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VACUUM POLARIZATION EFFECTS OF HADRONS. -

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Strong interaction effects of hadrons constitute a menace to pure QED even if the interacting particles are carefully prepared to eliminate strongly interacting particles from the initial state. This is due to the fact that the photon being a $J^{PC} = 1^{--}$ meson can undergo virtual transitions into heavier $J^{PC} = 1^{--}$ neutral mesons such as the ρ^0 , ω , ϕ . It is therefore necessary to calculate these $\gamma \rightarrow V^0$ transition effects since any refined test of QED will eventually have to consider corrections of the order of magnitude of those due to hadronic modification of the photon propagator. Accordingly we present in this note a simple method for calculating fairly accurately vacuum polarization (VP) corrections of hadrons in $e^+ - e^-$ colliding beam reactions. We also comment briefly on the $\rho^0 - \omega$ interference in such processes as

$$e^+ + e^- \longrightarrow \pi^+ + \pi^-$$

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which Donnachie⁽¹⁾ suggested could show marked peaks sitting on the mass of the ω . We consider the second order ($O(e^2)$) hadronic modification of the photon propagator due to emission and subsequent absorption of hadrons as illustrated in fig. (1).

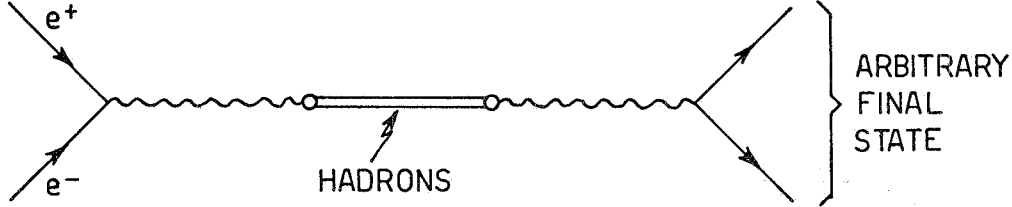
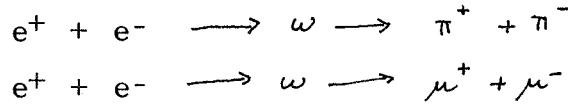


FIG. 1

When the hadronic modification is schematised as in fig. (1) all hadronic contributions are considered coherently and the corresponding correction factor is calculated once and for all and applied to all $e^+ - e^-$ annihilations irrespective of the particular final state. In other words the VP correction factor is unique and depends only on the available CM energy of the annihilating $e^+ - e^-$ pair. Consequently processes such as



must be (correctly) regarded as part of the vacuum polarization corrections to the corresponding lowest order purely electrodynamic processes $e^+ + e^- \longrightarrow \pi^+ + \pi^-$ and $e^+ + e^- \longrightarrow \mu^+ + \mu^-$ respectively.

The modified photon propagator is given in the Källén-Lehmann form by

$$D_{\mu\nu}(q^2) = \frac{\delta_{\mu\nu}}{q^2} + 4\pi\alpha \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \int_0^\infty \frac{S(-x) dx}{x^2 (x+q^2-i\epsilon)} \quad (1)$$

and the spectral function $S(q^2)$ is defined in terms of matrix elements of the hadronic electromagnetic current $j_\mu(x)$ by

$$\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) S(q^2) = \sum_n (2\pi)^4 \delta^4(q - P_n) \langle 0 | j_\mu(0) | n \rangle \langle n | j_\nu(0) | 0 \rangle$$

The $|n\rangle$ are sets of hadronic states satisfying 4-momentum conditions as implied by the delta function in (2). To evaluate the integral in (1), which means having a compact analytic expression for $S(q^2)$, we appeal to field-current identity (FCI)⁽²⁾ which allows one to relate $S(q^2)$ to two other spectral functions $S_\rho(q^2)$, $S_{\phi-\omega}(q^2)$ on which readily acceptable simplicity assumptions can be made.

