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A. F. Grillo: SOME REMARKS ON Q. E. D. BREAKING. -

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I. - INTRODUCTION. -

In this note we shall do some considerations on the hypothesis of a breakdown of quantum electrodynamics (Q. E. D.), a hypothesis that in last the years has taken on some weight in view of the growing possibility of testing electrodynamics at ever higer energies.

These remarks will be especially devoted to the study of possible experiments with colliding beams, following two different, but complementary, directions :

a) In Section II we shall examine the possibility that the electron does not behave like a point particle, attributing to it a magnetic as well as an electric form factor. This is analogous to what one does for protons, the structure of which is expressed by the Rosenbluth formula for  $e p \rightarrow e p$  scattering.

We consider especially the reactions of the "one photon channel"  
 $e^+ e^- \rightarrow n$  hadrons

and their cross sections will be examined on the grounds of the symmetry properties of e. m. interactions.

From the results of this section we can conclude that the electron structure can be tested by measurements of the angular distribution of the final particles: in an Adone experiment at maximum energy we have, for  $e^+ e^- \rightarrow \pi^+ \pi^-$ , an angular distribution like  $\sin^2\theta + \alpha \cos^2\theta$

2.

(instead of  $\sin^2 \theta$ ) where  $\alpha$  is essentially the product of the electron magnetic form factor and an amplification factor  $\gamma^2 = E^2/m^2 \approx 10^7$  that allows a measurement of a magnetic form factor  $\mu(-4E^2) \geq 10^{-4}$ .

b) In Section III we shall examine the consequences of modifying the photon propagator, following the old fashioned idea of a "regularized" propagator, proposed in a new interpretation by T. D. Lee and G. C. Wick<sup>(1)</sup>.

We shall see that, writing  $L(q^2)$  as the photon propagator :

$$L(q^2) = \frac{1}{q^2} - \frac{1}{q^2 + M^2}$$

and in the mass range  $3.3 \text{ GeV} \leq M \leq 10 \text{ GeV}$  there will be measurable deviations in the total cross section of e.g.  $e^+e^- \rightarrow \mu^+\mu^-$  and in the differential cross section of  $e^+e^- \rightarrow e^+e^-$ .

c) In the Appendix will be given the details of the calculations of Section II.

## II. - ONE PHOTON CHANNEL. -

In this section we shall firstly give a simple argument to prove that we expect some deviation in the angular distribution of emitted particles from that predicted by applying Q. E. D. to point particles; then we shall calculate the cross section for a reaction like :

$$(1) \quad e^+e^- \rightarrow a\bar{a}$$

on the ground of invariance properties of the theory. This will allow us to put in more precise form our first argument and to make a number of definite predictions of experimental results.

Suppose one write, for the matrix element of the electron e.m. current operator<sup>(x)</sup>

$$(2) \quad J_\mu = (0 | J_\mu(0) | e^+ p_2, e^- p_1)_{in} = \\ = e \bar{v}(p_2) \left[ \gamma(q^2) \gamma_\mu + \frac{\mu(q^2)}{2m} \epsilon_{\mu\nu} q_\nu \right] u(p_1)$$

(It is possible to prove that (2) is the most general expression, if we require Lorentz, gauge, C and P invariance).

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(x) -  $q = p_1 + p_2$ ; the normalization is  $\bar{u}u = -\bar{v}v = m/E$  where  $m$  and  $E$  are the electron mass and energy.

This can also be written:

$$(3) \quad J_\mu = \frac{e}{4} \left\{ \gamma(q^2) \bar{v}(p_2) \left[ (1 + \gamma_5) \gamma_\mu (1 - \gamma_5) + (1 - \gamma_5) \gamma_\mu (1 + \gamma_5) \right] u(p_1) + \right. \\ \left. + \frac{\mu(q^2)}{2m} q_\nu \bar{v}(p_2) \left[ (1 + \gamma_5) \gamma_{\mu\nu} (1 - \gamma_5) + (1 - \gamma_5) \gamma_{\mu\nu} (1 + \gamma_5) \right] u(p_2) \right\}$$

$$\text{since } [\gamma_5, \gamma_{\mu\nu}] = 0 \quad \text{and} \quad \{\gamma_5, \gamma_\mu\} = 0.$$

In the ultrarelativistic limit  $(1 + \gamma_5)/2$  is the  $-1$  helicity operator, and  $(1 - \gamma_5)/2$  is the  $+1$  one: the first term allows the coupling of  $+1$  ( $-1$ ) helicity electrons with  $-1$  ( $+1$ ) helicity positrons while the second term couples electrons and positrons with the same helicity.

In annihilation, the angular momentum created by the first term is  $+1$  and oriented in the direction of motion of the electron, while the second term corresponds to an angular momentum perpendicular to the direction of motion.

If we consider now an annihilation reaction, e. g.  $e^+e^- \rightarrow \pi^+\pi^-$  we have the following two cases (owing to the angular momentum conservation, the pions are in a state with  $L = 1$ ):

1) If  $\mu = 0$  (as it would be for a point particle) the pions and the electrons go cannot go in the same direction without violating angular momentum conservation, since the pions are spinless particles and their total angular momentum must be orthogonal to their direction of motion. We expect, then, that the cross-section behaves like  $\sin^2\theta$  (where  $\theta$  is the angle of, say, the outgoing  $\pi^+$  with respect to the incoming  $e^+$ ).

2) If  $\gamma = 0$  the pions cannot be produced at right angles to the momentum of the electrons: in this case we expect an angular distribution like  $\cos^2\theta$ .

It is clear that, if both form factors are different from zero, the angular distribution will be different from that predicted from pure Q. E. D..

We note again that our argument is completely general: if we write

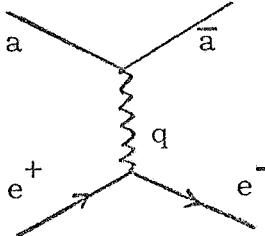
$$j_\mu = \bar{v}(p_2) \left[ \sum_i F_i(q^2) \Gamma_\mu \right] u(p_1)$$

where the  $\Gamma_\mu$  are combination of Dirac matrices and particle momenta, it is clear that terms such that  $[\Gamma_\mu, \gamma_5] = 0$  will give an anomalous contribution to the angular distribution and terms such that

$\{\Gamma_\mu, \gamma_5\} = 0$  will give a normal one.

4.

