is essentially a constant, one can generate the  $n$ -th order scattering term, corresponding to  $n$  Pomeron exchanges:

$$(5) \quad A_n^t(s, t) \underset{\substack{\text{large } s \\ \text{small } t}}{\propto} \zeta \frac{1}{n!} \left[ \frac{-\zeta}{\log(s/s_0) - i\pi/2} \right]^{n-1} \left[ \frac{s}{s_0} e^{-i \frac{\pi}{2}} \right]^n \alpha_n(t)$$

where

$$\alpha_n(t) = 1 + \frac{\alpha(t)}{n}.$$

This result, which is valid only at high energies and small angles, can be also obtained by analysing the contributions to the scattering amplitude from a certain class of branch points in the complex  $j$ -plane<sup>(10)</sup>, with the help of some results of Regge cuts discontinuities<sup>(11)</sup>. In a previous paper<sup>(9)</sup>, furthermore, it was suggested how to approximate in a crossing symmetric way the contribution of the  $j$ -plane branch points, disregarding complications due to the signature factors. Next step will be therefore to generalize formula (1) in the way suggested in ref. (9), to include the multiple Pomeron exchange in both  $t$  and  $u$  channels, consistently with the eikonal approximation results.

We write the complete amplitude which includes all the Pome-

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ranchon effects as follows:

$$(6) \quad A(u, t) = - \frac{\beta}{\Gamma(1+i)} \sum_{m, n=1}^{\infty} \frac{\Gamma[1 - \alpha_m(t) + i] \Gamma[1 - \alpha_n(u) + i]}{\Gamma[1 - \alpha_m(t) - \alpha_n(u) + i]} x$$

$$x \frac{1}{m m!} [-\beta_m(u, t)]^{m-1} \frac{1}{n n!} [-\beta_n(t, u)]^{n-1}$$

where  $\alpha_m(x) = 1 + (\alpha' x / m)$ . The functions  $\beta_m(u, t)$  and  $\beta_n(t, u)$  are requested to satisfy:

$$(7a) \quad \lim_{\substack{s \rightarrow \infty \\ t \rightarrow 0}} \beta_m(u, t) = \frac{\beta}{\lg[-\alpha(u) + i] - i\pi/2}$$

$$(7b) \quad \lim_{\substack{s \rightarrow \infty \\ u \rightarrow 0}} \beta_n(t, u) = \frac{\beta}{\lg[-\alpha(t) + i] - i\pi/2}$$

in order to reproduce the eikonal results (5). The whole  $t$  and  $u$  dependence however, which reflects the structure of the Regge cuts also outside the large  $s$ , small  $t$  or small  $u$  regions, is completely unknown. We can nevertheless try to guess, with again the help of the eikonal approximation. The  $n$ -th order term, corresponding to  $n$  Pomeron exchanges, with total momentum transfer  $t$ , see eq. (5), can be visualized as a single pole exchange carrying out a momentum transfer  $t/n$ , with a "residue" function proportional to

$$\left\{ -\gamma / [\lg(s/s_0) - i\pi/2] \right\}^{n-1}.$$

Consequently we parametrize the function  $\beta_m(u, t)$  as follows:

$$(8) \quad \beta_m(u, t) = \frac{\beta |1 - \alpha_m(t) + i|}{\lg[-\alpha(u_m) + i] - i\frac{\pi}{2} \alpha_m(t)}$$

where  $u_m = -s + 4M^2 - t/m$ , and  $|1 - \alpha_m(t) + i|$  takes approximately care of an eventual further dependence on  $t/m$ . The same is applied the  $u$  channel exchanges.

We arrive finally to the amplitude:

$$\begin{aligned}
A(u, t) = & - \frac{\beta}{\Gamma(1+i)} \sum_{m, n=1}^{\infty} \frac{\Gamma[1 - \alpha_m(t) + i] \Gamma[1 - \alpha_n(u) + i]}{\Gamma[1 - \alpha_m(t) - \alpha_n(u) + i]} x \\
(9) \quad & \times \frac{1}{m m!} \left\{ \frac{- |1 - \alpha_m(t) + i| \beta}{\lg[-\alpha(u_m) + i] - i \frac{\pi}{2} \alpha_m(t)} \right\}^{n-1} x \\
& \times \frac{1}{n n!} \left\{ \frac{- |1 - \alpha_n(u) + i| \beta}{\lg[-\alpha(t_n) + i] - i \frac{\pi}{2} \alpha_n(u)} \right\}^{n-1},
\end{aligned}$$

where again  $u_m = -s + 4M^2 - (t/m)$  and  $t_n = -s + 4M^2 - (u/m)$ .

