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ABSTRACT. -

A model for the isovector form factor of the nucleon is proposed in analogy with the Veneziano model for scattering amplitudes. The agreement with the experimental data is quite good and provides additional evidence for the linearity of the Regge trajectories up to  $t = -25 \text{ (GeV/c)}^2$ .

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The existence of the now well known vector mesons  $\rho$ ,  $\omega$  and  $\phi$  was first suggested, before their discovery, in order to explain some features of the electromagnetic form factors of the nucleon<sup>(1)</sup>. In the subsequent years a considerable amount of effort has been expended to provide a coherent formulation of the idea of vector dominance and universality<sup>(2)</sup>.

Unfortunately after the discovery of the suggested vector mesons, it turned out that the models based on their dominance were unable to describe in a simple way the experimental behavior of the nucleon form factors<sup>(x)</sup>. From the theoretical point of view the failure

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(x) - See e. g. Ref. 3 and references quoted there.

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of these simple models is hardly surprising. In fact while a pole approximation should be very good near the position of the pole in the time-like region, it is not at all clear why such a pole on an unphysical sheet should still dominate the dispersive integral in the space-like region. In fact no justification has ever been given to neglect the contribution of the higher mass states in the dispersive integral.

The aim of the present Letter is to present a model for the isovector form factor of the nucleon that takes into account the contribution of such higher mass states. Let us review some of the experimental information on form factors.

The following relations have been shown to be approximately valid experimentally:

$$(1) \quad G_E^p(t) = \frac{G_M^p(t)}{\mu_p} = \frac{G_M^n(t)}{\mu_n} = G(t)$$

and

$$(2) \quad G_E^n(t) = 0$$

where  $G_E(G_M)$  is the electric (magnetic) form factor and n, p stands for neutron, proton<sup>(x)</sup>. Moreover for large space-like  $t$  the function  $G(t)$  decreases like  $t^{-2}$  or faster.

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(x) - The relation  $G_E^p(t) = (G_M^p(t))/\mu_p = G(t)$  seems satisfied for  $|t| \leq 4(\text{GeV}/c)^2$  within the limits of the errors: see e. g. Ref. (4). However recent precision measurements<sup>(5)</sup> have given the first possibly statistically significant indication that the "scaling law" may be violated. The new Boun data<sup>(5)</sup> cover a range up to  $t = -2(\text{GeV}/c)^2$  and can be fitted by an equation of the form

$$G_E^p(t) = \frac{G_M^p(t)}{\mu_p} \left[ 1 + (0.063 \pm 0.018)t \right]$$

Considering the difficulties of these measurements also the authors of this experiment do not claim that this deviation is necessarily significant. The relation  $G_M^n/\mu_n = G(t)$  has been tested for  $|t| \leq 1, 2(\text{GeV}/c)^2$ . See e. g. Ref. (4).

The data on the electric form factor of the neutron remain in an extremely unsatisfactory state, but are compatible with being close to zero everywhere: see e. g. Ref. (6).

The properties of the form factors are determined by the strong interaction dynamics. Very interesting developments have recently taken place in this field in the framework of the Regge pole theory and the new bootstrap program. The extreme idealization of this point of view, based on linearly rising Regge trajectories

$$(3) \quad \alpha(t) = at + b$$

led to the beautiful representation proposed by Veneziano<sup>(7)</sup>. Moreover it has been shown that any Regge trajectory  $\alpha(t)$  must be accompanied by an infinite set of daughters whose intercepts at  $t=0$  are  $\alpha(0)-n^{(x)}$ . These daughter trajectories are automatically contained in the Veneziano amplitude; The final outcome of these studies on Regge pole theory is then that an approximate description of the real world can be obtained in terms of families of infinite linearly rising Regge trajectories, which interpolate an infinite number of equally spaced resonances of zero width (unitarity is of course violated).

We will now propose in analogy with the Veneziano model an expression for the isovector form factor that:

1) contains an infinite number of equally spaced poles, corresponding to zero width  $1^-$  resonances lying on the daughter trajectories of the  $\rho$  meson: the same poles are contained in the Veneziano amplitude;

2) satisfies the dispersion relation

$$(4) \quad G(t) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\text{Im} G(t')}{t'-t} dt'$$

3) behaves like a power for  $|t| \rightarrow \infty$ ,  $\arg t \neq 0$ .