Going back to the reaction (1), described by the following Feynman graph



we can write (we use the general formulae and the notation given in ref. (2))

$$(4) \quad \langle a, \bar{a}(\text{out}) | S | e^+ p_2, e^- p_1(\text{in}) \rangle = \frac{2\pi}{q^2} \langle 0 | J_\mu(0) | e^+ p_2, e^- p_1(\text{in}) \rangle \times \\ \times \langle a, \bar{a}(\text{out}) | J_\mu(0) | 0 \rangle \delta^4(p_1 + p_2 - a - \bar{a})$$

(in the following we will omit the (in), (out) suffixes), where  $J_\mu(x)$  is the e. m. current operator. For the cross section we write, for unpolarized ingoing and outgoing particles

$$\sigma = \frac{2(2\pi)^4}{(4E)^4} \int d^3 a d^3 \bar{a} \delta(E_a + E_{\bar{a}} - 2E) \delta^3(\vec{a} + \vec{\bar{a}}) \mathcal{T}_{\mu\nu} \mathcal{T}'_{\mu\nu}$$

where we have defined the e. m. current matrices

$$(5) \quad \begin{aligned} \mathcal{T}_{\mu\nu} &= S_p \langle 0 | J_\mu | e^+ e^- \rangle^* \langle 0 | J_\nu | e^+ e^- \rangle \\ \mathcal{T}'_{\mu\nu} &= -S_p \langle a, \bar{a} | J_\mu | 0 \rangle^* \langle a, \bar{a} | J_\nu | 0 \rangle \end{aligned}$$

(\* - means complex conjugation) and  $S_p$  is the sum over the polarization and spin states of the particles; of course there will be one  $\mathcal{T}_{\mu\nu}$  for every polarization measurement.

The  $\mathcal{T}_{\mu\nu}$  can be specialized by noting that <sup>(x)</sup>

$$J_\mu^+(x) = \sum (\mu) J_\mu(x)$$

We have then

$$\mathcal{T}_{\mu\nu} = \frac{E(\mu)}{E_1 E_2} T_{\mu\nu} = E(\mu) S_p (e^+ e^- | J_\mu | 0) (0 | J_\nu | e^+ e^-).$$

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(x) - We define  $x_\mu^* = E(\mu) x_\mu$  (we use complex metric)

We have, thus, defined a quantity that describes the e. m. vertices and that is directly related to the cross section; in terms of  $T_{\mu\nu}$  we can put the considerations of Q. E. D. breaking in a form that is directly related to experimental quantities(x).

The most general formula of  $T_{\mu\nu}$  on the ground of Lorentz covariance is

$$T_{\mu\nu} = a_1 P_\mu P_\nu + a_2 q_\mu q_\nu + a_3 P_\nu q_\mu + a_4 P_\mu q_\nu + \\ + a_5 \delta_{\mu\nu} + i a_6 \epsilon_{\mu\nu\rho\sigma} P_\rho q_\sigma$$

where  $P_\mu = p_{1\mu} - p_{2\mu}$  and  $a_i = a_i(q^2) / i = 1, \dots, 6 /$  since  $(p, q) = 0$  and  $q^2 = -P^2 - 4m^2$ .

We have the following limitations on  $T_{\mu\nu}$

1) Current conservation implies

$$q_\mu T_{\mu\nu} = 0, \quad q_\nu T_{\mu\nu} = 0$$

and then  $a_3 = a_4 = 0$  and  $a_5 = -q^2 a_2$ .

2) Reality

$$T_{\mu\nu}^* = \epsilon(\mu) \epsilon(\nu) T_{\mu\nu}$$

and then we can write

$$(6) \quad T_{\mu\nu} = A(q^2) P_\mu P_\nu + B(q^2) (q_\mu q_\nu - q^2 \delta_{\mu\nu}) + C(q^2) \epsilon_{\mu\nu\rho\sigma} P_\rho q_\sigma$$

where A, B, C are real functions of  $q^2$ .

Further conditions on  $T_{\mu\nu}$  are derived from the discrete symmetries CP, C, P.

From the results of the Appendix we see that CP invariance is verified by (6), while the  $C(q^2)$  term violate C and P separately: so that we can write

$$T_{\mu\nu} = t_{\mu\nu}(p, q) + \theta_{\mu\nu}(p, q)$$

where  $t_{\mu\nu}$  is the term symmetric in  $\mu$  and  $\nu$  and is C, P conserving

$$t_{\mu\nu} = A P_\mu P_\nu + B (q_\mu q_\nu - q^2 \delta_{\mu\nu})$$

(x) - All this holds generally for n particles in the final state, where as the following considerations hold only for  $e^+e^- \rightarrow a\bar{a}$ .

6.

and  $\theta_{\mu\nu}$  is the (C, P violating, CP conserving) antisymmetric term.

We have, then, for the cross section in the center mass system :  $E = E_a = E_{\bar{a}}$  :

$$\begin{aligned}\frac{d\sigma}{d(\cos \theta)} &= \frac{\beta}{4} \frac{(2\pi)^5}{64E^6} T_{\mu\nu}(e^+e^-) T'_{\mu\nu}(a\bar{a}) = \\ &= \frac{\beta}{4} \frac{(2\pi)^5}{64E^6} \left[ t_{\mu\nu} t'_{\mu\nu} + \theta_{\mu\nu} \theta'_{\mu\nu} \right]\end{aligned}$$

where  $\beta$  is the speed of, say,  $a$ .

Remembering the expressions for  $t_{\mu\nu}$  and  $\theta_{\mu\nu}$  we have (the primed quantities refer to  $T'_{\mu\nu}(a, \bar{a})$ ):

$$(7) \quad \frac{d\sigma}{d\cos \theta} = \frac{(2\pi)^5 \beta}{16E^2} \left\{ \beta^2 A'(A \cos^2 \theta + B) + B'(3B + A) - 4\beta CC' \cos \theta \right\}$$

We can now put our previous argument in a quantitative form; namely, if electrons behave as point particles, we have e.g. for  $e^+e^- \rightarrow \pi^+\pi^-$  :  $T_{\mu\nu}(e^+e^-)$ :  $A = -B$ ,  $C = 0$ ;  $T'_{\mu\nu}$ :  $A' \neq 0$ ,  $B' = C' = 0$ , and then

$$\frac{d\sigma}{d\cos \theta} = \frac{(2\pi)^5 \beta^3}{16E^2} A' B \sin^2 \theta$$

otherwise we will observe deviations in the angular distribution.

Before proceeding further, we note that we can get a large amount of information on the structure of Q.E.D. in reactions like (1) by studying the angular distribution of final particles: therefore we remark on the usefulness of "hunting" possible violations by turning the experimental set-up.

