COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 69/19
24 Marzo 1969

A. Małecki and P. Picchi: NUCLEAR RECOIL CORRECTION AND SHORT RANGE NUCLEON-NUCLEON CORRELATIONS IN ELASTIC ELECTRON SCATTERING FROM LIGHT NUCLEI.

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A. Małecki[(x)] and P. Picchi: NUCLEAR RECOIL CORRECTION AND SHORT RANGE NUCLEON-NUCLEON CORRELATIONS IN ELASTIC ELECTRON SCATTERING FROM LIGHT NUCLEI. -

I. - INTRODUCTION. -

It is well known that the harmonic oscillator well model, which is succesful in predicting the electron scattering form factor of the light nuclei, fails for Li^6.

It is likely that for this nucleus the p-nucleons are bound a good deal less firmly that in heavier nuclei (C^{12}, O^{16}) in the p-shell. This can be simulated by allowing the s- and p-nucleons to move in different potential wells as suggested by Elton(1).

This model provided a good fit to the experimental data at low momentum transfer q. However after having extended the measurements(15)

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to large q's, the model with two different oscillator wells was not able to explain the experimental results anymore.

The situation may be remedied by taking into the effect of two-body correlations arising from repulsive core of the nucleon-nucleon interaction. This was suggested by Ciofi degli Atti(2). The influence of the short-range correlations on the elastic form factor has been also investigated by the present authors(3).

The purpose of this note is twofold:

1) in Chapter II and in Appendix we calculate in the exact way (by means of the Gartenhaus-Schwartz(4) transformation) the nuclear center-of-mass motion correction to the elastic form factor in the model with the two different oscillator wells. The model was used not only for Li^6, but also for the C^{12} and O^{16} nuclei(5) in order to obtain a coherent interpretation of the elastic and quasi-free electron scattering data(6). We have found that there is quite a substantial difference between the exact result and the usually used c.m. correction(1), taken by analogy with the model of a common oscillator well for all nucleons,

2) in Chapter III we perform the short-range correlation calculations. We obtain a good fit to the Li^6 elastic electron scattering data in the model with the two different oscillator wells. It turns however that this model (with appreciably different oscillator strengths) is no more succesful for C^{12} and O^{16}. Good fits for these nuclei may be obtained in a simpler model, with the single oscillator potential well.

II. - NUCLEAR CENTER-OF-MASS MOTION CORRECTION. -

We consider elastic electron scattering from spin zero nuclei (as C^{12}, O^{16}) and from nuclei with a small quadrupole moment (as Li^6). The cross-section for the elastic scattering of an electron with energy through an angle θ is given then by the following formula(x):

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 e^4 \cos^2\theta/2}{4\,\mathcal{E}^2 \sin^4\theta/2}\left(1+\frac{2\mathcal{E}}{M_T}\sin^2\theta/2\right)^{-1}\left|F_{ch}(q)\right|^2 \qquad (1)$$

where Z and M_T are the charge number and mass of the target nucleus.

(x) - We use units c=h=1, e^2=1/137. Since we consider high-energy electrons the rest mass of the electron is neglected.

The Quantity $F_{ch}(q)$ in Eq. (1) is called the charge elastic form factor of the nucleus.

Using the nonrelativistic form of the nuclear charge operator[8] (with terms to the order $1/M^2$; M being the nuclear mass) we obtain in the first Born approximation:

$$F_{ch}(q^2) = f(q^2)\frac{1}{Z}\langle\psi|\sum_{j=1}^{A} e_j e^{i\vec{q}\cdot\vec{r}_j}|\psi\rangle \tag{2}$$

where

$$f(q^2) = (G_{Ep}+G_{En})\left(1+\frac{q^2}{8M^2}\right)$$
$$q^2 = -q^2\left(1-\frac{q^2}{4M_T^2}\right) \tag{3}$$

In Eq. (2) one sums over all the nucleons in the target nucleus and e_j, r_j are the charge and position operators for j-th nucleon. ψ is the nuclear wave function of the ground state.

In Eq. (3) we have introduced the electric form factors of the nucleons[9]. $f(q^2)$ represents the correction due to finite nucleon sizes.

If ψ is not translationally invariant a nuclear center-of-mass motion correction in Eq. (2) has to be applied. The correction can be easily evaluated in the shell model with the harmonic oscillator potential well[7]. In this case one obtains

$$C_{c.m.} = \exp\left(\frac{q^2}{4A\alpha^2}\right) \tag{4}$$

as the additional factor in the r.h.s. of Eq. (2); α being the oscillator parameter.