The expression that we propose is the following

$$(5) \quad G(t) = \gamma \frac{\Gamma(1 - \alpha_s(t))}{(1 - \alpha_s(t) + \beta)}$$

Assuming the validity of the relations (1) and (2) all the data

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(x) - Here and in following the term "daughter trajectory" is not used in the group theoretical sense, but only to indicate any subsidiary trajectory which, at  $t=0$ , is integer spaced from the parent trajectory.

can be described in terms of the single isovector form factor  $G(t)^{(x)}$ . It should however be kept in mind that the validity of (1) and (2) is only approximate and has been tested only at relatively low values of  $-t$ .

In the comparison with the experimental data, our formula (5) has an important advantage over similar expressions for scattering amplitudes. In fact the finite width corrections that are crucial in the time-like region are not expected to play an important part in the space-like region, where the form-factors have been measured. Of course in the time-like region the expression (5) has to be somehow modified, the position of the poles being shifted on to an unphysical sheet. In the following our point of view will be that the expression (5) can be reasonably accurate even in the time-like region, provided that  $\alpha_\rho(t)$  is not too close to an integer. For the trajectory  $\alpha_\rho(t)$  we take the form (8) derived by imposing the PCAC requirement on a Veneziano-like scattering amplitude<sup>(o)</sup>

$$(6) \quad \alpha_\rho(t) = \frac{1}{2} + \frac{t}{2m_\rho^2}$$

The normalization condition  $G(0)=1$  enables us to express  $\gamma$  in terms of  $\beta$  in (5): we are therefore left with the single parameter  $\beta$ . However if the expression (5) and the relations (1) and (2) are taken seriously, the acceptable values of  $\beta$  are severely restricted. In fact it is well known that  $G_E(t)$  and  $G_M(t)$  are essentially helicity amplitudes for the transition  $\gamma \rightarrow N\bar{N}$ : they must therefore satisfy a constraint at the threshold  $t=4M^2$ , where  $M$  is the nucleon mass. The constraint turns out to be

$$(7) \quad G_E(t=4M^2) = G_M(t=4M^2)$$

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(x) - Note that if  $\beta = n$ , a positive integer,  $G(t)$  has only  $n$  poles. In particular for  $\beta = 1$  the simpler form of vector dominance and universality is recovered.

The expression (5) is suggested by the following consideration. The  $I=1$  partial wave amplitude with  $l=1$  for a process like  $\pi - \pi$  scattering, say, is proportional to  $\Gamma(1 - \alpha_\rho(t))$ . The gamma function in the denominator is then introduced in order to have a power behavior at infinity.

(o) - We are neglecting the pion mass. The inclusion of a finite value for  $m_\pi$  would not alter substantially our results, but would have the only effect of substituting number like  $1/2$  with more exotic one, like 0.483.

If the relations (1) and (2) were exact, the only way of satisfying eq. (7) is

$$(8) \quad G(t=4M^2) = 0$$

Of course since (1) and (2) are only approximate there is no compelling reason for imposing eq. (8). However in the spirit of our approximations we will assume eq. (8) so that essentially no free parameter is left. This of course must be verified a posteriori. Since the point  $\alpha_\rho(4M^2)$  is not too close to an integer, we can safely use the expression (5) for  $G(t)$  and we get<sup>(\*)</sup> from eq. (8)

$$(9) \quad \beta = \alpha_\rho(4M^2) - 1 - n = \frac{5}{2} - n$$

where the value  $m_\rho = 765$  MeV has been used.

According to the experimentally known behavior of the form factors for large momentum transfer the only value eventually acceptable will be  $\beta = 5/2$ . Now everything is fixed and we can compare our expression with the experimental results. This is done in Figs. 1, 2 and 3 for small, intermediate and large momentum transfers respectively,