Finally, we go to a definite expression for  $(0 | J_\mu(0) | e^+e^-)$  and write

$$(8) \quad (0 | J_\mu(0) | e^+e^-) = e \bar{v}(p_2) \left[ \gamma(q^2) \gamma_\mu + \frac{\mu(q^2)}{2m} \tilde{\epsilon}_{\mu\nu} q_\nu \right] u(p_1)$$

The  $T_{\mu\nu}$  is of the form (6) with

$$A(q^2) = \frac{e^2}{2} \left[ |\gamma(q^2)|^2 + q^2 \left| \frac{\mu(q^2)}{2m} \right|^2 \right]$$

$$B(q^2) = -\frac{e^2}{2} \left| \gamma(q^2) + \mu(q^2) \right|^2$$

$$C(q^2) = 0$$

We note that the result  $C(q^2) = 0$  could have been expected, since the matrix element (2) implies Q. E. D. invariance under both C and P.

Going back to a definite physical problem, consider the reaction  $e^+e^- \rightarrow \pi^+\pi^-$ ; we have

$$\langle \pi^+\pi^- | J_\mu(0) | 0 \rangle = \frac{e}{(2\pi)^3} (4E_{\pi^+} E_{\pi^-})^{-1/2} F(q^2) K_\mu$$

where  $K_\mu = P_\mu^{\pi^+} - P_\mu^{\pi^-}$  and  $F(q^2)$  is the e. m. form factor of the pion; and for  $T_{\mu\nu}$

$$T_{\mu\nu} = -\frac{e^2}{4(2\pi)^6} |F(q^2)|^2 K_\mu K_\nu$$

(note that in the c. m. s.  $E_{\pi^+} = E_{\pi^-} = E$ ).

We have then

$$\frac{d\sigma}{d\cos\theta} = \frac{(2\pi)^5 \beta^3}{16E^2} \left[ A(A\cos^2\theta + B) \right] = \frac{\pi \beta^3 \alpha^2}{16E^2} |F(q^2)|^2 \phi(\theta)$$

where we have defined the angular distribution function  $\phi(\theta)$

$$(9) \quad \phi(\theta) = |\gamma(q^2)|^2 \sin^2\theta + |\mu(q^2)|^2 \left( 1 + \frac{E^2}{m^2} \cos^2\theta \right) + 2 \operatorname{Re}(\gamma^* \mu).$$

Let us now comment briefly on this formula and the assumptions made in its derivation; the first thing to note is that, if we take the values  $\gamma(q^2) = 1$ ,  $\mu(q^2) = 0$  (point-like electron) we have  $\phi \sim \sin^2\theta$ , as expected from normal Q. E. D.

If both form factors are different from zero (the electric one from 1) there is indeed a Q. E. D. breakdown: if we want to continue in our analogy with the nucleon case, we must conclude that an expression like (2) shows the contribution of other interactions than the e. m. one.

Going back to consider the experimental importance of the formula (9) above, suppose for simplicity that

$$|\mu|^2 \left( 1 + \frac{E^2}{m^2} \cos^2\theta \right) \gg 2 \operatorname{Re}(\gamma^* \mu)$$

(this is clearly connected with the asymptotic behaviour of  $\gamma$  and  $\mu$ ; nevertheless this assumption seems reasonable) we will write then

$$\phi(\theta) \approx |\gamma(q^2)|^2 \sin^2\theta + |\mu(q^2)|^2 \frac{E^2}{m^2} \cos^2\theta.$$

If we do an experiment at  $\theta = 0$ , and with an accuracy e. g.  $\lambda = 5\%$ , we will have a measurable correction if

$$|\mu(-4E^2)| > \sqrt{\lambda} \frac{m}{E} \simeq 10^{-4} \quad (\text{for } E = 15 \text{ GeV})$$

(note that, owing to its dependence on  $\sqrt{\lambda}$  this result is rather independent of the "true" value of  $\lambda$ ).

It is useful to note again the great "amplification" factor  $\gamma^2 = (E/m)^2 \simeq 10^7$  in an ADONE experiment at the maximum energy: this factor comes from the particular kind of Q. E. D. breaking, but also from the fact that we consider, experimentally, reactions in the c. m. s. and so is peculiar to colliding beam experiments.

### III. - REGULARIZED PROPAGATOR. -

We examine now the problem of the Q. E. D. structure from a different viewpoint.

Recently T. D. Lee and G. C. Wick<sup>(1)</sup> have shown that the old idea for removing divergences from Q. E. D. writing for the photon a regularized propagator

$$L(q^2) = \frac{1}{q^2} - \frac{1}{q^2 + M^2}$$

(where, from the experimental data we must have  $M \geq 3.3$  GeV), can be formulated in a consistent way.

We are not interested here in the theoretical details and in the interesting consequences of the Lee hypothesis: we must only note that, if we consider the second term in the propagator  $L(q^2)$  as the propagator of a "heavy photon"  $\Gamma^0$  the pole of this particle is on the physical sheet of the complex energy plane, i. e. if we consider  $L$  at all orders we have for the mass  $\bar{M} = M + i/2\Gamma$  where  $\Gamma$  is the width of  $\Gamma^0$ .

We go now to the experimental implications of this proposal: we can show that in experiments with colliding beams we can test both the presence and the sign of the  $\Gamma^0$  term.

For sake of simplicity we shall examine two reactions:  $e^+e^- \rightarrow \mu^+\mu^-$  and  $e^+e^- \rightarrow e^+e^-$  and we suppose that the interaction of these particles is point-like.

For  $e^+e^- \rightarrow \mu^+\mu^-$  we can use the formalism developed in the preceding section: it is then evident that the ratio of the Lee cross section to the normal cross section is simply the ratio of modulus squared propagators. We have then three cases: one corresponding to the

normal cross section, one corresponding to the minus sign in the propagator and the other to the plus sign in the propagator; correspondingly we have

$$\bar{\sigma}_- = \frac{M^4 + \Gamma^2 M^2}{(M^2 - 4E^2)^2 + \Gamma^2 M^2} \bar{\sigma}_0$$

$$\bar{\sigma}_+ = \frac{(M^2 - 8E^2)^2 + \Gamma^2 M^2}{(M^2 - 4E^2)^2 + \Gamma^2 M^2} \bar{\sigma}_0$$

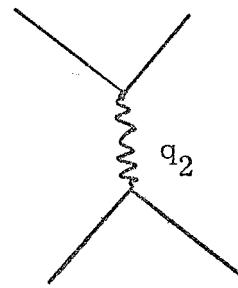
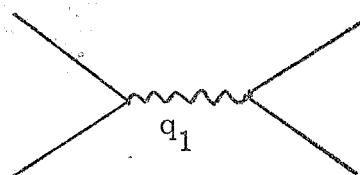
where  $\bar{\sigma}_0$  is the normal cross section for  $e^+e^- \rightarrow \mu^+\mu^-$  as given, e.g. in ref. (3).