In the model with the two different oscillator wells ($\alpha_s \neq \alpha_p$) usually one uses the same type corrections[1] as the factors multiplying contributions from the s- and p-shell, respectively. More precisely, for nuclei with two protons in the s-shell and Z-2 protons in the p-shell one obtains for the form factor:

$$F = \frac{2}{Z}\left[\exp\left(-\frac{q^2}{4\alpha_s^2}\right)\exp\left(\frac{q^2}{4A\alpha_s^2}\right) + \frac{Z-2}{6}\left(3-\frac{q^2}{2\alpha_p^2}\right)\exp\left(-\frac{q^2}{4\alpha_p^2}\right)\exp\left(\frac{q^2}{4A\alpha_p^2}\right)\right] f(q_\mu^2) \tag{5}$$

One might suggest also another c.m. corrections for this model putting in Eq. (4) an average value of the α-parameter obtained e.g. either by averaging of α_s^2 and $\alpha_p^{2(3)}$

(6) $$\bar{\alpha}^2 = \frac{2}{Z}\left(\alpha_s^2 + \frac{Z-2}{2}\alpha_p^2\right)$$

or by averaging r.m.s. radii for the two shells:

(7) $$\bar{\bar{\alpha}}^2 = (5Z-4)\left[\frac{6}{\alpha_s^2} + \frac{5(Z-2)}{\alpha_p^2}\right]^{-1}$$

All such c.m. corrections seem to be plausible but in fact are chosen quite arbitrarily without an exact justification.

The exact way of calculating the c.m. motion effect on the form factor is to perform in the expression (2) for the latter the so-called Gartenhaus-Schwartz transformation. Then, in a shell model yielding the nuclear wave function Ψ_{SM}, the charge elastic form factor is given by:

(8) $$F_{GS} = f(q_\mu^2)\frac{1}{Z}\left\langle\Psi_{SM}\right|\sum_{j=1}^{A} e_j e^{i\vec{q}(\vec{r}_j - \vec{R})}\left|\Psi_{SM}\right\rangle, \quad \vec{R} = \frac{1}{A}\sum_{k=1}^{A}\vec{r}_k$$

It may easily be shown that

(9) $$\left\langle\Psi_{SM}\right|\sum_{j=1}^{A} e_j e^{i\vec{q}(\vec{r}_j - \vec{k})}\left|\Psi_{SM}\right\rangle = \sum_{j,k=1}^{A}\langle\alpha_j|e \cdot$$

$$\cdot \exp\left(i\frac{A-1}{A}\vec{q}\cdot\vec{r}\right)|\alpha_k\rangle(-1)^{j+k} M_{jk}$$

where M_{jk} is the determinant of order A-1 which is constructed from the determinant

$$D = \left\|\langle\alpha_j|\exp\left(-\frac{iq\cdot r}{A}\right)|\alpha_k\rangle\right\|$$

by removing in the latter the j-th row and k-th column; $|\alpha_j\rangle, |\alpha_k\rangle$ being single particle states of the shell model.

One can also write

$$(10) \qquad \langle \Psi_{SM} | \sum_{j=1}^{A} e_j e^{i\vec{q}(\vec{r}_j - \vec{R})} | \Psi_{SM} \rangle = \sum_{j=1}^{A} D_j$$

where D_j is the determinant of order A which is formed from D by replacing the j-th row of D through

$$\langle \alpha_j | e \exp(i \frac{A-1}{A} \vec{q} \cdot \vec{r}) | \alpha_1 \rangle \langle \alpha_j | e \exp(i \frac{A-1}{A} \vec{q} \cdot \vec{r}) | \alpha_2 \rangle \ldots \langle \alpha_j | e \exp(i \frac{A-1}{A} \vec{q} \cdot \vec{r}) | \alpha_A \rangle$$

We have calculated (see Appendix) F_{GS} for the Li^6, C^{12} and O^{16} nuclei in the model with two different oscillator potential wells. We have compared the results with the expression F - see Eq. (5). In Fig. 1 is presented the ratio $R = F_{GS}^2/F^2$ in the function of momentum transfer for the three nuclei.

Evidently (and perhaps surprisingly) these is a remarkable difference between F_{GS} and F. The mathematical structure of the ratio R(q) depends strongly on the parameters α_s, α_p one has chosen and may become quite complicated. The difference between the two form factors will be most accentuated at the values of q where either $F_{GS}(q)$ or F(q) has a minimum. In general, one may say that the discrepancy between F_{GS} and F becomes larger as the difference between the oscillator parameters α_s, α_p increases (if $\alpha_s = \alpha_p$ one has $F_{GS} = F$).

The difference between F_{GS} and expressions for the form factor obtained by using other c.m. motion corrections as those mentioned above (Eqs. (6) and (7)) is even bigger.

All of this indicates that one must be careful when taking into account the c.m. motion effect in shell model calculations as various and apparently similar procedures may bring to very different results.

III. - SHORT RANGE NUCLEON-NUCLEON CORRELATIONS. -

In this chapter we study the influence of the short range nucleon-nucleon correlations on the elastic form factor in the model with two different oscillator wells.