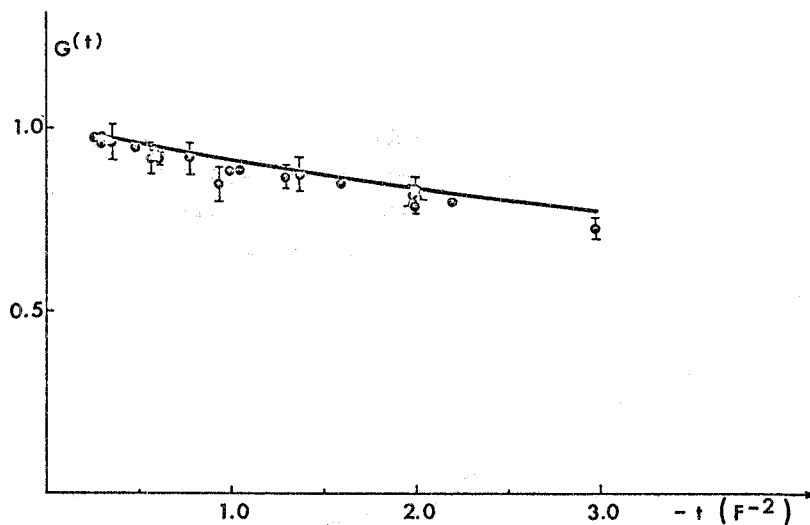


FIG. 1 - Comparison of our model with the low- $t$  data from Ref. (9).

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(\*) - We are assuming, as usual in the study of Veneziano-like amplitudes, that a single term of the form (5) is adequate and that no subsidiary terms are needed.

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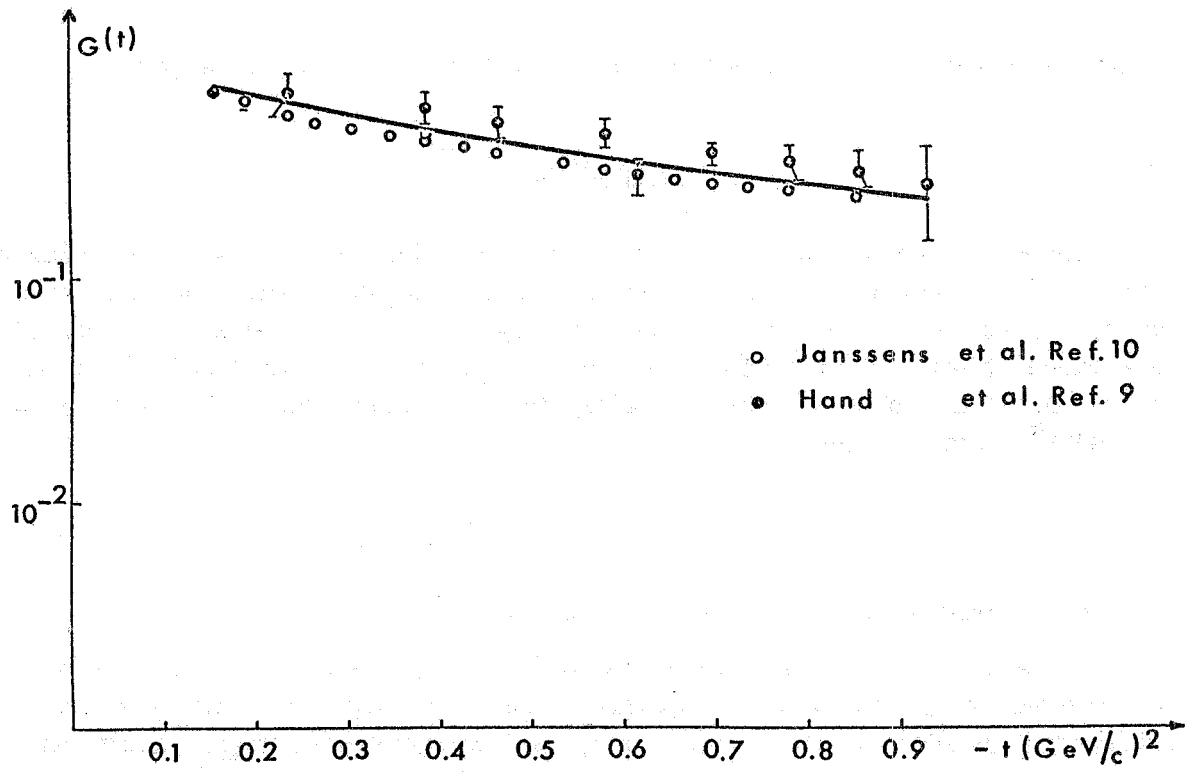


FIG. 2 - Data for  $-t \leq 1 (\text{GeV}/c)^2$  from Ref. (9) and (10)