These cross sections are given in figures 1, 2, 3, 4 for various values of  $M$ : we note the entirely different behaviour of  $\bar{\sigma}_-$  and  $\bar{\sigma}_+$  through the resonance that allows in principle to distinguish between the two cases (The form of these curves can be affected by variations due to the "true" value of  $\Gamma$  but only near to the resonance).

More interesting is the reaction  $e^+e^- \rightarrow e^+e^-$ , since we can explore the structure of the propagator by studying the angular distribution, owing to the interference term between the scattering and the annihilation channels.

In fact, if we are near enough to the resonance, the annihilation graph is dominated by the  $\Gamma^0$  term, while the scattering propagator is essentially the normal one: we see, then, that the interference between these processes gives directly the sign of the  $\Gamma^0$  term in the propagator.

The involved graphs are



where  $q_1 = 4E^2 \sin^2 \theta$  and  $q_2 = -4E^2$ .

For the cross section we have

$$\begin{aligned} \frac{d\bar{\sigma}}{d \cos \theta} = r_0^2 E^2 & \left\{ \left(1 + \cos^4 \frac{\theta}{2}\right) \left| L(q_1^2) \right|^2 + 2 \cos^4 \frac{\theta}{2} \operatorname{Re} [L(q_1) L^*(q_2)] + \right. \\ & \left. + \frac{1}{2} (1 + \cos^2 \theta) \left| L(q_2^2) \right|^2 \right\}. \end{aligned}$$

The results are plotted in figures 5, 6, 7, 8.

10.

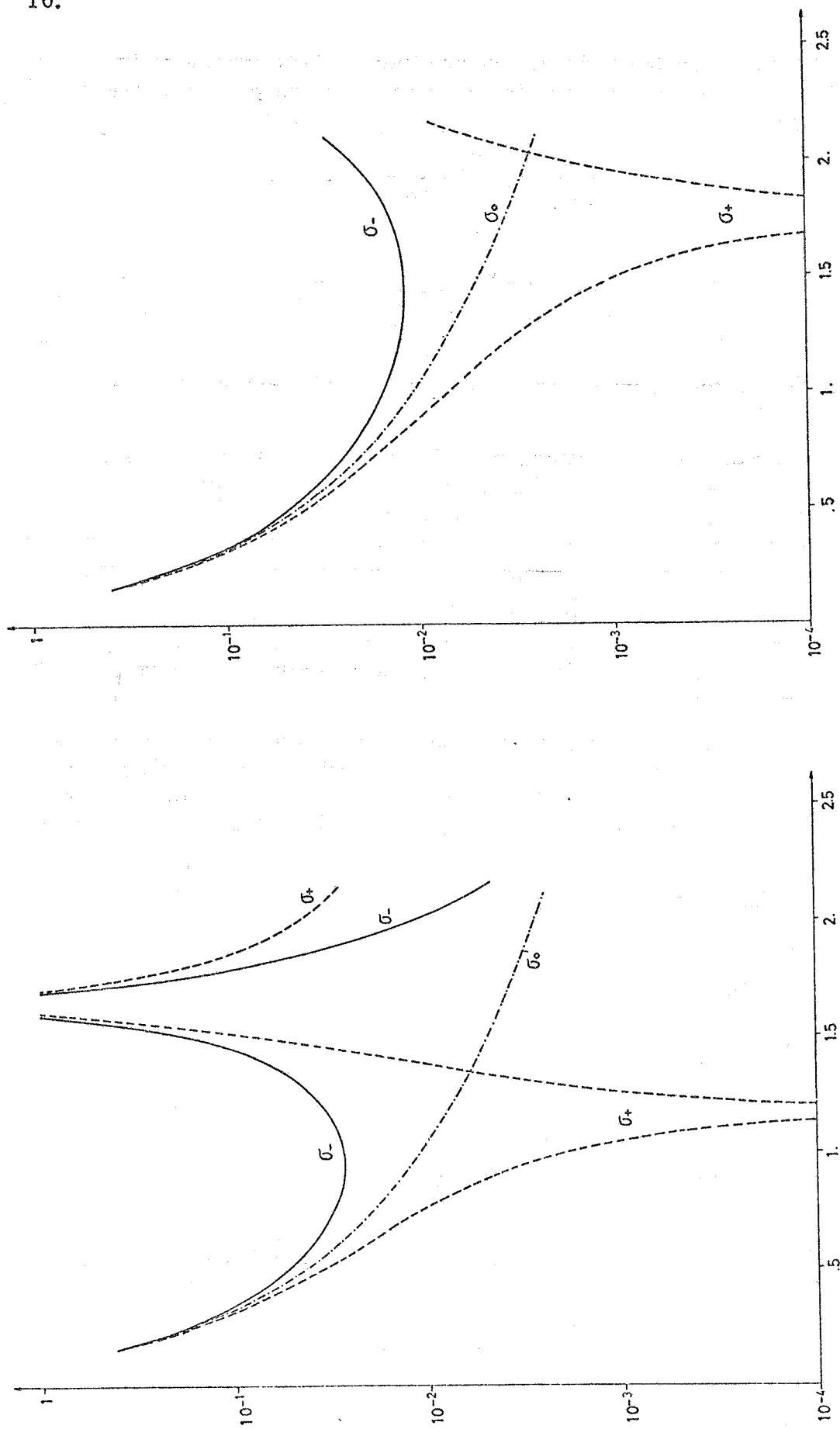


FIG. 2 - Total cross section of  $e^+e^- \rightarrow \mu^+\mu^-$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\Gamma^0$  mass  $M = 5 \text{ GeV}$ .

FIG. 1 - Total cross section of  $e^+e^- \rightarrow \gamma^0$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\Gamma^0$  mass  $M = 3.3 \text{ GeV}$ .

