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LARGE ANGLE p-p SCATTERING, CERULUS-MARTIN BOUND AND THE VENEZIANO MODEL

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Abstract

The Veneziano model, modified in order to satisfy the Cerulus-Martin bound, is shown to give a very good description of p-p elastic scattering over the full range of t, but the diffraction peak.

A great deal of both experimental and theoretical efforts has been devoted in the past few years to study the p-p elastic scattering at high energies and large momentum transfers. By looking at the experimental differential cross section $d\sigma/dt$ (1-5) one can distinguish three different regions.

In the first one, for values of $-t \lesssim 1 \div 2$ $(GeV/c)^2$ the diffe rential cross section goes down as e^{At}. Theoretically this is completely understood in terms of diffraction-like scattering, and it is both qualitatively and quantitatively described by means of few Regge poles[6].

In the second region, for values of $-t$ lying between ~ 2 and ~ 6 $(GeV/c)^2$, it has been observed[7] that the fixed s experimental data lie approximately on straight lines, when they are plotted as function of $\sqrt{-t}$. This is shown in Fig. 1. Theoretical attempts[8-10] to explain these data in terms of Regge cuts have partially succeded.

The transition from the second to the third region give rise to the well known break. The data still lie, at least roughly, on a straight line but the slope is different. To be more precise the following formula:

$$\frac{d\sigma}{dt} \sim e^{-\frac{\sqrt{s}}{T}} \left\{ e^{-\frac{\sqrt{|t|}}{T}} + e^{-\frac{\sqrt{|u|}}{T}} \right\} \qquad (1)$$

derived on the basis of general thermodynamical assumptions, has been shown[11] to be in very good agreement with the experiments.

The most interesting feature of equation (1) is that, besides the t and u symmetry which takes care of the identical nature of the particles involved, the variables s and -t play a completely symmetric role. This feature reflects the physical fact that at large angles the values of $|t|$ become comparable with s. Recalling now that the usual Regge theory is supposed to be good at small angles, where $|t| \ll s$, if we want to get a satisfactory description of elastic scattering over the full range of t, we have to be able to perform both limits, $s \to \infty$ for t fixed, and $s, |t| \to \infty$.

Recently a very interesting model for a crossing symmetric relativistic scattering amplitude has been proposed by Veneziano[12]. Once linearly rising trajectories are assumed, the model exhibits Regge behaviour, saturation of finite energy sum rules, daughters, etc. This model has been introduced to describe a very specific process (namely $\pi\pi \to \pi\omega$), but, due to the fact that all the properties listed above are supposed to be shared by any relativistic scattering amplitude, we are encouraged to guess that its structure has to have a very much wider validity. The great advantage of the Veneziano model compared with former Regge pole models is that the kinematical variables s and t can be treated on the same footing.

In a pure Veneziano-like model the limits discussed above both go like e^{tk}, where k is a function of the scattering angle Θ, and for the t-fixed limit, contains also a term like log s. This fact is very unsatisfactory if we recall the phenomenological discussion at the beginning of this paper. From a theoretical point of view the reason of the failure can be related to the fact that the Cerulus-Martin[13] bound is not satisfied.

Recent works however [9,10] have shown how, still keeping linearly rising trajectories, these asymptotic limits can saturate the Cerulus-Martin bound by the introduction of Regge cuts.

Let us now consider the p-$\bar{p}$ elastic scattering, which is a process crossing symmetric in the variables u and t, where u is the energy variable and t is the momentum transfer.

In the region of the high values of u and $|t|$ where the Pomeranchon poles and its cuts can be supposed to be dominant, it is not unreasonable to approximate cuts for away to the left in the j-plane, by poles at their branch points.

Assuming for the residues of these poles a parametrization consistent with the works on Regge cuts discontinuities [14,15] we suggest the following generalization of the Veneziano amplitude:

$$A(u,t) = \sum_{m,n}^{1,\infty} \frac{\Gamma[1-\alpha_m(u)]\,\Gamma[1-\alpha_n(t)]}{\Gamma[1-\alpha_m(u)-\alpha_n(t)]} (-\beta)^{m+n-2} a_{mn} \qquad (2)$$

where (*)

$$\alpha_n(x) = 1 + \frac{\alpha' x}{n}$$

. We note that being the s dependence missing in eq.(2), we are ignoring the contributions coming from the baryon number two channel but this is considered to be a very reasonable approximation in this process.

(*) The form of the a_{mn}'s has to be such that the signature factors come out properly, in case with the help of the baryon number two channel contributions.