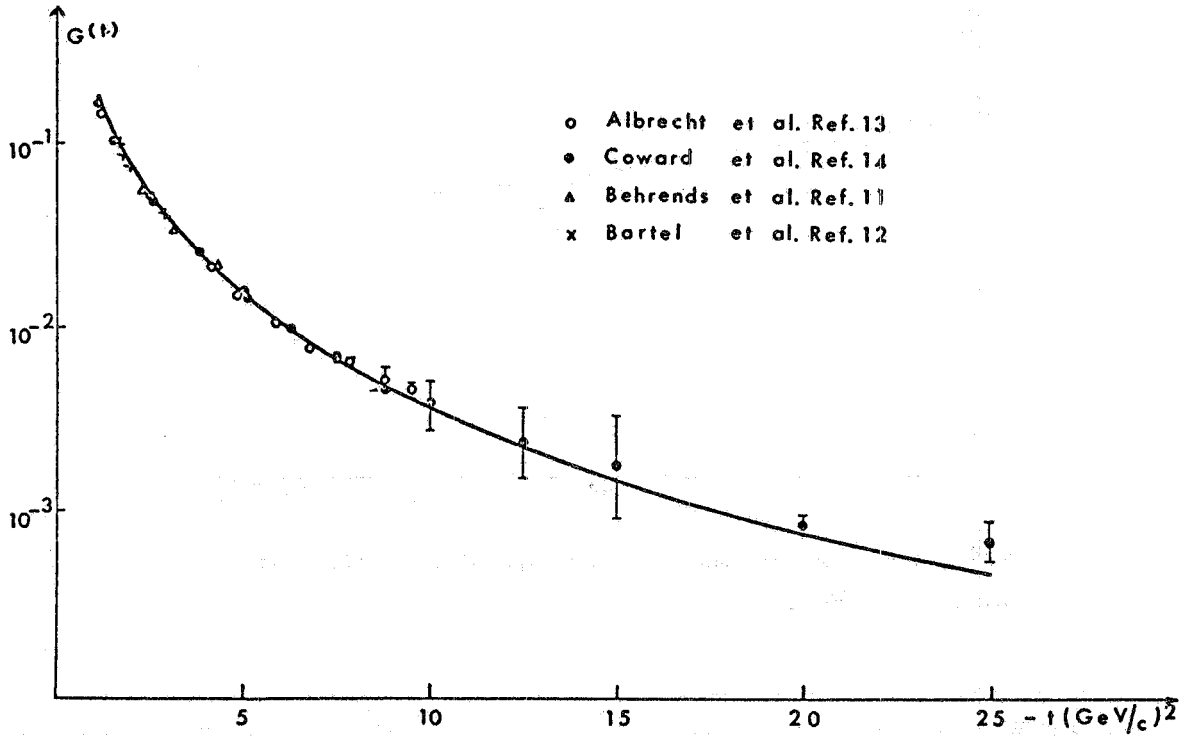


FIG. 3 - Data for  $-t \leq 1 (\text{GeV}/c)^2$  from Ref. (11, 12, 13) and (14).

The agreement obtained is extremely good particularly in view of the following considerations:

1) the experimental data from the various groups are not entirely consistent with each other: the quoted errors are only statistical and systematic uncertainties have not been included;

2) our model is based on the relations (1) and (2), that have been approximately checked only in a limited region of momentum transfer;

3) we are using a zero width approximation.

In the time-like region, there are two experiments<sup>(15)</sup> on the annihilation  $p+\bar{p} \rightarrow e^+e^-$  and  $p+\bar{p} \rightarrow \mu^+\mu^-$  that put the limits

$$|G_E| \lesssim 0.2 \qquad |G_M| \lesssim 0.2$$

at  $t=5.1$  and  $6.8 (\text{GeV}/c)^2$ . These limits are satisfied by our expression.

With the inclusion of finite width effects our model can give predictions on the  $e^+e^-$  storage ring reactions. We hope to discuss this question in a future publication, together with other applications of our model.

As a concluding remark we note that if this model can be taken seriously, it provides a good evidence for the linear (or almost linear) decrease of the  $\rho$  trajectory up to  $t=-25 (\text{GeV}/c)^2$ . Moreover no other Regge trajectory with negative signature,  $I^G=1^+$ , is required, beside the one associated with the  $\rho$  meson.

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