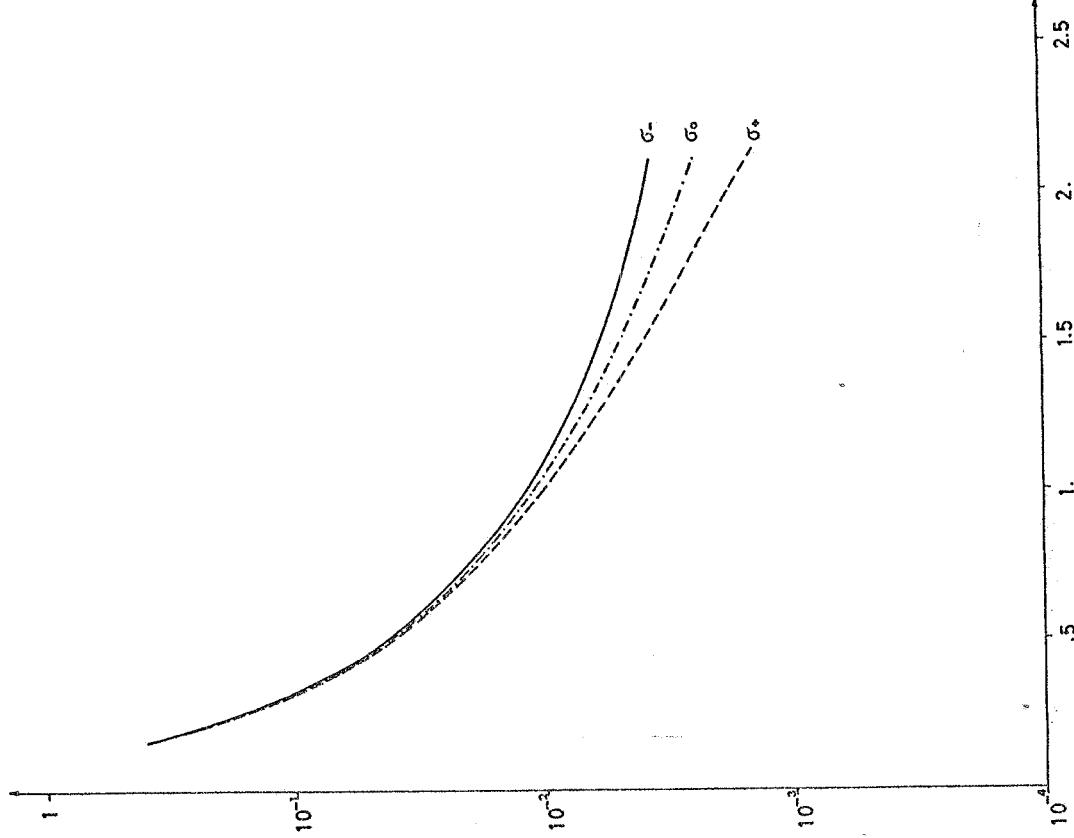


FIG. 4 - Total cross section of  $e^+e^- \rightarrow \mu^+\mu^-$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\Gamma_0$  mass  $M = 10 \text{ GeV}$ .

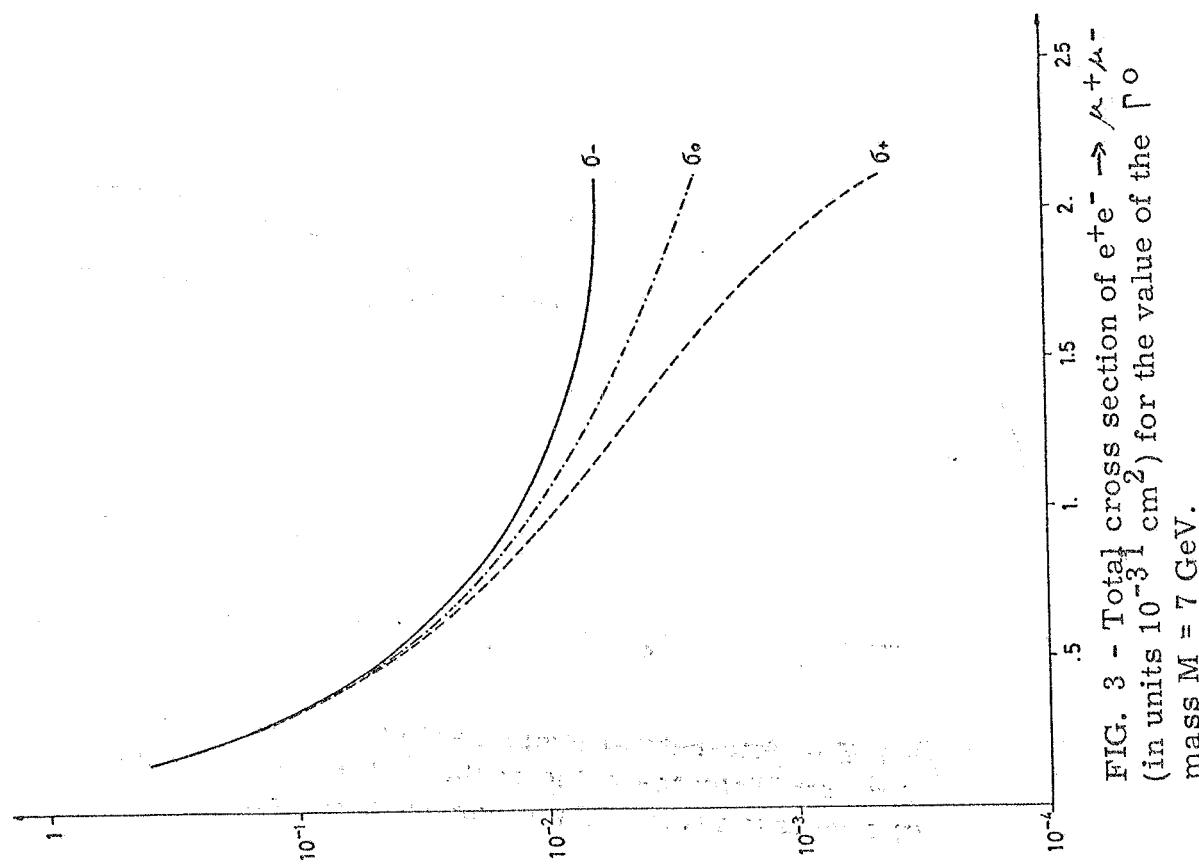


FIG. 3 - Total cross section of  $e^+e^- \rightarrow \mu^+\mu^-$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\Gamma_0$  mass  $M = 7 \text{ GeV}$ .