We emphasize however that eq. (8) is just a guess to check experimentally eq. (6), which does not change the essential features of the amplitude and is justified a posteriori (see below) by the very good agreement with the experimental data.

The amplitude (9), apart a phase factor, is not too much different from the one given in ref. (9), so the main conclusions on the asymptotic behaviours in the fixed  $t$ , and large angle regions are substantially unchanged.

Finally, for the sake of completeness, we must add to eq. (9) the amplitude corresponding to the exchange of ordinary trajectories in  $u$  and  $t$  channels. These will be degenerate, giving no contribution to the forward imaginary amplitude, and the amplitude is represented by a Veneziano formula<sup>(7)</sup>:

$$(10) \quad A_d(u, t) = \gamma \frac{\Gamma[1 - \alpha_d(t)] \Gamma[1 - \alpha_d(u)]}{\Gamma[1 - \alpha_d(t) - \alpha_d(u)]}$$

where  $\gamma$  is a constant and  $\alpha_d(t)$  is the  $\zeta$ - $P'$  trajectory.

We shall compare now with the  $p$ - $p$  data our complete amplitude, which is essentially the helicity non-flip amplitude, having completely neglected any spin dependence. We have in practice only two parameters,  $\alpha'$  and  $\beta$  to fit the  $t$  dependence of  $d\sigma/dt$  in the full angular range, and the energy dependence of  $\sigma_{pp}^{\text{tot}}$ ,  $\text{Re}A(0)/\text{Im}A(0)$  and  $d\sigma/dt$ . In order to reproduce however the diffraction peak a normal slope of  $1 \text{ GeV}^{-2}$  is required, and this reduces to only one parameter the freedom of our amplitude. The  $\zeta$ - $P'$  contribution (10) is completely negligible at 20 GeV, while it ameliorates the ratio  $\text{Re}A(0)/\text{Im}A(0)$  at smaller energies. The double sum (9) has been evaluated by computer

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and the best results are obtained with  $\beta = 5.0 \pm 0.5$ . Let us discuss them in detail.

i) The differential cross sections  $d\sigma/dt$  are shown in Fig. 1. As one can see from there, the agreement is excellent, in the full range of  $t$  and for different values of the laboratory momentum. The diffraction peak strikes suitably as  $s$  increases, and the intermediate and large angle features previously discussed<sup>(9, 12)</sup> are reproduced. The role of the secondary trajectories is limited at a very small angular ( $-t \lesssim 1 \text{ GeV/c}^2$ ) and energy ( $p_L \lesssim 15 \text{ GeV}$ ) range. The effect of the helicity flip amplitudes cannot change the essential feature of the model, but can however further ameliorate our results, for example near  $t \approx -1.5 \text{ (GeV/c)}^2$ . At very high energies (Serpukhov and CERN-ISR experiments) a further fall of  $d\sigma/dt$  is predicted.

ii) Our model, with  $\alpha(0)=1$ , gives a logarithmical rise of  $\sigma_{pp}^{\text{tot}}$  as  $s$  increases. From 36 mb at 12 GeV and 40 mb at 20 GeV,  $\sigma_{pp}^{\text{tot}}$  will rise to  $\approx 50$  mb at infinite energy. The physical reason of this rising is well known<sup>(8, 13)</sup>, and is due to the Glauber shadowing of the multiple scattering. The disagreement with the small, observed decrease of  $\sigma_{pp}^{\text{tot}}$  at present energies can be attributed at one of the following reasons: a) direct channel resonances are not completely negligible; b) the Regge cuts contributions are more complicated than those guessed in our amplitude; c)  $\alpha(0)$  deviates from 1.

iii) The ratio  $\text{Re}A(0)/\text{Im}A(0)$  for the forward non flip amplitude is  $-0.27$  at 20 GeV and decreases logarithmically to  $\approx -0.35$  at infinite energy. At laboratory energies smaller than 20 GeV, the right magnitude and the rising behaviour is obtained with the help of the secondary trajectories contribution (10), which is computed by fixing the  $\zeta$ - $P'$  parameters at the standard values  $\alpha_{\zeta}(0)=0.5$ ,  $\alpha'_{\zeta}=1 \text{ GeV}^{-2}$ , and  $\gamma$ , which is free, equal to  $\sim 1$ .