Since the correlations between nucleons affect only the intrinsic part of the nuclear wave function we assume the same center-of-mass correction as in the model without correlations. We write the expression

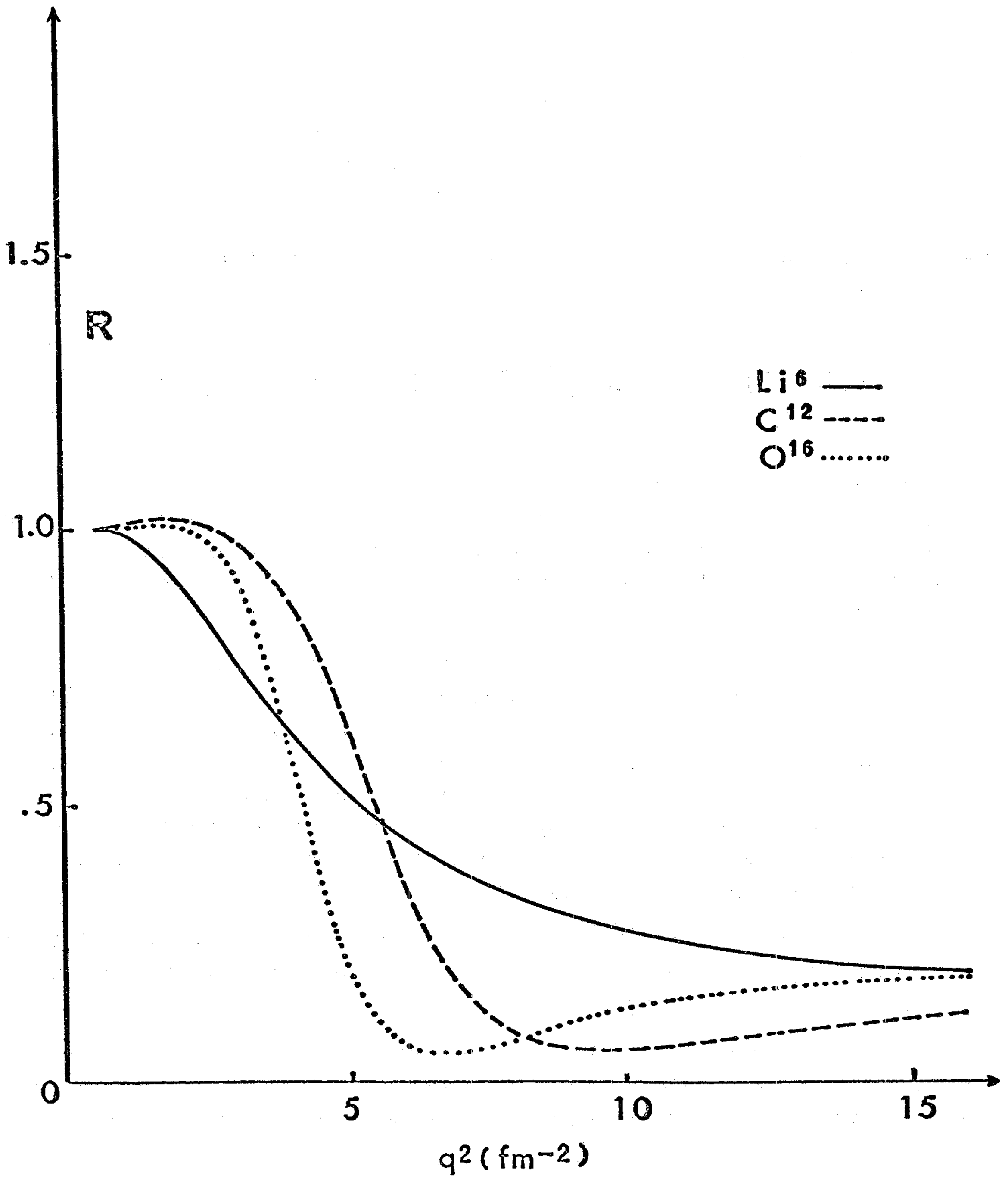

FIG. 1 - Comparison of form factors (shell model with two oscillator wells; no short-range correlations) calculated with two different nuclear c.m. motion corrections. F_{GS}-c.m. correction calculated exactly (by means of the Gartenhaus-Schwartz transformation), F - c.m. correction calculated as in Ref. (1); $R = (F_{GS}/\bar{F})^2$. The oscillator parameters are: for Li^6: $\alpha_s = 130.517$ MeV, $\alpha_p = 98.588$ MeV; for C^{12}: $\alpha_s = 170$ MeV; $\alpha_p = 130$ MeV; for O^{16}: $\alpha_s = 170$ MeV, $\alpha_p = 120$ MeV.

for the "correlated" elastic form factor as follows:

(11) $$\tilde{F} = \frac{F_{GS}}{F_{SM}} (F_{SM} + \Delta F)$$

where F_{SM} is the so-called shell model form factor

(12) $$F_{SM} = \frac{1}{Z} \langle \Psi_{SM} | \sum_{j=1}^{A} e_j e^{i\vec{q}\cdot\vec{r}_j} | \Psi_{SM} \rangle$$

ΔF is the correction to F_{SM} which accounts for nucleon-nucleon correla_ tions.

We calculate the correlation correction as in Ref. (3). The cor_ relations are introduced into the nuclear wave function by means of a unita_ ry transformation $\Psi = U\Psi_{SM}$. Of course, one can work still with the shell model wave function and use the unitary transformed operator of the elec-tron-nucleus interaction.

As we consider the short-range correlations arising from the hard core repulsion between nucleons we can take into account the two-par_ ticle correlations only. We neglect then the probability of simultaneous modification of the wave functions of more than two particles as the pro-bability for three and more nucleons to come close together should be very small.

In order to account for the two-particle correlations we construct U as a Jastrow product[10] of unitary operators of two particles.