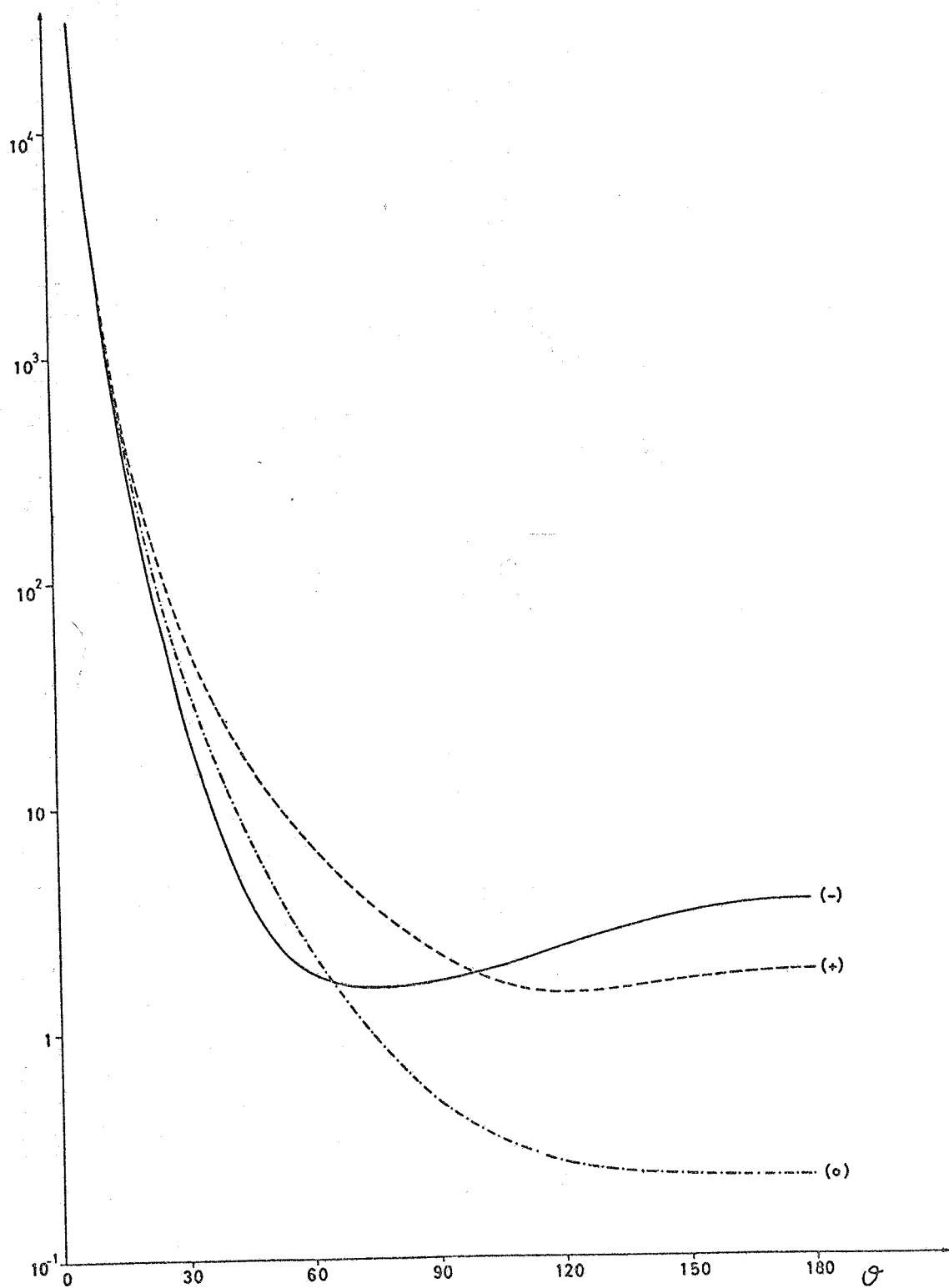


FIG. 5 - Differential cross section of  $e^+e^- \rightarrow e^+e^-$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\pi^0$  mass  $M = 3.3 \text{ GeV}$ . The (o) sign is referred to the normal propagator.

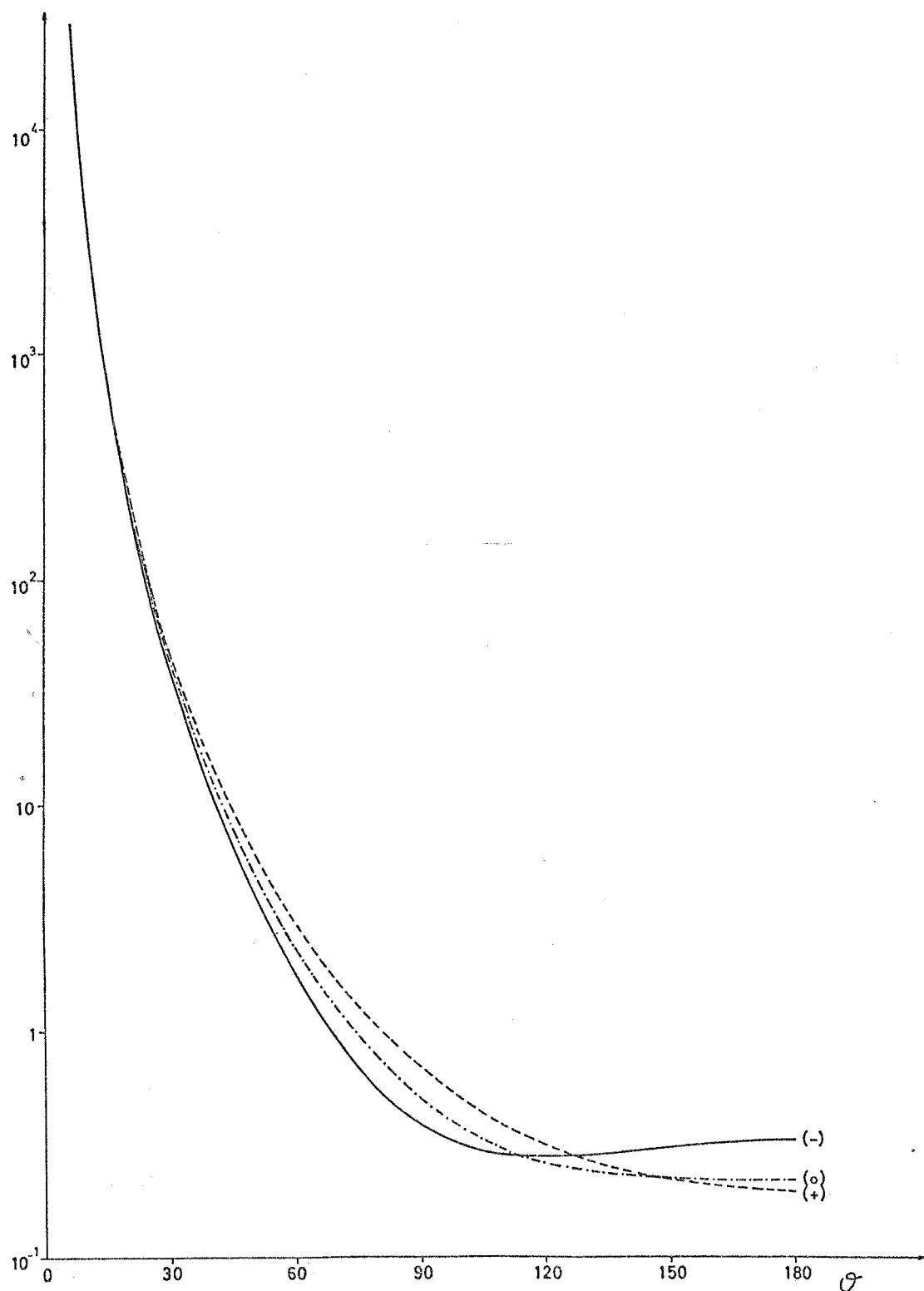


FIG. 6 - Differential cross section of  $e^+e^- \rightarrow e^+e^-$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\Gamma^0$  mass  $M = 5 \text{ GeV}$ . The (o) sign is referred to the normal propagator.