These assumptions are inherent in all vector dominance model calculations and usually yield reasonably good results. The decomposition of $S(q^2)$ into two terms is a consequence of the decomposition of the hadronic electromagnetic current into an isovector and an isoscalar part. The relationship between these spectral functions is⁽²⁾

$$S(q^2) = g_\rho^{-2} m_\rho^4 S_\rho(q^2) + \frac{1}{4} \left[g^{-1} M^2 S_{\phi-\omega}(q^2) M^2 \tilde{g}^{-1} \right]_{1,1} \quad (3)$$

g_ρ is related to the γ - ρ^0 coupling constant, m_ρ the ρ^0 mass and

$$S_\rho(q^2) = \frac{1}{3} \sum_{\vec{z}} \left| \langle 0 | \vec{p}(0) | \vec{z} \rangle \right|^2 \quad (4)$$

with $\vec{p}(x)$ the spatial component of the ρ^0 -meson field $p_\mu(x)$ and $|\vec{z}\rangle$ is a spin one state of zero 3-momentum and energy $m_{\vec{z}}$. \tilde{g} (with \tilde{g} its transpose), M , $S_{\phi-\omega}(q^2)$ are 2×2 matrices generalising g_ρ , m_ρ , $S_\rho(q^2)$ respectively in the case of the ϕ - ω complex, $[\dots]_{1,1}$ denotes the $(1,1)^{\text{th}}$ matrix element. A convenient representation of these matrices is given in ref. (2). The spectral function $S_\rho(q^2)$ and matrix $S_{\phi-\omega}(q^2)$ have Dirac delta contributions at the masses of ρ^0 , ω , ϕ of the form $Z_0 \delta(q^2 + m_R^2)$ with Z_0 of the order of unity. In evaluating the integral in (1) we retain only these delta function contributions and obtain a simple relationship between the photon propagator and the propagators of the neutral vector mesons

$$D_{\mu\nu}(q^2) \approx \frac{\delta_{\mu\nu}}{q^2} + 4\pi d Z_0 \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \cdot \left[g_\rho^{-2} \frac{1}{m_\rho^2 + q^2 - i m_\rho \Gamma_\rho} + \frac{g_Y^{-2}}{4} \left(\frac{\cos^2 \theta_Y}{m_\phi^2 + q^2 - i m_\phi \Gamma_\phi} + \frac{\sin^2 \theta_Y}{m_\omega^2 + q^2 - i m_\omega \Gamma_\omega} \right) \right] \quad (5)$$

g_Y is the hypercharge coupling constant and θ_Y the corresponding mixing angle⁽²⁾. Eq. (5) is one of the important results of FCI, which stated simply means that there is an exact relationship in the strong interactions between the second order ($\mathcal{O}(e^2)$) hadronic contributions to the photon propagator and the vector meson propagators. In other words eq. (5) suffices to represent the effect of all hadrons and not only of the neutral mesons ρ^0 , ω , ϕ . As a result of current conservation the term proportional to $q_\mu q_\nu$ does not contribute to the VP corrections, consequently the VP correction factor $C_{VP}(E)$ is defined as

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$$C_{VP}(E) = \left| 1 - 16\pi\alpha Z_0 E^2 \left[\frac{g_p^{-2}}{m_p^2 - 4E^2 - im_p\Gamma_p} + \right. \right. \quad (6)$$

$$\left. \left. + g_y^{-2} \cdot \left(\frac{\cos^2\theta_y}{m_\phi^2 - 4E^2 - im_\phi\Gamma_\phi} + \frac{\sin^2\theta_y}{m_\omega^2 - 4E^2 - im_\omega\Gamma_\omega} \right) \right] \right|^2$$

q^2 has been replaced by $-4E^2$, with $2E$ the total energy of the $e^+ - e^-$ pair in CM. Eq. (6) is plotted in fig. (2) (full curve) as a function of $2E$ and for $Z_0 = 1$.