(13) $$U = \prod_{j>k=1}^{A} \mathcal{U}(j,k)$$

Let us consider the unitary transformation of an one-body opera-tor like that in Eq. (12).

Because of unitarity one has

(14) $$U^{+} \sum_{j=1}^{A} 0(j)\, U = \sum_{j=1}^{A} \prod_{k(\neq j)}^{A} \mathcal{U}^{-1}(j,k)\, 0(j)\, \mathcal{U}(j,k)$$

as the operators for different particles are naturally supposed to commute.

Assuming now only the two-particle correlations we obtain

$$(15)\qquad U^{+} \sum_{j=1}^{A} 0(j)\, U \cong \sum_{j\neq k}^{A} \mu^{+}(j,k)\, 0(j)\, \mu(j,k) - (A-2) \sum_{j=1}^{A} 0(j)$$

where one has subtracted the contributions from these k's which are not "correlated" to a given j.

Applying (15) to Eq. (12) one obtains the correlation correction to the elastic form factor:

$$(16)\qquad \Delta F = \frac{1}{Z} \sum_{\alpha B} \Big[\big\langle \widetilde{\alpha B}(1,2) \big| e_1\, e^{i\vec{q}\cdot\vec{r}_1} \big| \widetilde{\alpha B}(1,2) - \widetilde{B\alpha}(1,2) \big\rangle - \big\langle \alpha(1)\, B(2) \big| e_1\, e^{i\vec{q}\cdot\vec{r}_1} \big| \alpha(1)\, B(2) - B(1)\, \alpha(2) \big\rangle \Big]$$

where the summation extends over all occupied single-particle states of the shell model.

In (16) we have introduced the correlated two-particle states

$$(17)\qquad \big| \widetilde{\alpha B}(j,k) \big\rangle = \mu(j,k) \big| \alpha(j)\, B(k) \big\rangle$$

Performing in Eq. (17) the summation over the spin and isospin quantum numbers one has:

$$(18)\qquad \Delta F = \frac{2}{Z} \sum_{ab} \Big[4\, \Delta \big\langle ab \big| e^{i\vec{q}\cdot\vec{r}_1} \big| ab \big\rangle - \Delta \big\langle ab\; e^{i\vec{q}\cdot\vec{r}_1} \big| ba \big\rangle \Big]$$

where $\Delta(\ldots)$ denotes the difference between correlated and uncorrelated magnitudes. The single particle spatial quantum numbers we denote a, b, . . ; the one-particle orbital state is $|a\rangle = |n_a\, l_a\, m_a\rangle$.

In the case of harmonic oscillator wave functions, it is possible to define a transformation that takes us from motion of two particles about a common center to a description of the relative and "center-of-mass" motion of the two particles. "Following Moshinsky[11, 12] this transformation may be written:

$$(19)\qquad \big| n_1 l_1, n_2 l_2, \lambda\mu \big\rangle = \sum_{nl\,NL} \big\{ nl, NL, \lambda \big| n_1 l_1, n_2 l_2, \lambda \big\} \big| nl, NL, \lambda\mu \big\rangle$$

where (nlm) are the quantum numbers of relative motion and (NLM) are the quantum numbers of the c.m. motion.

The relative and the c.m. coordinates are defined as follows:

$$\tilde{r} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2), \qquad R = \sqrt{2}\, \frac{\alpha_1^2 r_1 + \alpha_2^2 r_2}{\alpha_1^2 + \alpha_2^2} \tag{20}$$

In the case of a single oscillator parameter ($\alpha_1 = \alpha_2$) the transformation bracket in (19) is independent of α, but in general depends on the oscillator frequencies of the two particles.

We introduce the two-particle correlations in (18) modifying (in a unitary way thus preserving normalizations and orthogonalities[13]) the radial functions of the relative motion of a nucleon-nucleon pair:

$$\begin{aligned} |\widetilde{nlm}\rangle &= \frac{g(r)}{\sqrt{N_{nl}}} R_{nl}(r)\, Y_{lm}(\theta, \phi), \\ N_{nl} &= \int_0^\infty dr\, r^2 R_{nl}^2\, g^2(r) \end{aligned} \tag{21}$$

where g(r) is the "correlation" function which modifies the radial oscillator function $R_{nl}(r)$.

Employing the Moshinsky technique[11] one obtains from (18) the following two-particle correlation correction to the elastic form factor:

$$\begin{aligned} Z\Delta F = {} & 6\exp(-t_s)\Delta\langle 000, 000;q;s\rangle + (Z-2)\exp(-t_p)\Big[(3-4t_p+t_p^2)\Delta\langle 000, 000;q;p\rangle \\ & + \frac{1}{2}\Delta\langle 100, 100;q;p\rangle - \left(\frac{2}{3}\right)^{1/2} t_p \Delta\langle 100, 000;q;p\rangle \\ & + \frac{5}{3}\Delta\sum_m \langle 01m, 01m;q;p\rangle - \frac{10}{3} t_p \Delta\langle 011, 011;q;p\rangle \\ & + \frac{1}{2}\Delta\sum_m \langle 02m, 02m;q;p\rangle + \left(\frac{4}{3}\right)^{1/2} t_p \Delta\langle 020, 000;q;p\rangle\Big] \\ & + \frac{1}{2}(Z-2)\exp(-v_s v_p t_{sp})\Big\{(3-2v_s v_p t_{sp})v_s\Big[\Delta\langle 000, 000;qv_s;sp\rangle \\ & + \Delta\langle 000, 000;qv_p;sp\rangle\Big] + \frac{5}{3} v_p\Big[\Delta\sum_m \langle 01m, 01m;qv_s;sp\rangle \\ & + \Delta\sum_m \langle 01m, 01m;qv_p;sp\rangle\Big]\Big\} \end{aligned} \tag{22}$$