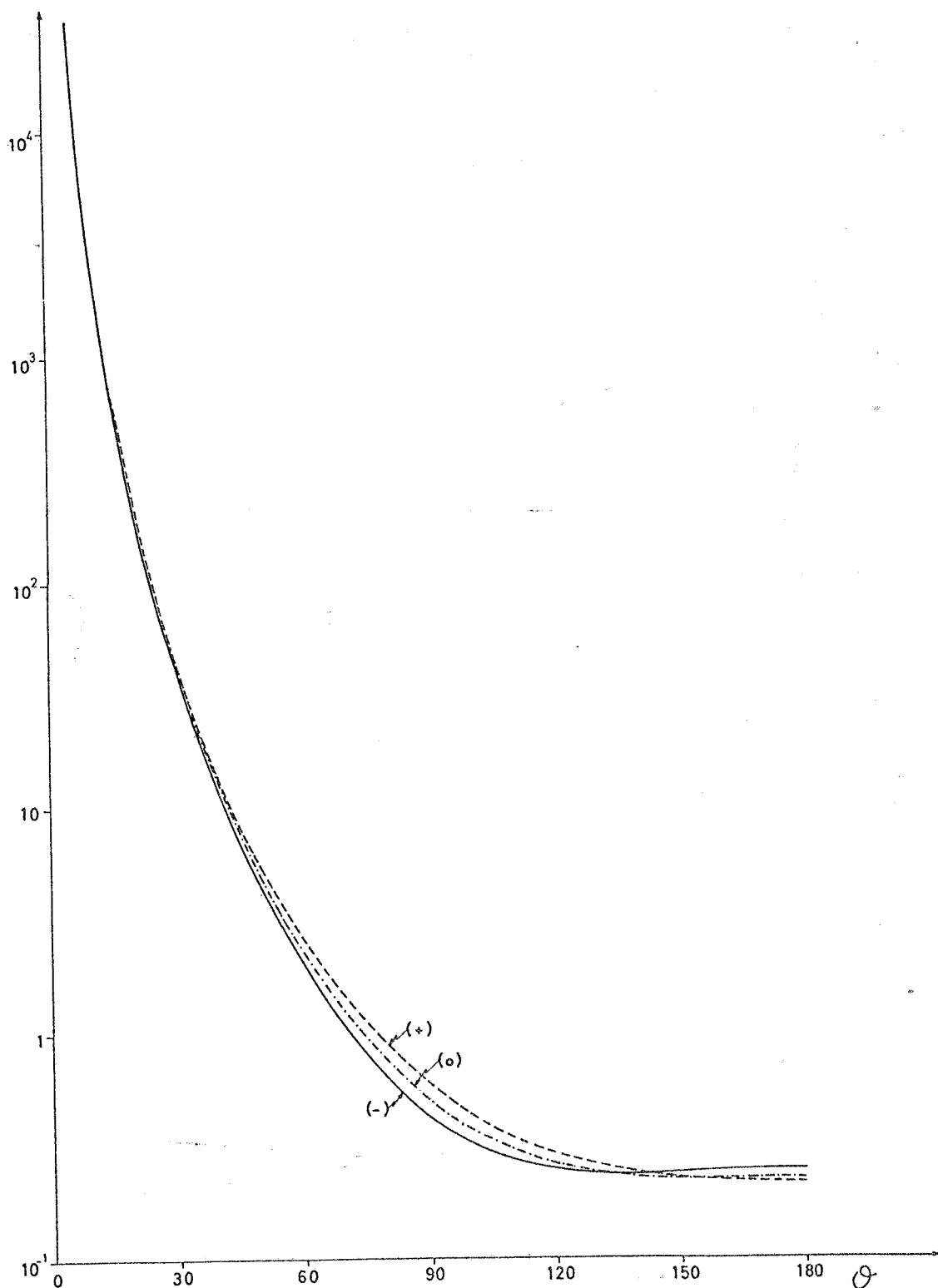


FIG. 7 - Differential cross section of  $e^+e^- \rightarrow e^+e^-$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\Gamma^0$  mass  $M = 7 \text{ GeV}$ . The  $(\circ)$  sign is referred to the normal propagator.