The values of the constants used have been obtained as

follows:

$$\frac{g_p^2}{4\pi} = 2.3^{(2)}$$

$$\frac{g_y^2}{4\pi} = \frac{1.4 \cos^2(\theta_N - \theta_y)^{(2)}}{\cos^2 \theta_N}$$

$$\frac{m_\omega \tan \theta_y}{m_\phi} = \frac{m_\phi \tan \theta_N}{m_\omega} = \tan \theta, \quad (2, 3)$$

$$\theta = 35^\circ \quad (4) \quad \Gamma_\phi = 4.2 \text{ MeV}$$

$$\Gamma_p = 108 \text{ MeV}, \quad \Gamma_\omega = 12.2 \text{ MeV}$$

$$m_p = 765 \text{ MeV} \quad m_\omega = 780 \text{ MeV} \quad m_\phi = 1020 \quad (3)$$

The curve manifests interesting structures around the resonant masses and shows that VP corrections become very important for $2E$ comparable with the mass of the ϕ . Even for $2E$ around the ρ^0 , ω masses they are not negligible varying between 5% and 10%.

It is well known that vector dominance as a dispersion theoretic description crystalizes into FCI in the language of local lagrangian field theory (2). It is therefore to be expected, at least as a consistency requirement, that dispersion theory yield eq. (5). This can be shown quite simply by considering a single resonant contribution to $D_{\mu\nu}(q^2)$. We define an amplitude $f_R(q^2)$ for $e^+ - e^-$ annihilation into hadrons passing through the said resonance R by the formula (suggested by eq. (6))

$$f_R(q^2) = N \frac{1}{m_R^2 + q^2 - im_R\Gamma_R} \quad (7)$$

⊕ fig. (2b) is a continuation of fig. 2a plotted on a different scale.

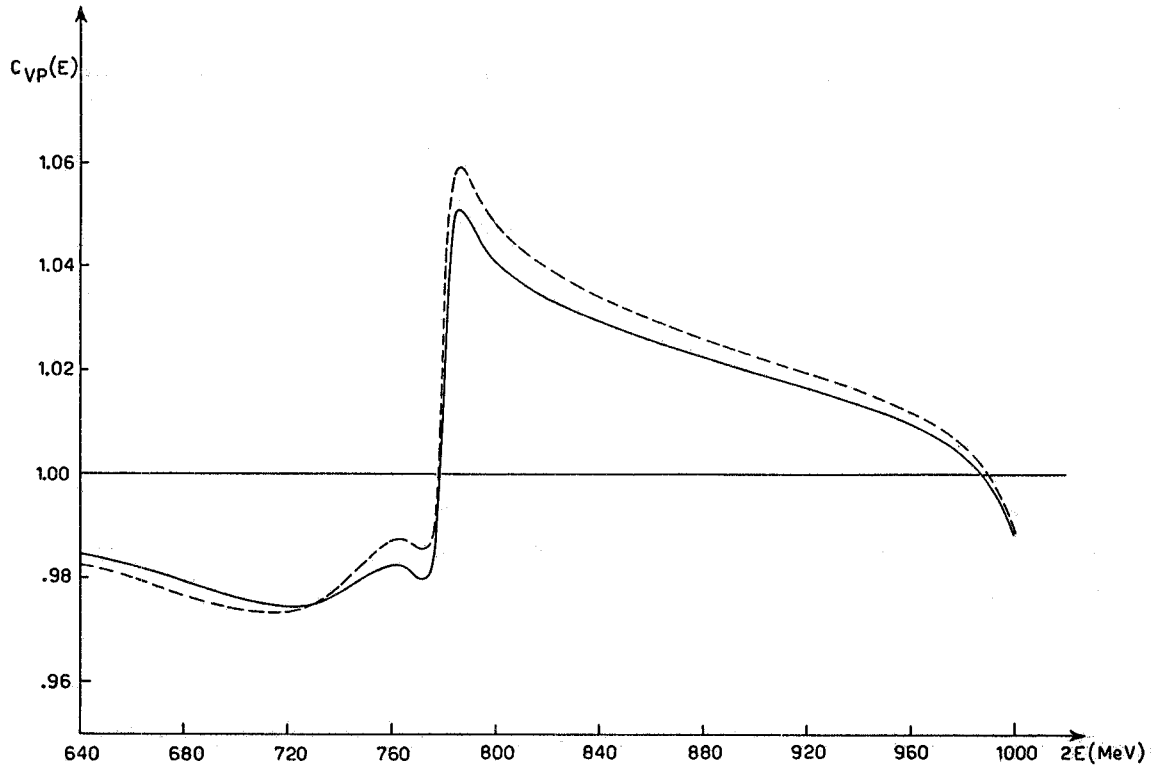


FIG. 2a - Plot of VP correction factor $C_{VP}(E)$ against $2E$ showing structure around ρ^0, ω masses.