where:

$$t_c = \frac{q^2}{8\alpha_c^2} \; ; \quad c = s, p, sp; \quad \alpha_{sp} = \left(\frac{2\alpha_s^2 \alpha_p^2}{\alpha_s^2 + \alpha_p^2}\right)^{1/2} \; ; \quad v_s = \left(\frac{\alpha_{sp}}{\alpha_s}\right)^2 ;$$

$$v_p = \left(\frac{\alpha_{sp}}{\alpha_p}\right)^2 ; \quad \langle nlm, n'l'm'; q; c\rangle = \langle (nlm)_c \left| \exp\left(\frac{iqZ}{\sqrt{2}}\right) \right| (n'l'm')_c \rangle$$

In Eq. (22) $|(nlm)_c\rangle$ is an harmonic oscillator state with the oscillator parameter α_c. The formula (22) is valid for nuclei with two protons in the s-shell and Z-2 protons in the p-shell. The oscillator parameters for the two shells are assumed to be different.

There is an improvement in the formula (22)[(x)] as compared to that in Ref. (3), namely the Moshinsky transformation for two nucleons from the different shells (terms with c=sp) has been performed in the exact way. Moreover some mistakes have been correct. These modifications in the formula for ΔF yield, however, only very small changes of the numerical results.

We have calculated the Li^6 elastic charge form factor from (11) and (22) using the following form of g(s)

$$g^2(s) = 1 - \exp\left(-\frac{1}{2}\Lambda^2 s^2\right) \tag{23}$$

Such a correlation function represents the soft-core repulsion between nucleons at small relative distances. The form (23) of g(s) enables us to perform all the integrations in (22) analytically.

The nucleon structure correction as in (3) has been used applying the experimental results of Ref. (15).

Our results are presented in Fig. 2 where we compare the correlated elastic charge form factor (full line) and the uncorrelated one (dashed line) with the experimental results for the Li^6 nucleus[(16)]. Including the short range correlations between nucleons in the model with two different oscillator wells we were able to obtain a good fit to the data over the large range of momentum transfer q. The fit presents a diffraction minimum at $q \simeq 3\ fm^{-1}$.

(x) - Let us call attention to the fact that in our preliminary work on the short range correlations[(14)] the formula for ΔF should read without a factor 2Z-1. The mistake has already been corrected in Ref. (3).