The asymptotic limits are

$$\lim_{\substack{u \to +\infty \\ t \to -\infty}} A(u,t) = F_1(u,t)\,|t|^{\frac{1}{4}}\, e^{-2\sqrt{\alpha' \lg(1/\beta)}\,\left\{\sqrt{|u|}\, f_1 + \sqrt{|t|}\, g_1\right\}} \tag{3}$$

and

$$\lim_{\substack{u \to +\infty \\ t \text{ fixed}}} A(u,t) = F_2(u,t)\, u\, |t|^{\frac{1}{4}}\, e^{-2\sqrt{\alpha' \lg(1/\beta)|t| \lg[\alpha' u \lg(1/\beta)]}} \tag{4}$$

where f_i and g_1 are two slowly varying, known functions of $\sqrt{|t|/u}$ only, while F_1 and F_2 contain also oscillating functions of $\sqrt{u}$ and $\sqrt{|t|}$. These results, have been derived with a double use of the saddle point method where the $(-1)^{m+n}_{amn}$ has been neglected. We shall discuss later this approximation.

Eq.(4) is consistent with that obtained previously by many other authors discussing p-p scattering,[(9-10)] while the result (3) is closely related to that given by Greco[(11)] in the case of high energy p-$\bar{p}$ elastic scattering.

Recalling from our previous discussion that formulas (3) and (4) have to describe two different regions of the $d\sigma/dt$ plot, a very simple explanation of the p-p "break" in terms of one parameter is suggested.

Our next step will be to consider the p-p scattering. This process is again symmetric in the variables t and u, where u now has the meaning of a momentum transfer, and we assume that it will still be described by amplitude (2).

However we want to point out that to ignore the baryonic number two channel contributions in both amplitudes, as we do, has different implications for p-p e $p\bar{p}$. Meanwhile eq.(2) should in principle describe $p\bar{p}$ scattering also in the diffraction region and its imaginary part in the forward direction should give the asymptotic total cross section, on the contrary the amplitude we have guessed for the p-p case, does not have any absorbitive part in the energy and therefore neither we can expect to describe the diffraction peak nor we can give estimates for the total cross sections.

The asymptotic behaviours of our amplitude, are now:

$$\lim_{\substack{u \to -\infty \\ t \to -\infty}} A(u,t) = G_1(u,t)\, |t|^{\frac{1}{4}}\, e^{-2\sqrt{\alpha' \lg(1/\beta)}} \left\{ \sqrt{|u|}\, f_2 + \sqrt{|t|}\, g_2 \right\} \tag{5}$$

$$\lim_{\substack{u \to -\infty \\ t \text{ fixed}}} A(u,t) = G_2(u,t)\, |u|\, |t|^{\frac{1}{4}}\, e^{-2\sqrt{\alpha' \lg(1/\beta)\, |t|\, \lg[\alpha' |u| \lg(1/\beta)]}} \tag{6}$$

Due to the different direction in which the u limit is taken, f_2, g_2, G_1 and G_2 are closely related to f_1, g_1, F_1 and F_2 introduced in (3) and (4), but not equal to them.

Just by inspecting these two formulae, one can see that all the previously discussed features of the large t p-p experimental data are reproduced. More explicitly the different behaviour of our amplitude in the fixed t and in the large angles regions give rise to the discontinuity of the differential cross section observed experimentally[3,4]. But the agreement goes very much deeper than that. Eq.(5), in fact, describes all the data in the 3^{th} region with an accuracy comparable with the experimental errors. This is shown in Fig. 2 where the experimental cross sections are compared with eq.(5), evaluated for $\sqrt{\alpha' \lg(1/\beta)} = 0.66\ (\mathrm{GeV/c})^{-1}$ (*).

Furthermore, with the value of $\sqrt{\alpha' \lg(1/\beta)}$ just given, eq.(6) predicts the slopes of the differential cross sections in the 2^{nd} region. The agreement shown in Table I is not any worse that other theoretical values available in the literature[10] but we want to stress that it has been obtained without free parameters and by considering G_1 and G_2 as constants.

We are aware that the oscillating terms that we have neglected play a very important role in modulating the amplitude[16]. A more careful evaluation of these terms is now in progress by numerically evaluating the whole series. But we think that our results are promising enough the believe that the main features are independent from these details.

Recently[10] in the framework of a Glauber-type eikonal approximation to large angle scattering it has been reconsidered the possibility that the Pomeranchon has a "normal" slope of about 1 GeV^{-2}. We

(*) This value is obtained by fitting the 90° data.

want to point out that, meanwhile our asymptotic formulae cannot give any direct information on α', the numerical evaluation of the whole series together with the above result for $\alpha' \lg(1/\beta)$, once compared with the experiments, will give values for α' and $\lg(1/\beta)$ to be checked against the asymptotic $p\bar{p}$ total cross section.