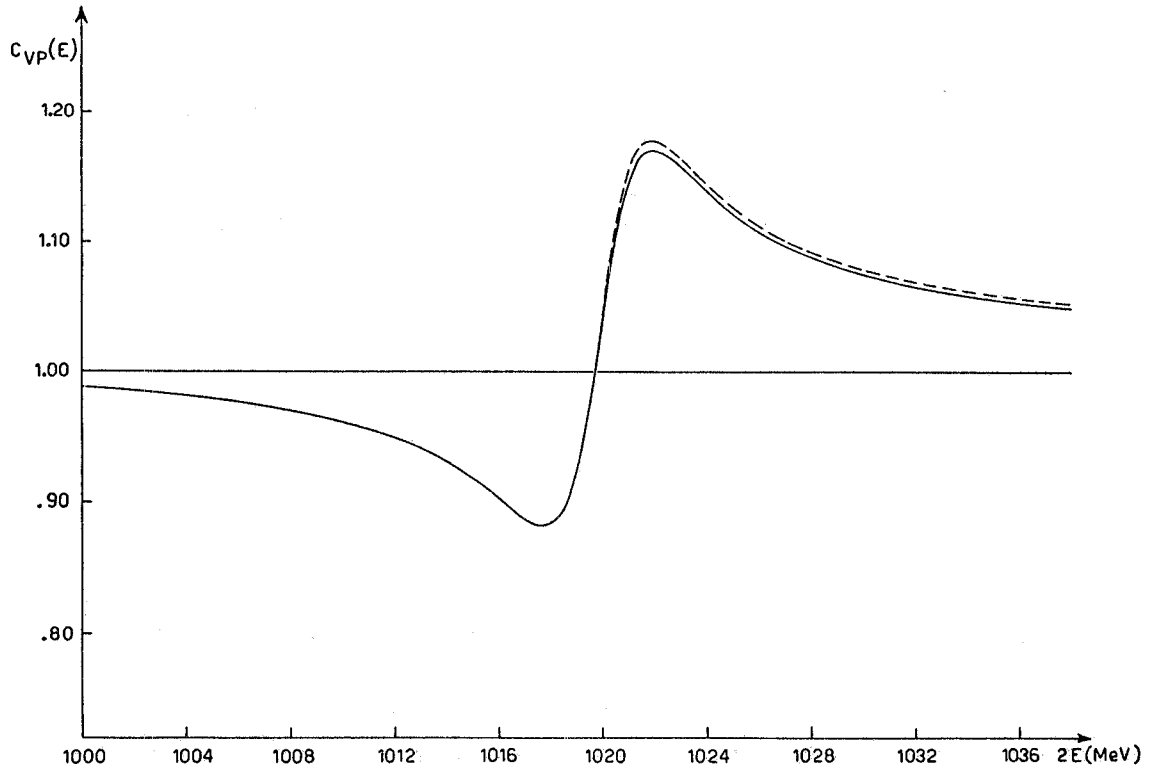


FIG. 2b - Plot of VP correction factor $C_{VP}(E)$ against $2E$ showing structure around ϕ mass.