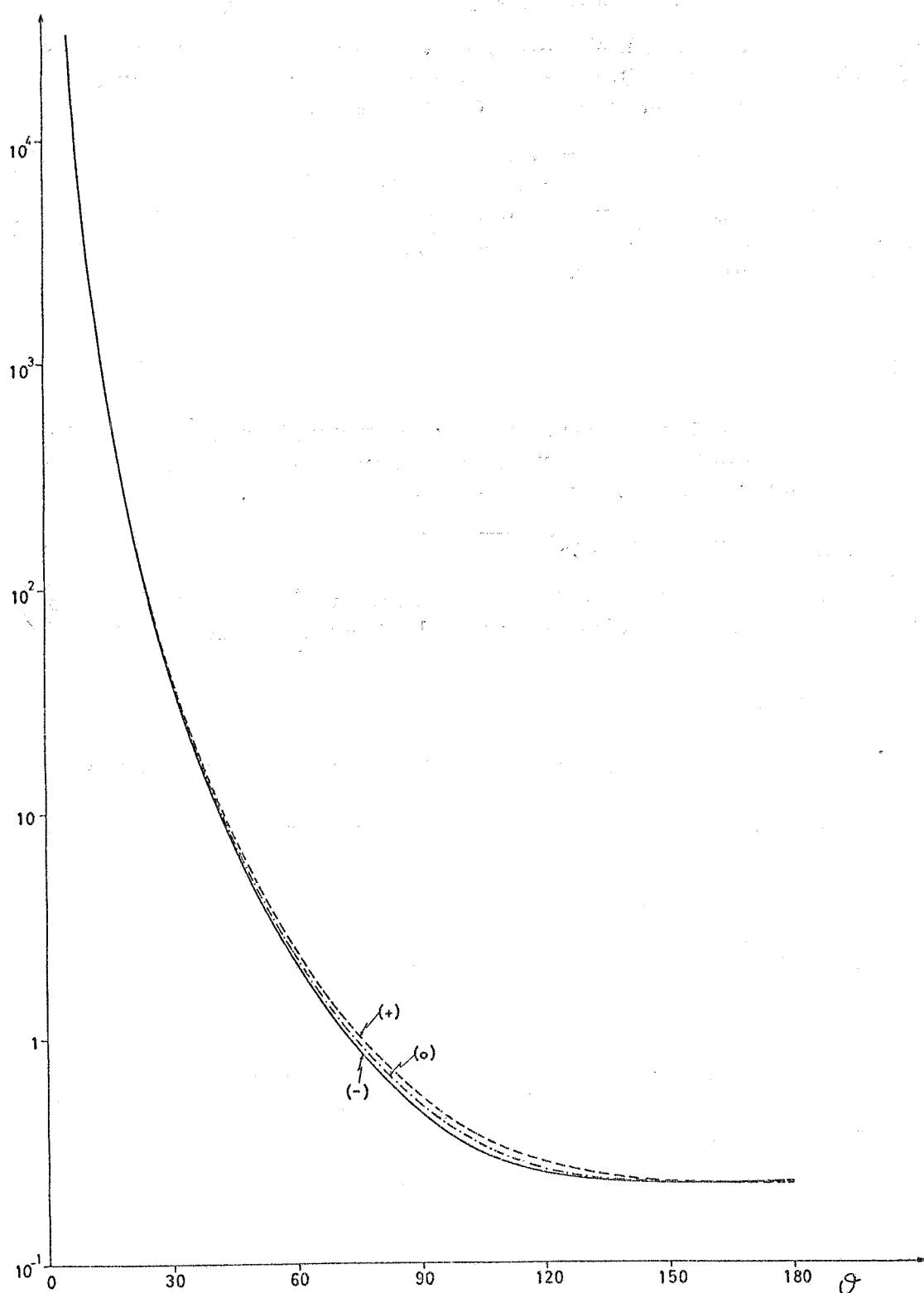


FIG. 8 - Differential cross section of  $e^+e^- \rightarrow e^+e^-$  (in units  $10^{-31} \text{ cm}^2$ ) for the value of the  $\rho^0$  mass  $M = 10 \text{ GeV}$ . The (o) sign is referred to the normal propagator.

We must make some comments on these results.

Firstly we note that in both reactions we have deviations of the order of 10%, even if  $M = 10 \text{ GeV}$  and bigger deviations if  $M$  is smaller, and that we can easily decide the sign of the propagator.

Secondly, our considerations are a little complicated if the particles have form factors: but it is easy to convince ourselves that, at least for the cases examined in the preceding section, a combined measurement of the total and differential cross section can give enough information on the propagator.

#### APPENDIX. -

This appendix is devoted to the discrete symmetries C, P, CP; our considerations are greatly simplified, owing to the fact that we speak about unpolarized particles only.

##### 1. - CHARGE CONJUGATION.

The C operation is the interchange of  $e^+$  and  $e^-$  and then all that we must do (for unpolarized particles) are the following substitutions in  $T_{\mu\nu}(e^+e^-)$ :  $P \rightarrow -P$ ,  $q \rightarrow q$ .

We have then

$$T_{\mu\nu}^{(C)} = T_{\mu\nu}(-p, q) = A P_\mu P_\nu + B(q_\mu q_\nu - q^2 \delta_{\mu\nu}) - C \epsilon_{\mu\nu\lambda\sigma} P_\lambda q_\sigma$$

Invariance of e. m. interactions requires

$$C J_\mu(0) C^{-1} = -J_\mu(0)$$

and then  $T_{\mu\nu}^{(C)} = T_{\mu\nu}$  which implies that  $C(q^2) = 0$ .

##### 2. - PARITY.

We must carry out the following substitutions on  $T_{\mu\nu}$ :  $P_\mu \rightarrow -\epsilon(\mu)P_\mu$ ;  $q_\mu \rightarrow -\epsilon(\mu)q_\mu$ ; moreover the e. m. interaction invariance requires

$$P J_\mu(0) P^{-1} = -\epsilon(\mu) J_\mu$$

so that we must have

$$T_{\mu\nu}^{(P)} = \epsilon(\mu) \epsilon(\nu) T_{\mu\nu} \left[ -\epsilon(\lambda) P_\lambda, -\epsilon(\lambda) q_\lambda \right] = T_{\mu\nu}$$

This implies

$$\begin{aligned} \mathcal{E}(\mu)\mathcal{E}(\nu) & \left[ A\mathcal{E}(\mu)\mathcal{E}(\nu)P_\mu P_\nu + B\mathcal{E}(\mu)\mathcal{E}(\nu)(q_\mu q_\nu - q^2 \delta_{\mu\nu}) - \right. \\ & \left. - C\mathcal{E}(\mu)\mathcal{E}(\nu)\mathcal{E}_{\mu\nu\beta\sigma} P_\beta q_\sigma \right] = T_{\mu\nu} \end{aligned}$$

(since, from the definitions of  $\mathcal{E}(\mu)$ ,  $\mathcal{E}_{\mu\nu\beta\sigma}$  we have:

$$\mathcal{E}(S)\mathcal{E}(G)\mathcal{E}_{\mu\nu\beta\sigma} = -\mathcal{E}(\mu)\mathcal{E}(\nu)\mathcal{E}_{\mu\nu\beta\sigma}$$

and then  $[\mathcal{E}(\mu)\mathcal{E}(\mu) = 1] : C(q^2) = 0$ .

### 3. - CP.

The CP operation is represented on  $T_{\mu\nu}$  in the following way:  $P_\mu \rightarrow \mathcal{E}(\mu)q_\mu$ ,  $q_\mu \rightarrow -\mathcal{E}(\mu)q_\mu$ ; we must have, moreover

$$(CP)J_\mu(0)(CP)^{-1} = \mathcal{E}(\mu)J_\mu(0).$$

This implies

$$T_{\mu\nu}^{(CP)} = \mathcal{E}(\mu)\mathcal{E}(\nu)T_{\mu\nu} [\mathcal{E}(\lambda)R_\lambda, -\mathcal{E}(\lambda)q_\lambda] = T_{\mu\nu}$$

All terms are allowed.

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