In conclusion we want to stress that, being an "universal" object like the Pomeranchon our main ingredient, any information about the specific structure of the proton seems to be missing, and this should strengthen the feeling that the high momentum transfers physics is sort of detached from the finer details of strong interaction dynamics as the success of the thermodynamical approach would suggest.

We should like to thank N.Cabibbo for his interest in our work and helpful criticism.

Figure Captions

Fig. 1 - p-p differential cross sections $(d\sigma/d\Omega)_{C.M.}$ in units of 10^{-30} cm^2/sr, as functions of $\sqrt{|t|}$. The experimental points are taken from Ref. 4 and 5. Dashed lines connect the experimental points with the same value of P_o.

Fig. 2 - p-p differential cross sections $(d\sigma/d\Omega)_{C.M.}$ in units of 10^{-30} cm^2/sr, as functions of $|t|$. The full lines result from eq.(5).

Table 1 - Experimental and theoretical slopes of the curves of fig. 1 versus the incoming lab. momentum P_o. Theoretical values result from eq.(6).//

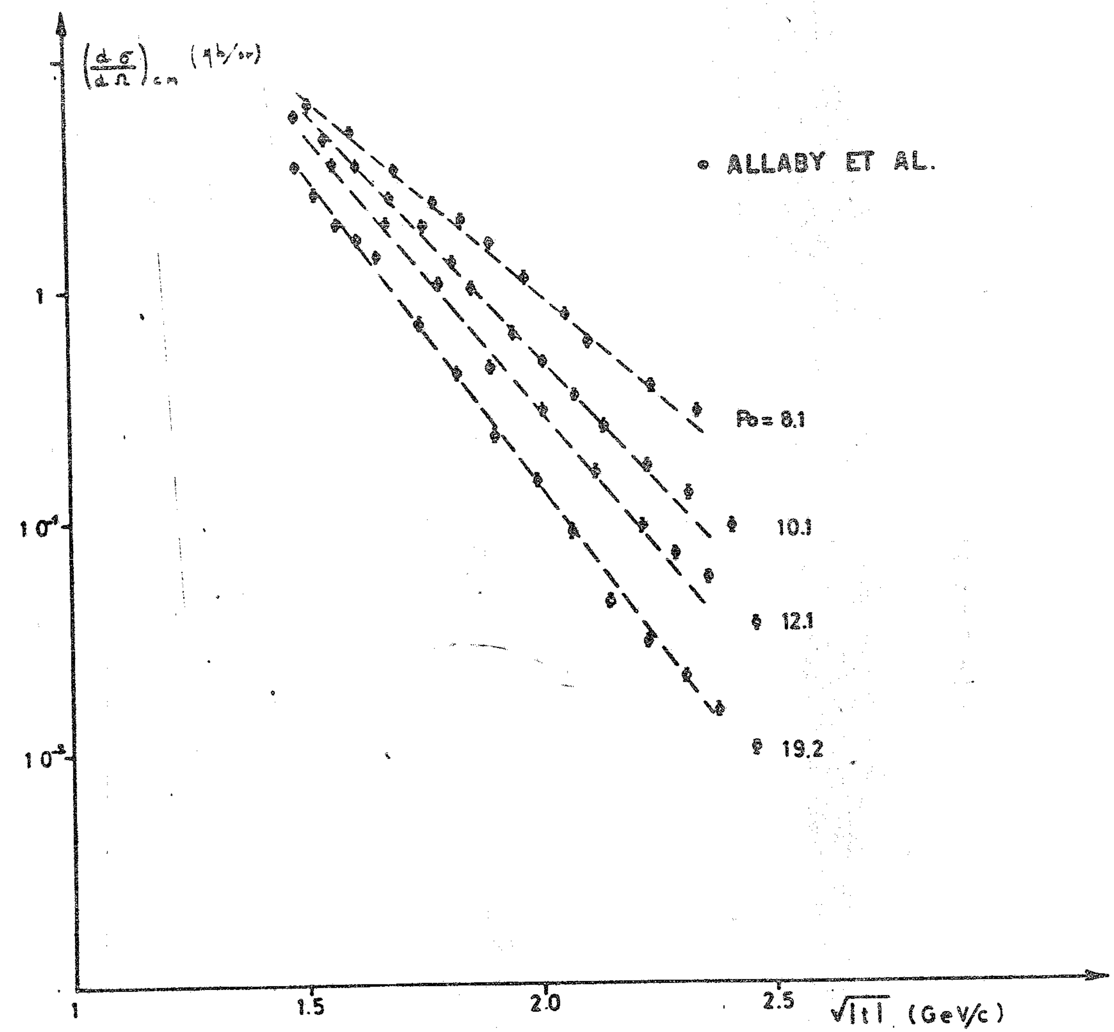
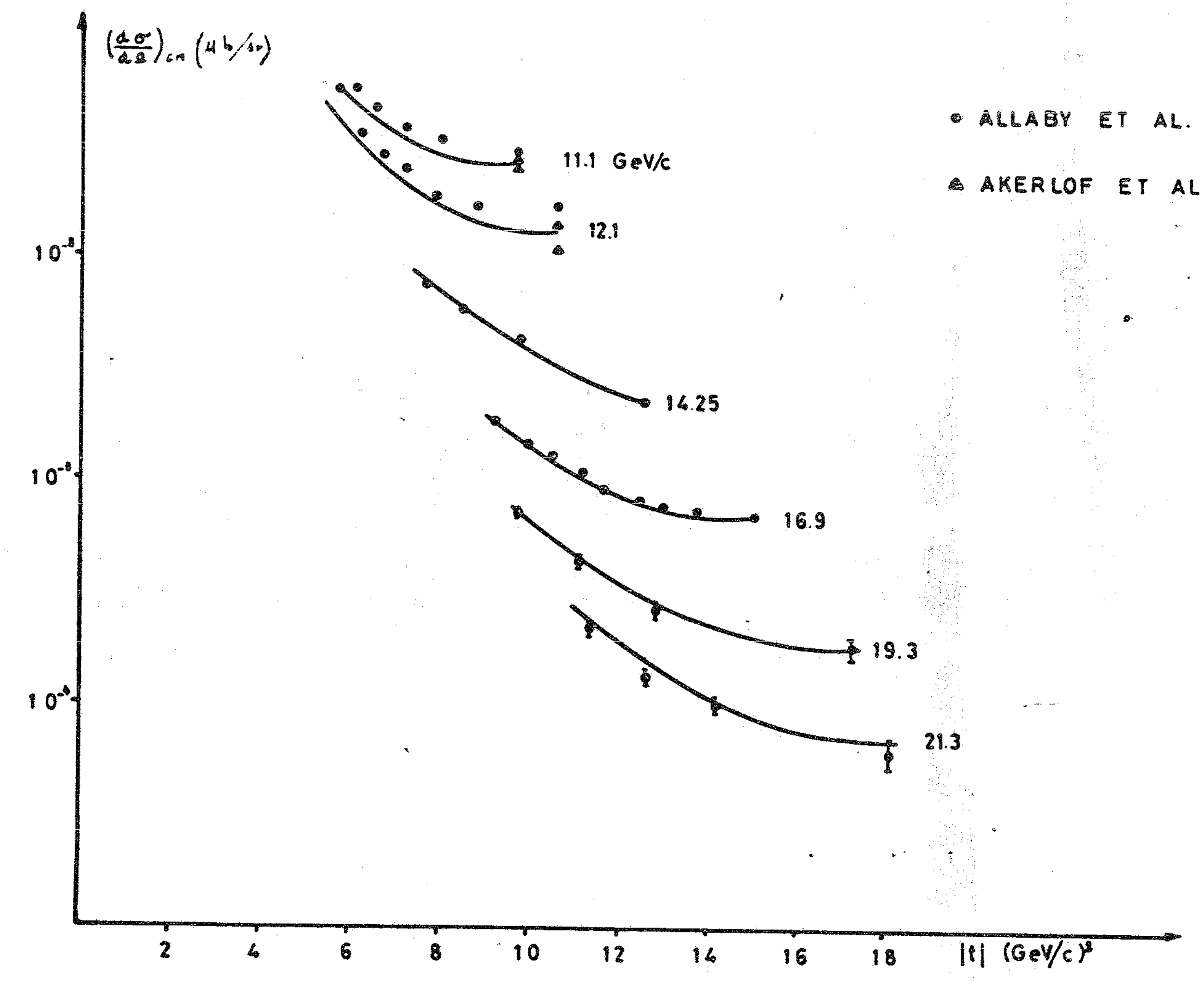

(dσ/dΩ)cm (μb/sr)
1
10⁻¹
10⁻²
• ALLABY ET AL.
Po = 8.1
10.1
12.1
19.2
1
1.5
2.0
2.5
√|t| (GeV/c)

FIG. 1



$\left(\frac{d\sigma}{d\Omega}\right)_{cm}$ (μb/sr)
ALLABY ET AL.
AKERLOF ET AL
11.1 GeV/c
12.1
14.25
16.9
19.3
21.3
10^{-3}
10^{-5}
10^{-6}
2
4
6
8
10
12
14
16
18
|t| (GeV/c)²

FIG. 2

TABLE I

P_o (GeV/c)	Exp. slope $(GeV/c)^{-1}$	Theor. slope $(GeV/c)^{-1}$
8.1	3.9	3.2
10.1	5.0	3.5
12.1	5.6	3.9
19.2	6.3	4.5

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