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where N is a normalization constant determined by the requirement

$$\text{Im } f_R (q^2 = -4E^2) = \sigma_R (E); \quad (8)$$

for the total cross-section $\sigma_R(E)$ for $e^+ - e^-$ annihilation through R we take a Breit-Wigner and obtain

$$N = 12 \pi \frac{\Gamma (R \rightarrow e^+ e^-)}{m_R} \quad (9)$$

$\Gamma (R \rightarrow e^+ e^-)$ is the partial width for the decay $R \rightarrow e^+ e^-$. Between $S(q^2)$ and the total cross-section $\sigma(q^2)$ there exists the relationship⁽⁵⁾

$$\sigma(q^2) = \frac{16 \pi^3 \alpha^2}{q^4} S(q^2) \quad (10)$$

which when combined with the assumption that $f_R(q^2)$ satisfies an unsubtracted dispersion relation of the form

$$f_R(q^2) = \frac{1}{\pi} \int_0^\infty \frac{\text{Im } f_R(x) dx}{x + q^2 - i\epsilon} \quad (11)$$

yields

$$\int_0^\infty \frac{S(-x) dx}{x^2 (x + q^2 - i\epsilon)} = \frac{3}{4\pi\alpha^2} \cdot \frac{\Gamma(R \rightarrow e^+ e^-)}{m_R^2 + q^2 - i m_R \Gamma_R} \quad (12)$$

Eq. (12) leads to a propagator correction equivalent to eq. (6) if occurring constants are identified; a comparison of equations (6) and (12) gives

$$\frac{Z_0}{g_R^2} = \frac{3}{4\pi\alpha^2} \frac{\Gamma(R \rightarrow e^+ e^-)}{m_R}$$

or equivalently

$$\left(\frac{g_R/2}{4\pi} \right)^2 = Z_0 \cdot \frac{\alpha^2}{12} \cdot \frac{m_R}{\Gamma(R \rightarrow e^+ e^-)} \quad (13)$$

which is exactly the well known relationship between the photon-vector meson coupling constant $\gamma_R = g_R/2$ and the partial decay width $\Gamma(R \rightarrow e^+ e^-)$. Eq. (13) reveals the necessity of putting $Z_0 = 1$. The dashed curve in fig. (2) has been obtained by substituting for $S(q^2)$ a sum of three Breit-Wigner shapes compatible with eqs (7) and (10) and the resulting integrals evaluated exactly by

the method of residues. The branching ratios used, $B(\rho \rightarrow e^+e^-) = 6.04 \times 10^{-5}$, and $B(\phi \rightarrow e^+e^-) = 3.55 \times 10^{-4}$ are from Ting while $B(\omega \rightarrow e^+e^-)$ is from ref. (6)

If the contributions of the three resonances ρ^0 , ω , ϕ to the photon propagator are considered singly the resultant form of $C_{VP}(E)$ for each has a minimum before the resonant mass and a maximum thereafter⁽⁷⁾. This is clearly seen around the ϕ mass. This minimum-maximum structure is absent around the ρ^0 mass as a result of the ρ^0 - ω interference. This fact emphasises the necessity of taking interference effects into account. Recently Donnachie proposed calculating coherent ρ^0 - ω interference in the reaction $e^+ + e^- \rightarrow \pi^+ + \pi^-$ as a means of extracting information on the decay $\omega \rightarrow \pi^+ + \pi^-$. The discussion outlined above suggests that such information may be difficult to obtain with storage rings because the intervention of ω in the process $e^+ + e^- \rightarrow \pi^+ + \pi^-$ re-enters as part of the VP corrections to the lowest order process $e^+ + e^- \rightarrow \pi^+ + \pi^-$. ρ^0 - ω interference in these storage ring reactions must therefore be understood as interference of VP contributions and it is incorrect to calculate the interference otherwise. In fact when VP corrections as given by eq. (6) are administered on the Novosibirsk⁽⁷⁾ and Orsay⁽⁴⁾ data (full curve) we obtain the dashed curve in fig. (3) which agrees with Roos's plot⁽⁸⁾ but is decidedly in disagreement with Donnachie's (dash-dot curve) plot calculated with

$$\frac{\Gamma(\omega \rightarrow \pi^+ + \pi^-)}{\Gamma(\omega \rightarrow \pi^+ + \pi^- + \pi^0)} = 0.03 \quad .$$