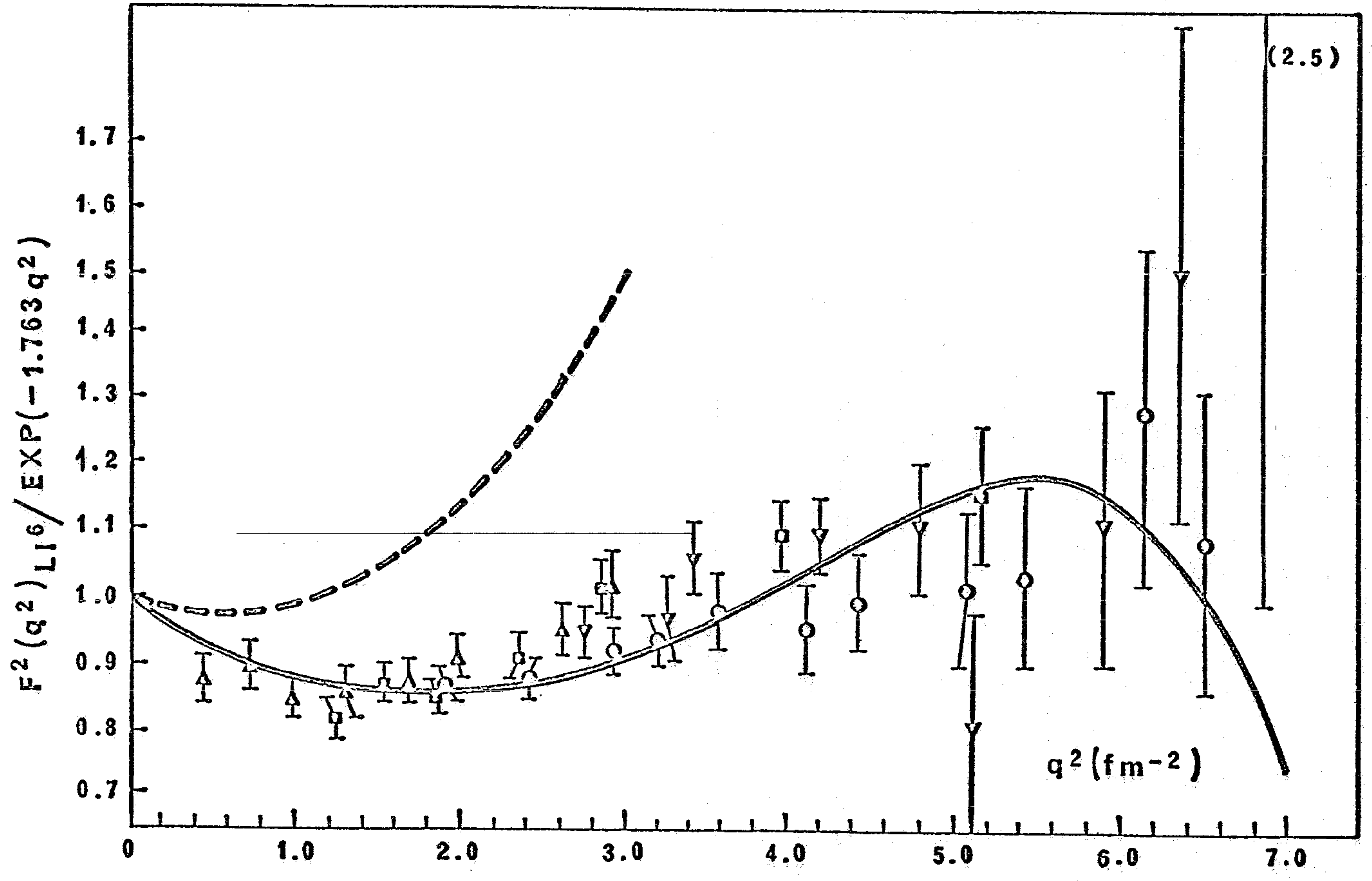

FIG. 2 - Li^6 charge form factor. Full line-short-range correlations included, dashed line no correlations. The values have been divided by $\exp(-1.763\,q^2)$ which corresponds to the square of the form factor for a Gaussian charge distribution of r.m.s. radius 2.3 fm. The parameters are: α_s=130.517 MeV, α_p=98.588 MeV, Λ=1.678 fm^{-1}. Experimental points from Ref. (16).

We have also performed the short range correlation calculations for the C^{12} and O^{16} nuclei. It turned out that in this case there is no need to use the shell model with two different oscillator potential wells. Satisfactory fits to the experimental data may be obtained in the simpler model, with a common oscillator well for the s- and p-nucleons.

These fits could be a bit improved by using two oscillator parameters with a small difference (of a few MeV) between them. However it seems that it would be rather hard to find a good fit to the data in the whole range of q if the two parameters differed appreciably from each other (say, by 20, 30 MeV). Although we did not make an exhausting search over the parameters we expect that in order to fit the data in the model with two appreciably different oscillator strengths it is essential to put a very strong (with very small value of Λ) correlation between nucleons. Such a "short-range" correlation would strongly affect the form factor also at low momentum transfers which is, physically, not very meaningful.

We have presented in Figs. 3 and 4 the correlated (full line) and uncorrelated (dashed line) charge form factors for C^{12} and O^{16} together with the existing experimental data[17].

The short range correlations have been introduced using the correlation function of the form (23).