We will add more details on this subject, as well discuss the implications of our model for  $p\bar{p}$  scattering, isobar production, and similar topic of experimental interest in a forthcoming paper. We conclude with a final comment about unitarity. Our amplitude (9) gives, a posteriori, a very good description of the single and multiple Pomeron exchanges which, according to a now accepted philosophy, must be considered as the unitary contributions to the physical amplitude. In this sense we shall speak of a "good" approximation of the true amplitude. On the other hand, the threshold conditions on the imaginary party of the amplitude<sup>(9)</sup>, with the above choice of  $\alpha'$  and  $\beta$ , are not fulfilled, as one can see by computing numerically the whole series. Nevertheless, a small deviation of  $\alpha(0)$  from 1, which conse-

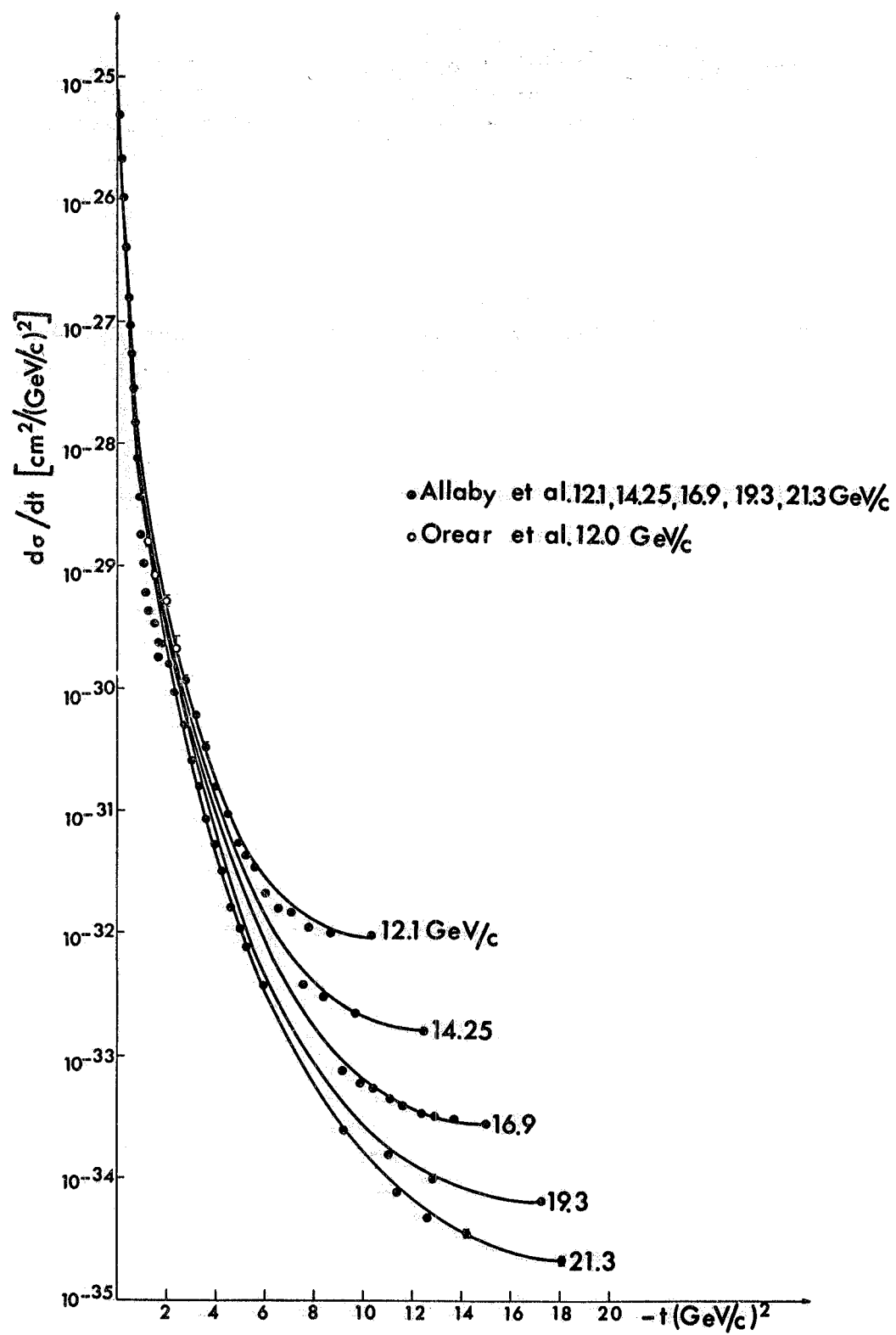


FIG. 1 - p-p differential cross sections  $d\sigma/dt$  in units of  $\text{cm}^2/(\text{GeV}/c)^2$ , as functions of  $-t$ . The experimental points are taken from Ref. (14-17). The full lines result from eq. (9).



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quently affects the trajectories  $\alpha_m(t)$  of the branch points in the  $j$ -plane, can lead to radically different result at the threshold, without however any appreciable change at high energies. Possible consequences on this point will be further discussed.

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