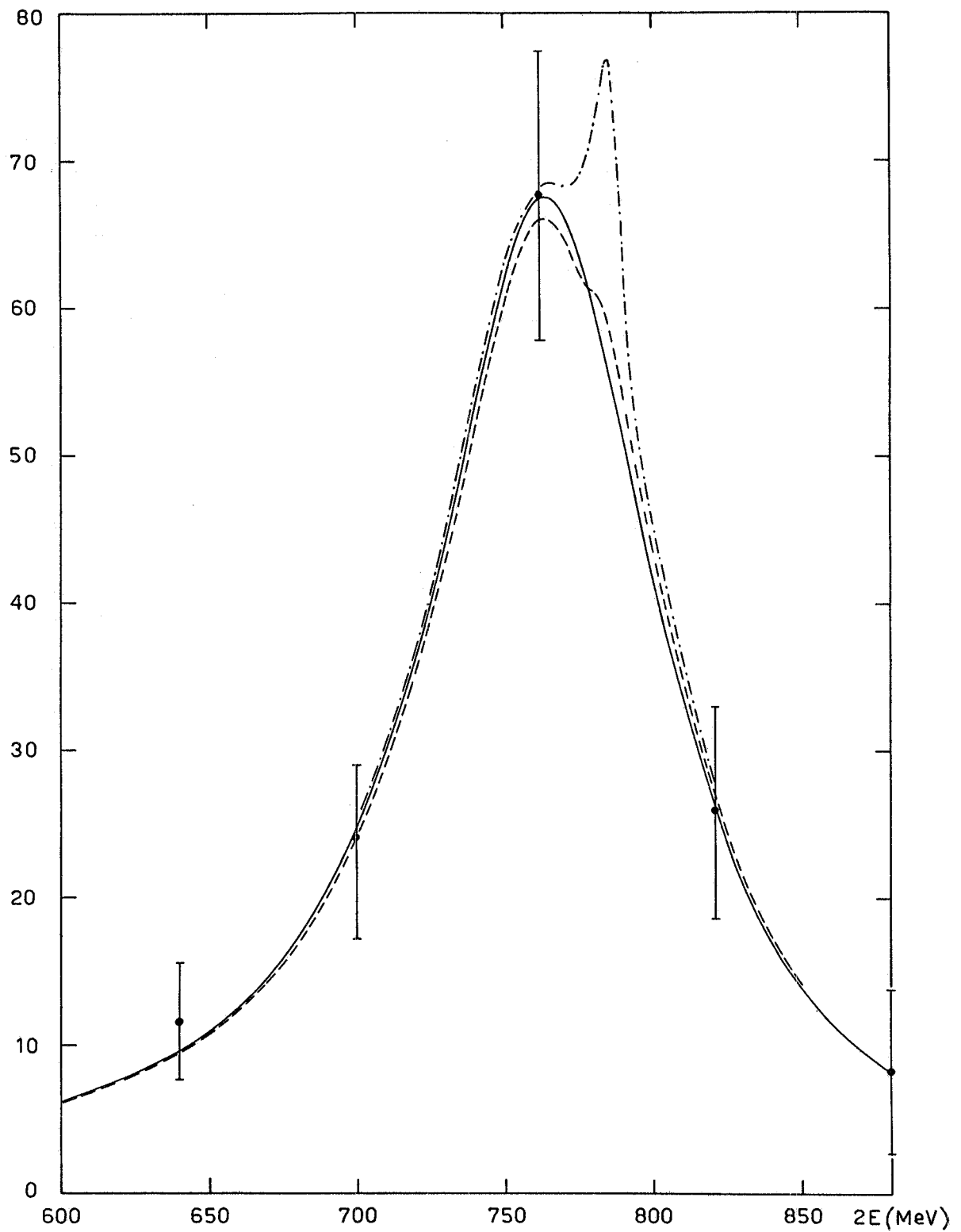


FIG. 3 - $e^+e^- \rightarrow \pi^+\pi^-$ resonant cross-section against CM energy: full curve: without VP correction; dashed curve: with VP corrections as calculated in the text; dash-dotted curve: result of Donnachie.

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