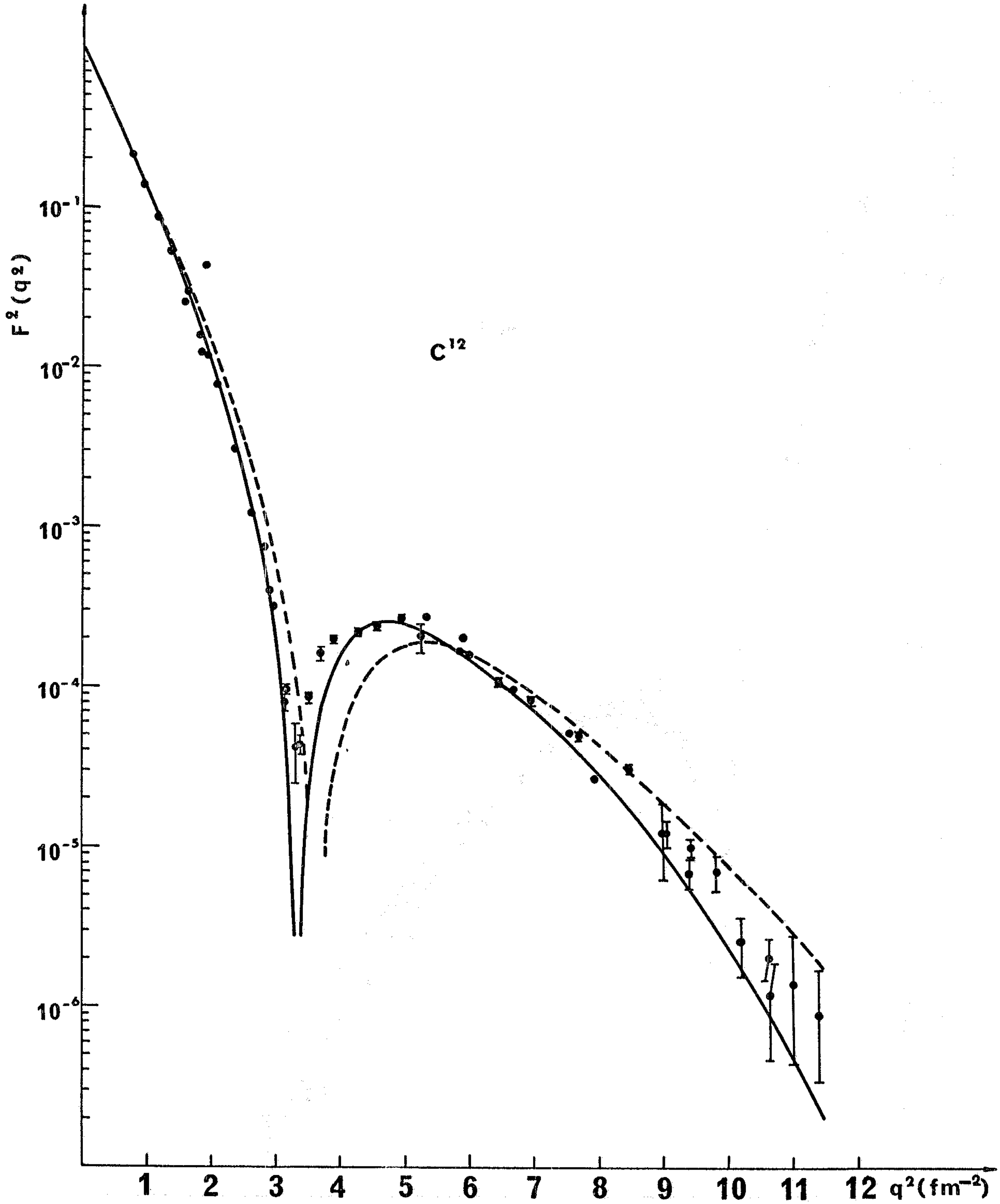

FIG. 3 - C^{12} charge form factor. Full line-short range correlations included, dashed line-no correlations. The parameters are $\alpha_s = \alpha_p =$ $= 126.38$ MeV, $\Lambda = 2.1475$ fm^{-1}. Experimental points from Ref. (17).

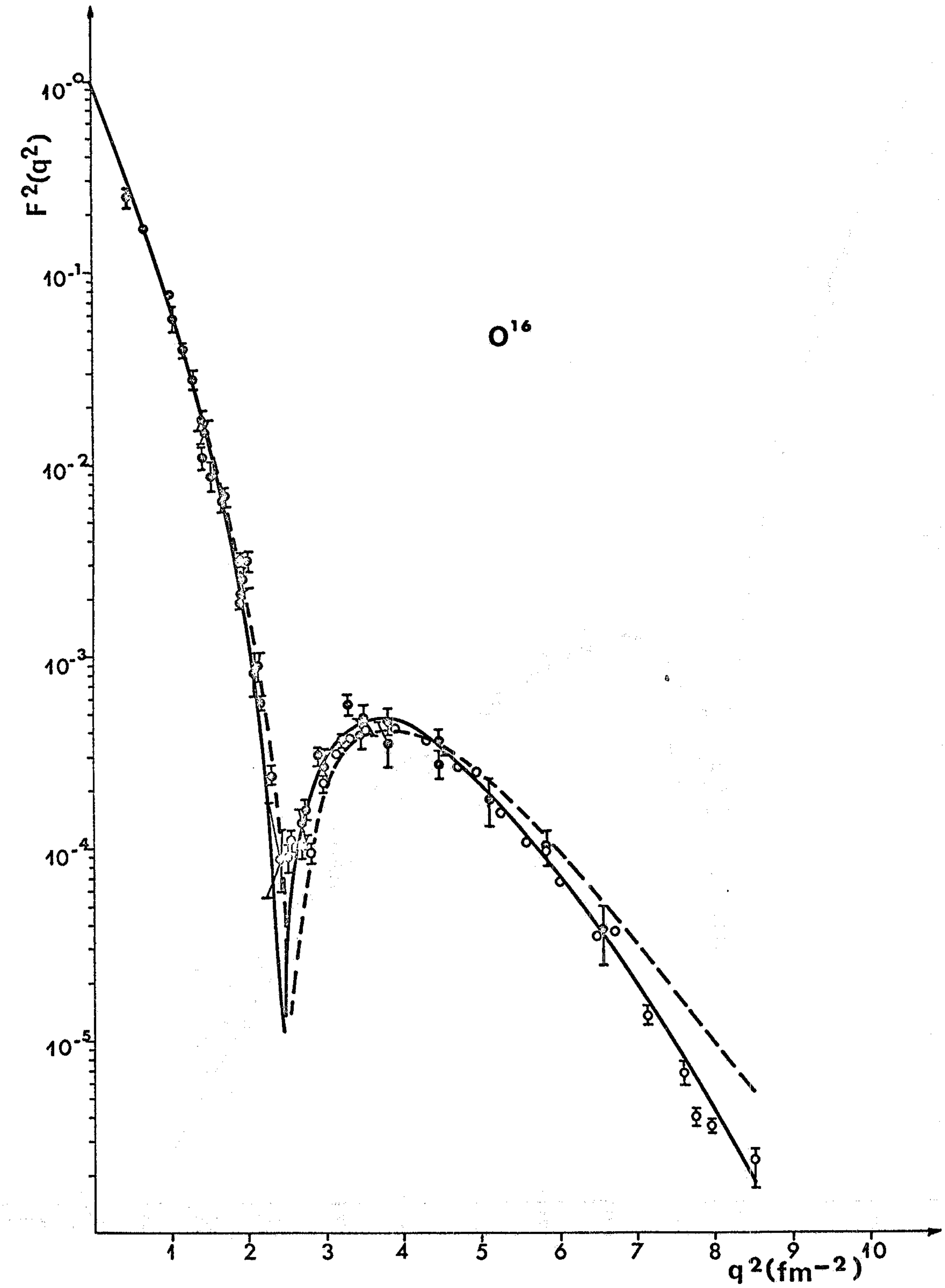

FIG. 4 - O^{16} charge form factor. Full line-short range correlations included, dashed line-no correlations. The parameters are $\alpha_s = \alpha_p = 112.27$ MeV, $\Lambda = 2.24$ fm^{-1}. Experimental points from Ref. (17).

APPENDIX. -

We present here the elastic form factors of Li^6, C^{12} and O^{16} in the model with the two different oscillator wells, corrected for the c.m. motion by means of the Gartenhaus-Schwartz transformation:

$$\frac{1}{3}\left\langle Li^6 \left| \sum_{j=1}^{A=6} e_j\, e^{i\vec{q}(\vec{r}_j-\vec{R})} \right| Li^6 \right\rangle =$$

$$= \frac{1}{9}\exp(-2t_s)\Big\{\Big[(1-2t_p)\exp(-t_s)\exp(-t_p)+2\tau_1\exp(-2\tau_2)\Big]\cdot \tag{A.1}$$

$$\cdot\Big[2(1-2t_p)\exp(-t_p)\exp(-T_s)-4(A-1)\tau_1\exp(-\tau_2)\exp(-\tau_3)+$$

$$+(1-2T_p)\exp(-t_s)\exp(-T_p)+2\tau_1\exp(t_s)\exp(-2\tau_2)\exp(-T_s)\Big]+$$

$$+2\exp(-t_s)\exp(-t_p)\Big[2\exp(-T_s)\exp(-t_p)+\exp(-T_p)\exp(-t_s)\Big]\Big\}$$

$$\frac{1}{6}\left\langle C^{12} \left| \sum_{j=1}^{A=12} e_j\, e^{i\vec{q}(\vec{r}_j-\vec{R})} \right| C^{12} \right\rangle =$$

$$= \frac{2}{9}\exp(-4t_p)\Big\{\Big[(1-2t_p)\exp(-t_s)\exp(-t_p)+2\tau_1\exp(-2\tau_2)\Big]^3 \tag{A.2}$$

$$\cdot\Big[(1-2t_p)\exp(-t_p)\exp(-T_s)-4(A-1)\tau_1\exp(-\tau_2)\exp(-\tau_3)+$$

$$+(1-2T_p)\exp(-t_s)\exp(-T_p)+(1-2t_p)\exp(-t_s)\exp(-T_p)+2\tau_1\exp(-T_p)\cdot$$

$$\cdot\exp(-2\tau_2)\exp(t_p)\Big]+\frac{1}{2}\exp(-3t_s)\exp(-3t_p)\Big[\exp(-t_p)\exp(-T_s)+$$

$$+2\exp(-t_s)\exp(-T_p)\Big]\Big\}$$

$$\frac{1}{8}\left\langle O^{16}\right|\sum_{j=1}^{A=16} e_j\, e^{i\vec{q}(\vec{r}_j-\vec{R})}\left|O^{16}\right\rangle$$

$$= \frac{1}{4}\exp(-8t_p)\Big[(1-2t_p)\exp(-t_s)\exp(-t_p)+2\tau_1\exp(-2\tau_2)\Big]^3\Big[(1-2t_p)\cdot$$

$$\cdot\exp(-t_p)\exp(-T_s)-4(A-1)\tau_1\exp(-\tau_2)\exp(-\tau_3)+(1-2T_p)\exp(-t_s)\cdot \tag{A.3}$$

$$\cdot\exp(-T_p)+4\tau_1\exp(-2\tau_2)\exp(-T_p)\exp(t_p)+2(1-2t_p)\exp(-t_s)\exp(-T_p)\Big].$$

where:

$$t_s=\frac{q^2}{4A^2\alpha_s^2}\,,\qquad t_p=\frac{q^2}{4A^2\alpha_p^2}$$

$$T_s=\frac{q^2}{4\alpha_s^2}\left(\frac{A-1}{A}\right)^2,\qquad T_p=\frac{q^2}{4\alpha_p^2}\left(\frac{A-1}{A}\right)^2$$

$$\tau_1 = 8\,\frac{\alpha_s^3\,\alpha_p^5\,q^2}{A^2(\alpha_s^2+\alpha_p^2)^5} \tag{A.4}$$

$$\tau_2=\frac{q^2}{2(\alpha_s^2+\alpha_p^2)A^2}\,,\qquad \tau_3=\frac{q^2}{2(\alpha_s^2+\alpha_p^2)}\left(\frac{A-1}{A}\right)^2$$

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